MICHAELHOUSE

# Mathematics Department

## Paper 1

## A BLOCK EXAMINATION

## AUGUST 2016

Examiner: Mr P. J. Stevens  Moderator: Mr A. Adlington-Corfield

Time: 3 hours  Marks: 150

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# PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages, an answer booklet and a separate Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions in the Answer Booklet provided.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.
6. It is in your own interest to write legibly and to present your work neatly.
7. Round all answers to **ONE decimal places** unless told to do otherwise.

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Do not write here:

| Q 1 | Q 2 | Q 3 | Q 4 | Q 5 | Q 6 | Q 7 | Q 8 | Q 9 | Q 10 | Q 11 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| **25** | **15** | **5** | **8** | **11** | **15** | **24** | **14** | **15** | **10** | **8** | **150** |

**QUESTION 1**

(a) Solve for $x$:

(1) $(x+5)(x-1)=7$

$x^2+4x-5=7$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6 \quad or \quad x=2$ (4)

(2) $3-x<2x^2$

$0<2x^2+x-3$

$0<(2x+3)(x-1)$

$x<-\frac{3}{2}\ or\ x>1$

(4)

(3) $\log(3x+1)=\log_2 8$

$\log(3x+1)=3$

$10^3=3x+1$

$1000=3x+1$

$999=3x$

$333=x$ (4)

(4) $\frac{3^{2x}}{3}=27^{x+1}$

$3^{2x-1}=3^{3x+3}$

$2x-1=3x+3$

$-4=x$ (3)

(b) (1) $2x^2 + 8x + p = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{64-8p}}{4}$

$x = \frac{-4 \pm \sqrt{16-2p}}{2}$ (3)

(2) For equal roots $\Delta = 0$

$64 - 8p = 0$

$64 = 8p$

$8 = p$ (2)

(c) $m(x-3) = 2x + m$

$mx - 3m = 2x + m$

$mx - 2x = 3m + m$

$x(m-2) = 4m$

$x = \frac{4m}{(m-2)}$ , $m \neq 2$ (5)

**[25]**

**QUESTION 2**

(a) Given: $f(x) = 3.2^x - 6$

(1) (1;0) and (0; -3) (2)

(2) $x = 3.2^y - 6$

$x + 6 = 3.2^y$

$\frac{x+6}{3} = 2^y$

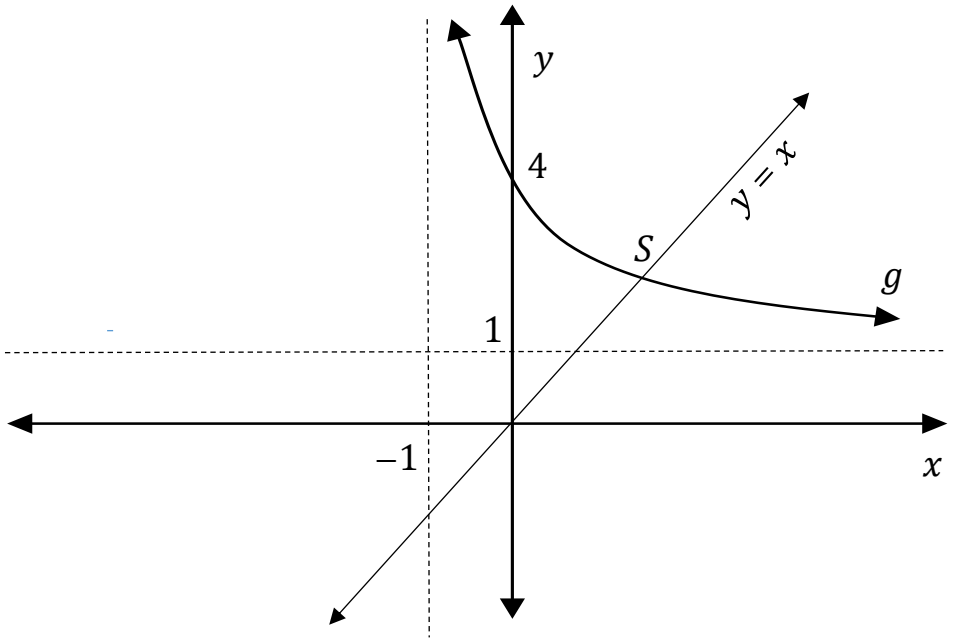
$y = log_2 \frac{x+6}{3}$ (3)

(3) Sketch the graph of $f^{-1}(x)$. (3)

(b) (1) $p = 1$ and $q = 1$ . (2)

(2) $S(2:2)$ (2)

(3) Sketch the graph of $g^{-1}(x)$ (3)




**[15]**

## QUESTION 3

In 2011 Sergey invested R 20 000 at 8,5% compounded monthly for his trip to the 2016 Rio Olympics. How many months will it take for him to reach his target of R30 000? Show all working.

$$30\,000 = 20\,000(1 + \frac{8{,}5\%}{12})^{12n}$$

$$\frac{30\,000}{20\,000} = (1 + \frac{8{,}5\%}{12})^{12n}$$

$$12n = log_{(1+\frac{8{,}5\%}{12})} \frac{3}{2}$$

$$= \mathbf{57{,}44\ months} \qquad \textbf{[5]}$$

## QUESTION 4

(a) Given: $f(x) = 2 - 3x^2$ .

$$f(x+h) = 2 - 3(x+h)^2$$
$$f(x+h) = 2 - 3x^2 - 6xh - 3h^2$$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{2-3x^2-6xh-3h^2-(2-3x^2)}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{-6xh-3h^2}{h}$$
$$f'(x) = \lim_{h\to 0} \frac{h(-6x-3h)}{h}$$
$$f'(x) = \lim_{h\to 0} -6x - 3h$$
$$f'(x) = -6x \qquad (4)$$

(b) $\frac{\sqrt{x}}{2} - \frac{1}{6x^3}$

$$= \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{6}x^{-3}$$

$$\frac{d}{dx} = \frac{1}{4}x^{-\frac{1}{2}} + \frac{1}{2}x^{-4}$$

$$\frac{d}{dx} = \frac{1}{4\sqrt{x}} + \frac{1}{2x^4} \qquad (4)$$

**[8]**

## QUESTION 5

(a) $a + 10d = 78$

$a + 4d = 42$

$6d = 36$

$d = 6$

$a = 18$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}[36 + 114]$

$S_{20} = 1500$ (4)

(b)

| Day | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| No of tourists | 10 000 | 22 000 | 44 000 | 76 000 |

(1) $a = 5\,000 \qquad c = 8\,000$

$T_n = an^2 + bn + c$

$10000 = 5000(1)^2 + b(1) + 8000$

$-3000 = b$

$T_n = 5000n^2 - 3000n + 8000$ (4)

(2) $480000 = 5000n^2 - 3000n + 8000$

$480000 = 5000n^2 - 3000n + 8000$

$0 = 5000n^2 - 3000n - 472000$

***By formula or by calculation n = 10 days*** (3)

**[11]**

**QUESTION 6**

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$

(a) $0 = x^2 - 7x - 8$

$0 = (x - 8)(x + 1)$ A(-1;0) B(8;0) (2)

(b) $-3x + 24 = -x^2 + 7x + 8$

$0 = -x^2 + 10x - 16$

$x^2 - 10x + 16 = 0$

$(x - 8)(x - 2) = 0$ $a = 2$ (3)

(c) ST= $-x^2 + 7x + 8 - (-3x + 24)$
ST= $-x^2 + 7x + 8 + 3x - 24$
ST= $-x^2 + 10x - 16$ (2)

(d) ST= $-x^2 + 10x - 16$

$\frac{dD}{dx} = -2x + 10$

$0 = -2x + 10$

$x = 5$

$D = 9\ units$ (3)

(e) Given $f(x) = -x^2 + 7x + k = 0$ ,

$k < 0$ for two positive roots. (2)

(f) $f'(x).g(x) \geq 0$

$(-2x + 7)(-3x + 24) \geq 0$

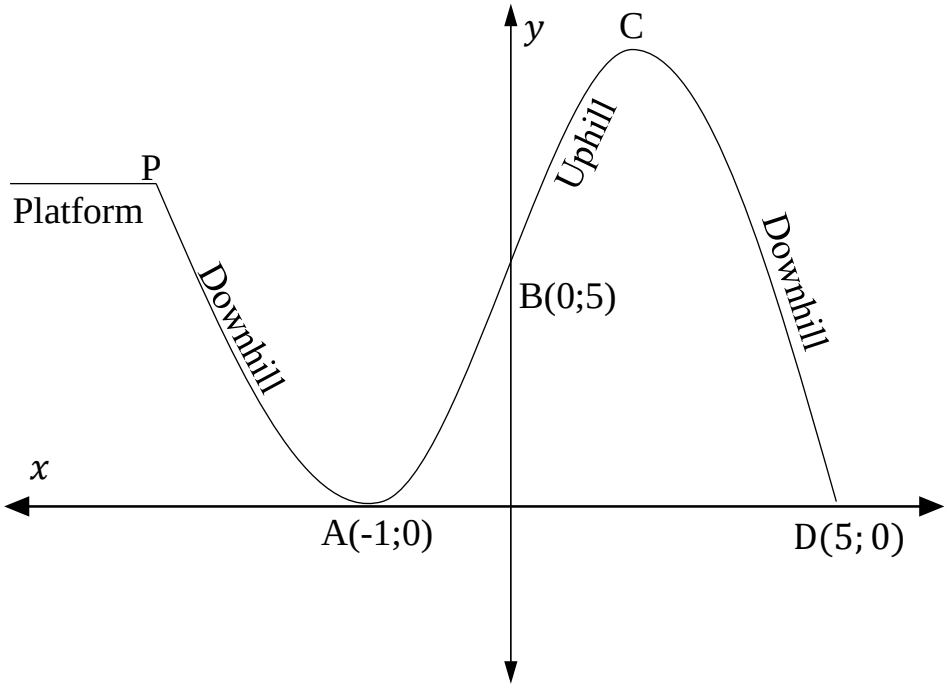
$(2x - 7)(x - 8) \geq 0$

***CVs***: $x = \frac{7}{2}$ *and* $x = 8$

$x \leq \frac{7}{2}$ *and* $x \geq 8$ (3)

**[15]**

**QUESTION 7**




(a) $(x+1)(x+1)(x-5)$

$= (x^2 + 2x + 1)(x - 5)$

$= x^3 - 3x^2 - 9x - 5$

$f(x) = -x^3 + 3x^2 + 9x + 5$ (5)

(b) $f(x) = -x^3 + 3x^2 + 9x + 5$

$f'(x) = -3x^2 + 6x + 9$

$0 = -3(x^2 - 2x - 3)$

$0 = -3(x - 3)(x + 1)$

$f(3) = -(-3)^3 + 3(-3)^2 + 9(-3) + 5$

$f(3) = 32$ **distance is 27 metres** (4)

(c) $7 = -x^3 + 3x^2 + 9x + 5$

$0 = -x^3 + 3x^2 + 9x - 2$

$0 = x^3 - 3x^2 - 9x + 2$

$0 = x^3 - 3x^2 - 9x + 2$

$0 = (x + 2)(x^2 - 5x + 1)$ **distance is 2 meters** (3)

(d) $m = \frac{5}{1}$ (3)

(e) $f(x) = -x^3 + 3x^2 + 9x + 5$

$f'(x) = -3x^2 + 6x + 9$

$f'(0) = -3(0)^2 + 6(0) + 9$

$m = 9$ (3)

(f) $f(x) = -x^3 + 3x^2 + 9x + 5$

$f'(x) = -3x^2 + 6x + 9$

$f''(x) = -6x + 6$

$0 = -6x + 6$

$x = 1$

$f(1) = -(1)^3 + 3(1)^2 + 9(1) + 5$

**Steepest point is just before (1;16)** (3)

(g) $f(x) = -x^3 + 3x^2 + 9x + 5$

$f'(x) = -3x^2 + 6x + 9$

$-15 = -3x^2 + 6x + 9$

$0 = -3x^2 + 6x + 24$

$0 = -3(x^2 - 2x - 8)$

$0 = -3(x - 4)(x + 2)$

$x = 4 \ or \ x = -2$ **distance is 0 metres or 6 metres** (3)

**[24]**

## QUESTION 8

Vladimir decided to support his fellow Russian athletes at the Rio Olympics. He took out a personal loan of ₽55 000 ( Russian Rubles) from RT Bank at an interest rate of 12%, compounded monthly, which is to be paid in equal instalments at the end of each month over a 10 year period.

(a) $$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$55000 = x\left[\frac{1-(1+\frac{12\%}{12})^{-120}}{\frac{12\%}{12}}\right]$$

$$\frac{55000}{\left[\frac{1-(1+\frac{12\%}{12})^{-120}}{\frac{12\%}{12}}\right]} = x$$

$$R789{,}09 = x \qquad (5)$$

(b) $$P_v = 789{,}09\left[\frac{1-(1+\frac{12\%}{12})^{-60}}{\frac{12\%}{12}}\right]$$

$$P_v = R35\ 473{,}57 \qquad (5)$$

(c) $$A = 55d000(1+\frac{12\%}{12})^{60}$$

$$A = 99918{,}32$$

$$F_v = 789{,}09\left[\frac{(1+\frac{12\%}{12})^{60}-1}{\frac{12\%}{12}}\right]$$

$$F_v = 64444{,}72$$

$$Interest = 99918{,}32 - 64444{,}72$$

$= 35473,60$

$55000 - 35473,57 = 19526,43$

$789,09 \times 60 = 47345,40$

(c) $\therefore Interest\ paid = 43745,40 - 19526,43$ (4)

$= 27818,97$

$\% = \frac{27818,97}{47345,40} \times 100 = 58,8\%$

**[14]**

**QUESTION 9**

(a) (1) $r = \frac{x-3}{2}$ (2)

(2) $-1 < \frac{x-3}{2} < 1$

$-2 < x - 3 < 2$

$\mathbf{1 < x < 5}$ (3)

(3) $S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{4(x-3)}{1-\frac{x-3}{2}}$

$S_\infty = \frac{4(x-3)}{\frac{2-x+3}{2}}$

$S_\infty = \frac{8x-24}{5-x}$ (3)

(4) $24 = \frac{8x-24}{5-x}$

$24(5-x) = 8x - 24$

$120 - 24x = 8x - 24$

$144 = 32x$

$\frac{9}{2} = x$ (2)

(b) $T_1 = \frac{1}{8}; T_2 = \frac{1}{4}; T_3 = \frac{1}{2}; T_4 = 1 \qquad a = \frac{1}{8}; r = 2$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$50 = \frac{\frac{1}{8}(2^k - 1)}{2 - 1}$

$400 = 2^k - 1$

$401 = 2^k$

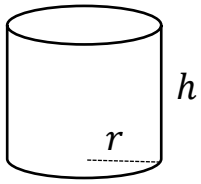
$k = \log_2 401$

$k = 8{,}65 \qquad \boldsymbol{k = 9}$ (5)

**[15]**

**QUESTION 10**

The suppliers of the 250ml Rio 2016 souvenir can would like to reduce the cost of the can by minimising the amount of aluminium used to produce it. The cost of aluminium is 2 centavos per $cm^2$ ( 100 centavos =1 Real ):




(a) $V = \pi r^2 h$

$250 = \pi r^2 h$

$\frac{250}{\pi r^2} = h$ (2)

(b) $C = 4\pi r^2 + \frac{1000}{r}$

$TSA = 2\pi r^2 + 2\pi r h$

$TSA = 2\pi r^2 + \frac{500}{r}$

$C = 4\pi r^2 + \frac{1000}{r}$ (4)

(c) $C = 4\pi r^2 + 1000r^{-1}$

$\frac{dC}{dr} = 8\pi r - \frac{1000}{r^2}$

$0 = 8\pi r - \frac{1000}{r^2}$

$\frac{1000}{r^2} = 8\pi r$

$\frac{1000}{8\pi} = r^3$

$3{,}413 = r$

$h = 6{,}8\ cms$ (4)

**[10]**

**QUESTION 11**

The Olympic Aquatics Stadium in Rio has a capacity of 15 000 spectators with ticket prices at R12 each. The organizers of the Olympics, which has been affected by the Zika virus, are keen to fill the stadium. A market survey indicates that for each Real (Brazilian currency) by which the ticket price is reduced, the expected attendance of 11 000 will increase by 1 000 spectators.

(a) $(11000 + 1000x) = People \quad (12 - x) = ticket\ price$

$T = (11000 + 1000x)(12 - x)$

$T = 132000 + 1000x - 1000x^2$ (3)

(b) $T = 132000 + 1000x - 1000x^2$

$\frac{dT}{dx} = 1000 - 2000x$

$0 = 1000 - 2000x$

$2000x = 1000$

$x = \frac{1}{2}$ (3)

(c) $T = 132000 + 1000x - 1000x^2$

$T = 132000 + 1000(\frac{1}{2}) - 1000(\frac{1}{2})^2$

$T = 132250$ (2)

**[8]**

**71 marks**

**Total: 150 marks**