MICHAELHOUSE

Mathematics Department

Paper 1

A BLOCK EXAMINATION

AUGUST 2016

Examiner: Mr P. J. Stevens Moderator: Mr A. Adlington-Corfield

Time: 3 hours Marks: 150

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages, an answer booklet and a separate Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions in the Answer Booklet provided.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.
6. It is in your own interest to write legibly and to present your work neatly.
7. Round all answers to **ONE decimal places** unless told to do otherwise.

Do not write here:

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Q 8	Q 9	Q 10	Q 11	Total
25	15	5	8	11	15	24	14	15	10	8	150

SECTION A

QUESTION 1

(a) Solve for x:

(1) $(x+5)(x-1)=7$ (4)

(2) $3-x<2x^2$ (4)

(3) $\log(3x+1)=\log_2 8$ (4)

(4) $\frac{3^{2x}}{3}=27^{x+1}$ (3)

(b) Given: $2x^2+8x+p=0$

(1) Solve for x in terms of p . (3)

(2) For which value(s) of p will the equation have equal roots? (2)

(c) Solve for x , stating any restrictions:

$m(x-3)=2x+m$ (5)

[25]

QUESTION 2

(a) Given: $f(x) = 3.2^{x} - 6$

(1) Determine the intercepts with the axes, correct to 2 decimal places (if necessary). (2)

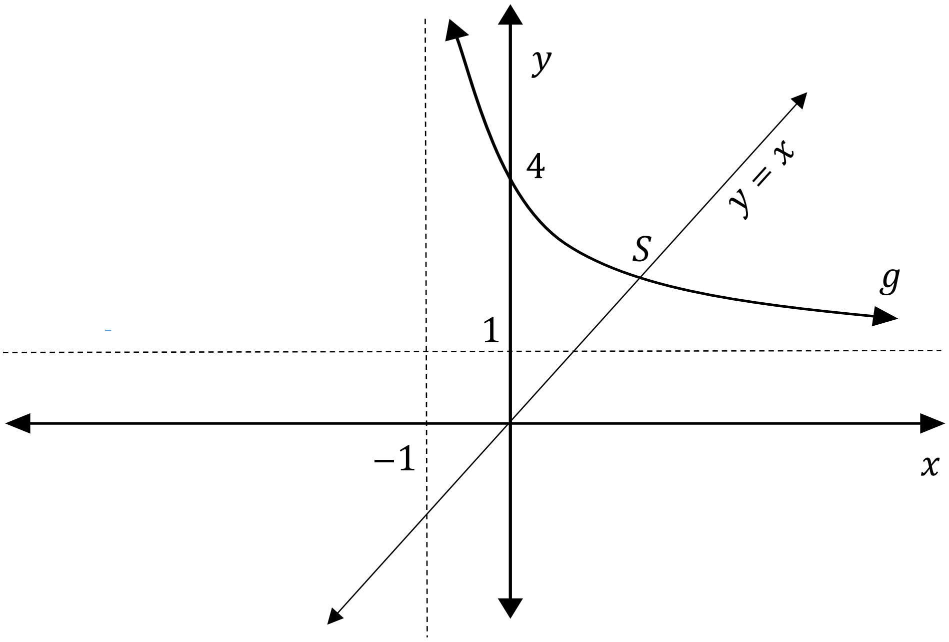
(2) Give the inverse function $f^{-1}(x)$. (3)

(3) Sketch the graph of $f^{-1}(x)$. Label clearly all asymptotes and intercepts with the axes. (3)

(b) The diagram below shows the hyperbola g defined by $g(x) = \frac{3}{x+p} + q$, with asymptotes $y = 1$ and $x = -1$ and where $x > -1$.

The graph of g intersects the y-axis at $(0; 4)$.

The line $y = x$ intersects the hyperbola at S.



(1) Write down the values of p and q. (2)

(2) Calculate the co-ordinates of S. (2)

(3) Sketch the graph of $g^{-1}(x)$, where $x > 0$, showing necessary asymptotes and intercepts with the axes. (3)

[15]

QUESTION 3

In 2011 Sergey invested R 20 000 at 8,5% compounded monthly for his trip to the 2016 Rio Olympics. How many months did it take for him to reach his target of R30 000? Show all working.

[5]

QUESTION 4

(a) Given: $f(x) = 2 - 3x^2$.

Determine $f'(x)$ from first principles. (4)

(b) Differentiate with respect to x : $\frac{\sqrt{x}}{2} - \frac{1}{6x^3}$

Leave your answer in positive exponents. (4)

[8]

QUESTION 5

(a) The fifth term of an arithmetic sequence is 42 and the eleventh term is 78.

Calculate the sum to 20 terms of this sequence. (4)

(b) The following table represents the number of tourists in Brazil on each day of the Rio Olympics.

Day	1	2	3	4
No of tourists	10 000	22 000	44 000	76 000

(1) Without the use of a calculator and showing all working, find a rule which satisfies the above information. (4)

(2) How long did it take for the number of tourists in Brazil to reach the expected 480 000 mark (to the closest day) ? (3)

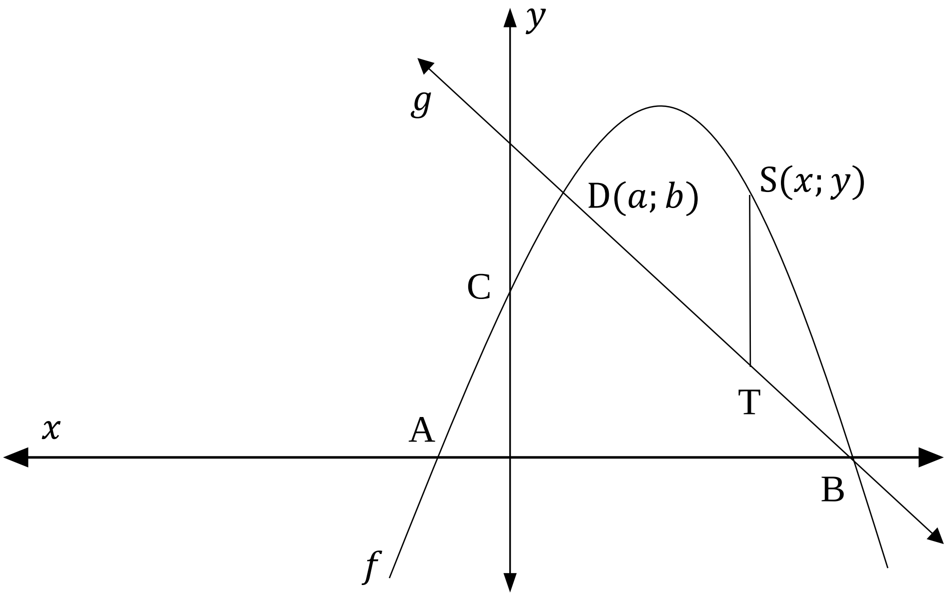
[11]

QUESTION 6

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$

are sketched below. f and g intersect in D and B.

A and B are the x – intercepts of f .



(a) Determine the co-ordinates of A and B. (2)

(b) Find the value of a. (3)

(c) S $(x; y)$ is a point on f where $a \leq x \leq 8$.

ST is drawn parallel to the y- axis with T on g.

Determine ST in terms of x. (2)

(d) Calculate the maximum length of ST. (3)

(e) Given $f(x) = -x^2 + 7x + k = 0$, find the value of k for which $f(x)$ has two positive roots. (2)

(f) Give the values for x when $f'(x).g(x) \geq 0$ (3)

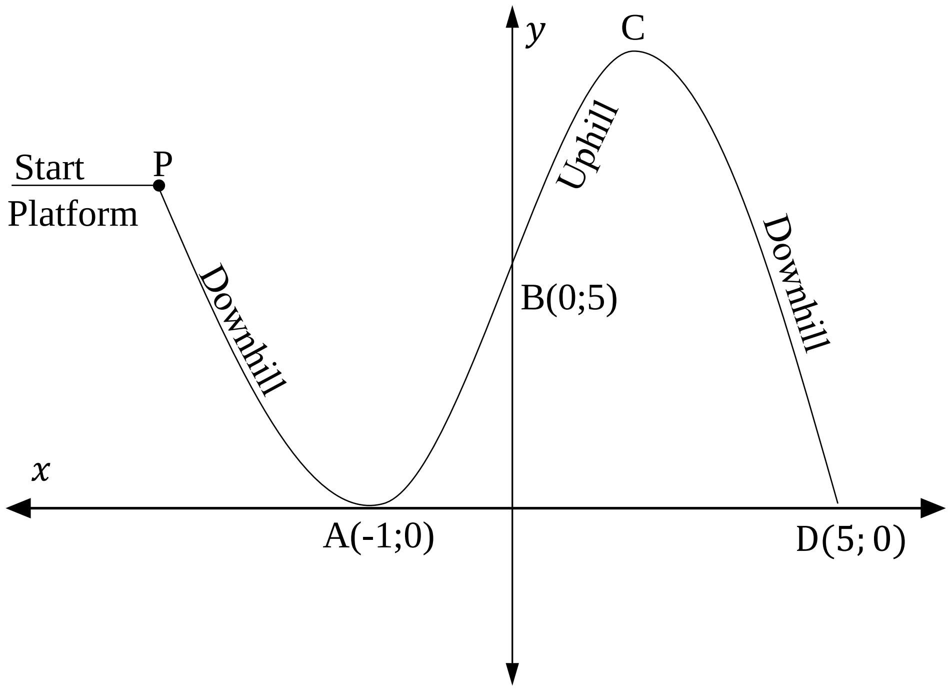
[15]

79 marks

SECTION B

QUESTION 7

A cross-section of the Deodora BMX track at the Rio Olympics is modelled by the function $f(x) = ax^3 + bx^2 + cx + d$.



(a) Show that $a = -1,\ b = 3,\ c = 9$ and $d = 5$ and hence give the function representing the route which the cyclists followed. (5)

(b) Give the vertical distance of the climb from B to C. (4)

(c) If the starting platform at P is 2 metres higher than point B, find the horizontal distance travelled from P to B. (3)

(d) Give the average gradient which the cyclists climbed between A and B. (3)

(e) Give the actual gradient at B. (3)

(f) Find the steepest uphill point on the track. (3)

(g) Give the horizontal distance(s) travelled from P to the point(s) where the downhill gradient is 15. (3)

[24]

QUESTION 8

Vladimir decided to support his fellow Russian athletes at the Rio Olympics. He took out a personal loan of ₽ 55 000 (Russian Rubles) from RT Bank at an interest rate of 12%, compounded monthly, which is to be paid in equal instalments at the end of each month over a 10 year period.

(a) Find the monthly instalment. (5)

(b) Calculate the outstanding balance of the loan after 5 years, showing all working. (5)

(c) Making reference to your answer in (b) above, give the percentage of the total monthly instalments over the first 5 years which went towards paying RT Bank's interest charges. (4)

[14]

QUESTION 9

(a) The first three terms of a convergent Geometric sequence are defined by:

$$4(x-3)\ ;\ 2(x-3)^2\ ;\ (x-3)^3$$

(1) Determine the common ratio. (2)

(2) Calculate the values of x for which the sequence converges. (3)

(3) Show by calculation that $S_\infty = \frac{8x-24}{5-x}$ (3)

(4) Find the value of x if the sum to infinity is 24. (2)

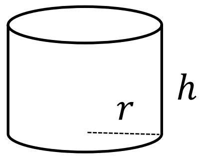
(b) Give the lowest value of k for which

$$\sum_{x=1}^{k} \frac{1}{8}(2)^{x-1} > 50$$ (5)

[15]

QUESTION 10

The suppliers of the 250ml Rio 2016 “souvenir can” reduced the cost of the can by minimising the amount of aluminium used to produce it. The cost of aluminium is 2 centavos per cm^2 (100 centavos =1 Real).



(a) Find an expression for h (the height) in terms of r (the radius). (2)

(b) Show that the cost is given as: $C = 4\pi r^2 + \frac{1000}{r}$. (4)

(c) Find the height which gives the minimum cost for the 250 ml can. (4)

[10]

QUESTION 11

The Olympic Aquatics Stadium in Rio has a capacity of 15 000 spectators with ticket prices at 12 Real each. The organizers of the Olympics, which was affected by the Zika virus, wanted to fill the stadium. A market survey indicated that for each Real (Brazilian currency) by which the ticket price was reduced, the expected attendance of 11 000 would increase by 1 000 spectators.

(a) If the ticket price was reduced by x Real**,** show that the income (T) from ticket sales can be expressed as:

$$T = 132\,000 + 1000x - 1000x^2 .$$ (3)

(b) Show how ticket prices were adjusted to maximise the revenue from ticket sales? (3)

(c) What was the maximum possible income from an Aquatics event? (2)

[8]

71 marks

Total: 150 marks