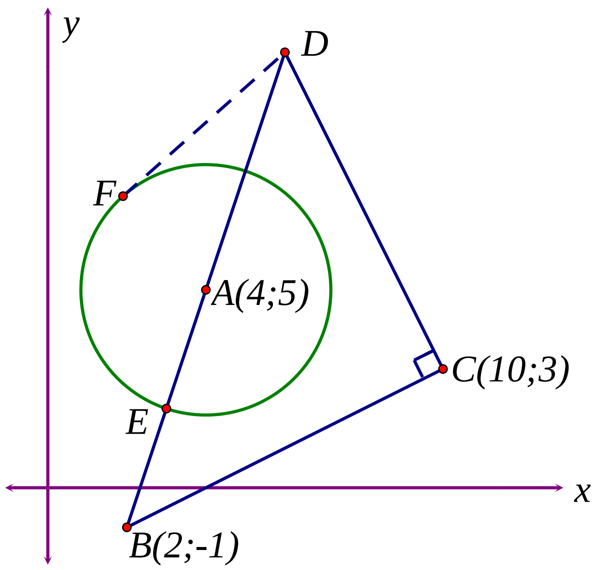
SECTION A

QUESTION 1

In the diagram below:

- $DC \perp BC$
- A is the centre of the circle.
- E is the midpoint of AB.
- The equation of line BA is: $y = 3x - 7$
- DF is a tangent to the circle at F.



(a) Find the co-ordinates of E. (2)

$$E\left(\frac{4+2}{2};\frac{5-1}{2}\right)$$

$$E(3;2)$$

(b) Determine the equation of the circle, centre A, passing through point E.

Give the equation on the form $(x-a)^2 + (y-b)^2 = r^2$ (3)

$$r^2 = (4-3)^2 + (5-2)^2$$

$$r^2 = 1 + 9$$

$$r^2 = 10$$

$$\therefore (x-4)^2 + (y-5)^2 = 10$$

(c) Find the gradient of line BC. (2)

$$m_{BC} = \frac{3+1}{10-2} = \frac{1}{2}$$

(d) Hence, or otherwise, determine the equation of line DC. (3)

$m_{DC} = -2$

$y = -2x + c$

$sub \quad (10;3)$

$3 = -2(10) + c$

$c = 23$

$\therefore y = -2x + 23$

(e) Show, by calculation, that the co-ordinates of D are $(6;11)$. (3)

$3x - 7 = -2x + 23$

$5x = 30$

$x = 6$

$\therefore y = 3(6) - 7 = 11$

$\text{D}(6;11)$

(f) Find the size of $E\hat{B}C$. (5)

$m_{EB} = 3$

$\tan\theta = 3$

$\theta = 71,6°$

$m_{BC} = \frac{1}{2}$

$\tan\alpha = \frac{1}{2}$

$\alpha = 26,6°$

$\therefore E\hat{B}C = 71,6° - 26,6° = 45°$

(g) Find the length of DF. (6)

$D\hat{F}A = 90°$ (tan *chord*)

$FA = \sqrt{10}$

$AD^2 = (6-4)^2 + (11-5)^2$

$AD^2 = 40$

$\therefore DF^2 = AD^2 - FA^2$ (*pythag*)

$DF^2 = 40 - 10$

$DF = \sqrt{30}$

[24]

QUESTION 2

(a) Simplify: $\dfrac{(\sin x - \cos x)^2 - 1}{\sin^2 x - 1}$ (4)

$$\frac{\sin^2 x - 2\sin x.\cos x + \cos^2 - 1}{-\cos^2 x}$$
$$= \frac{-2\sin x.\cos x}{-\cos^2 x}$$
$$= \frac{2\sin x}{\cos x}$$
$$= 2\tan x$$

(b) Solve for x in the interval $[-180°; 180°]$:

$\cos(x - 30°) = \sin 3x$. (7)

$$\cos(x - 30°) = \cos(90° - 3x)$$
$$\therefore x - 30° = \pm(90° - 3x) + k360°$$
$$x - 30° = 90° - 3x$$
$$4x = 120°$$
$$x = 30° + k90°$$

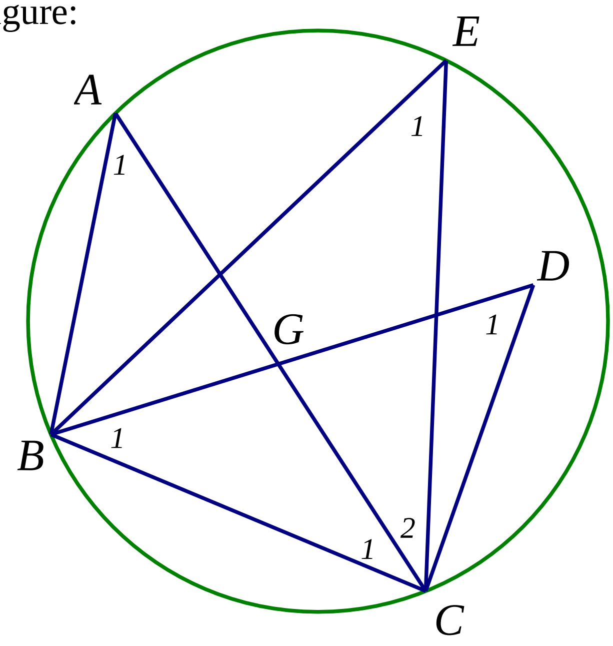
or

$$x - 30° = -(90° - 3x)$$
$$-2x = -60°$$
$$x = 30° + k180°$$
$$\therefore x = -150°; -60°; 30°; 120°$$

[11]

QUESTION 3

(a) Refer to the figure:



Below are three statements that refer to the above figure. What additional information would be required, if any, to make each individual statement true?

(1) $\hat{A}_1 = \hat{E}_1 = \hat{D}_1$

D must be on the circumference

(2) $\hat{B}_1 = \hat{C}_1$

GB = GC or G is the centre of the circle

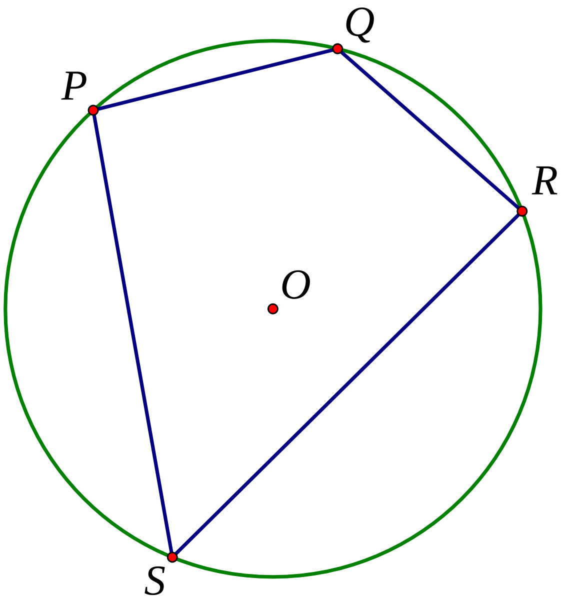
(3) $\hat{C}_1 + \hat{C}_2 = 90°$

BE is the diameter

(3)

(b) Use the figure below to prove that $\hat{S} + \hat{Q} = 180°$.

$\hat{O}$ is the centre of the circle. (6)



Given: Cyclic quad PQRS, O centre of the circle

RTP: $\hat{S} + \hat{Q} = 180°$

Const.: Join PO and OR

Proof:

$Let\ \hat{S} = x$

$P\hat{O}R = 2x \quad (2\angle at\ circum)$

$reflex P\hat{O}R = 360° - 2x \quad (\angle's\ around\ a\ point)$

$\hat{Q} = 180° - x \quad (half\ \angle\ at\ centre)$

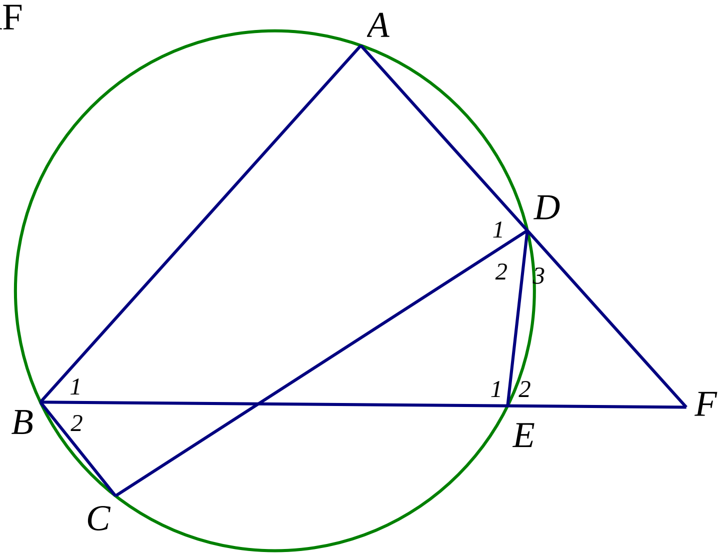
$\therefore \hat{S} + \hat{Q} = 180°$

[9]

QUESTION 4

In the figure:

- ABCD is a cyclic quadrilateral.
- AB = AF



(a) Prove that DE = EF. (3)

$Let\ \hat{F} = x$

$\therefore \hat{B}_1 = x \quad (AB = AF)$

$\hat{D}_3 = x \quad (ext \angle\ of\ cyclic\ quad)$

$\therefore ED = EF$

(b) If it is further given that ED bisects $C\hat{D}F$, prove that FB bisects $A\hat{B}C$. (3)

$\hat{D}_2 = x \quad (\angle\ bisected)$

$\therefore \hat{B}_2 = x \quad (\angle\ in\ same\ seg)$

$\therefore \hat{B}_1 = x = \hat{B}_2$

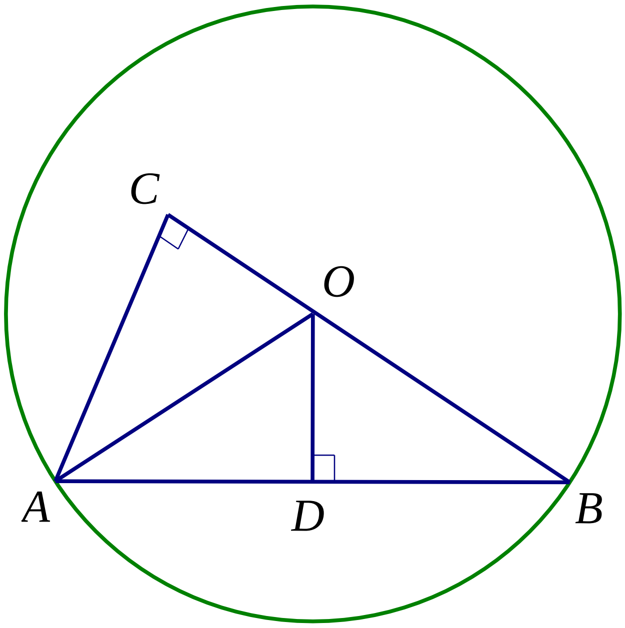
$\therefore FB\ bisects\ A\hat{B}C$

[6]

QUESTION 5

In the figure:

- O is the centre of the circle.
- $AC \perp CB$
- $OD \perp BA$



Prove that:

(a) $\Delta ODB /// \Delta ACB$ (3)

In ΔODB and ΔACB

1) $O\hat{B}D = 90° = \hat{C}$ $(given)$

2) $\hat{B} = \hat{B}$ $(common)$

3) $D\hat{O}B = C\hat{A}B$ $(3rd \angle \; of \; triangle)$

$\therefore \Delta ODB /// \Delta ACB$ (AAA)

(b) $2AD^2 = OA.BC$ (4)

$$\frac{OD}{AC} = \frac{DB}{CB} = \frac{OB}{AB} \quad (/// \Delta's)$$

$AD = DB$ $(OD \perp AB)$

$OB = OA$ $(radii)$

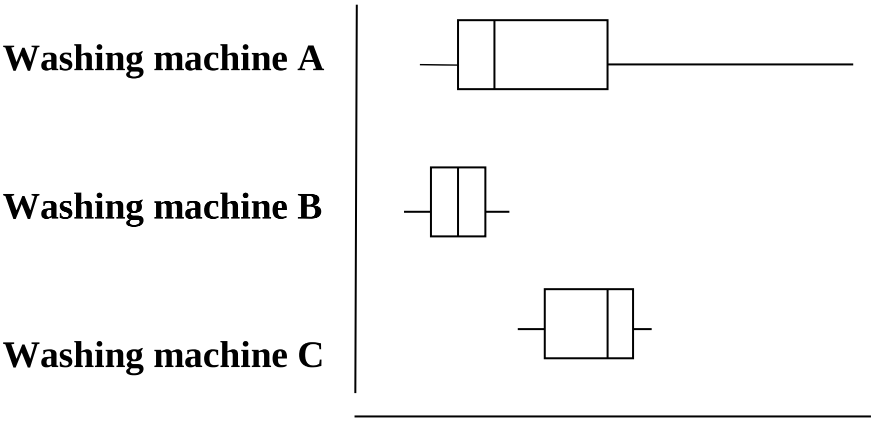
$$\therefore \frac{AD}{CB} = \frac{OA}{2AD}$$

$2AD^2 = OA.BC$

[7]

QUESTION 6

A consumer testing company studied three different brands of washing machines to see how much water was used during each wash. Each washing machine was tested 15 times. The box and whisker plots below show the results of this study.



Number of litres used by washing machine.

(a) Which brand of machine (A, B or C) used up most water on average? (1)

C

(b) Which brand of machine (A, B or C) is the most predictable? (1)

B

(c) The results of Washing Machine C are shown below:

81	85	85	88	89	90	92	101	104	105	106	106	108	112	112

(1) Determine the standard deviation of the litres of water used. (2)

10,4

(2) Based on this data, how many litres of water would be used in 67% of the washing loads? (2)

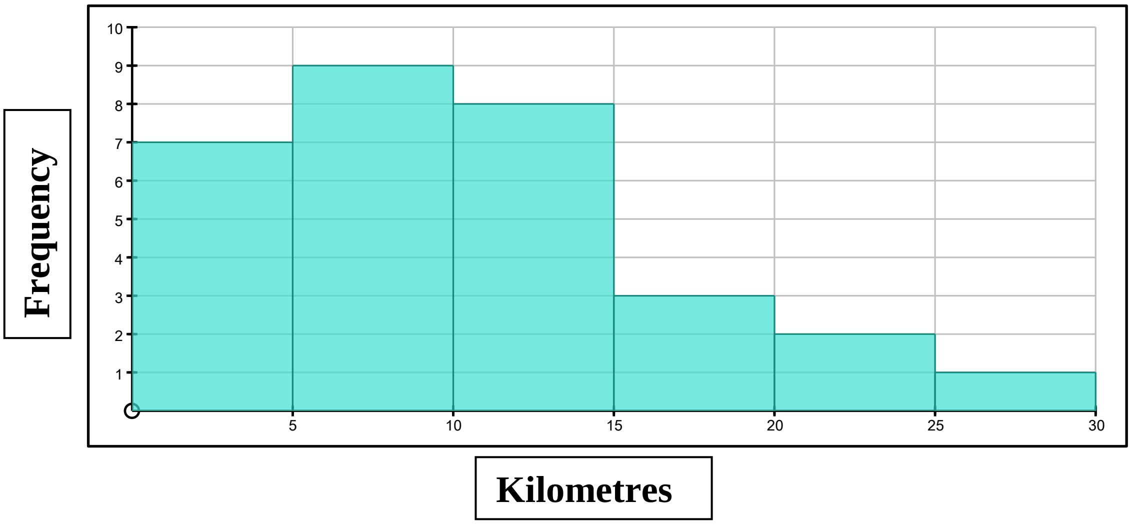
$[97,6-10,4 \quad ; \quad 97,6+10,4]$

$[87,2 \quad ; \quad 108]$

[6]

QUESTION 7

The distance (x) in kilometres that the staff at a certain school in Durban travel to work each day is summarised in the histogram below:



(a) Is the data positively or negatively skewed? (1)

Skewed to the right, positively skewed

(b) Which of the intervals is the modal interval? (1)

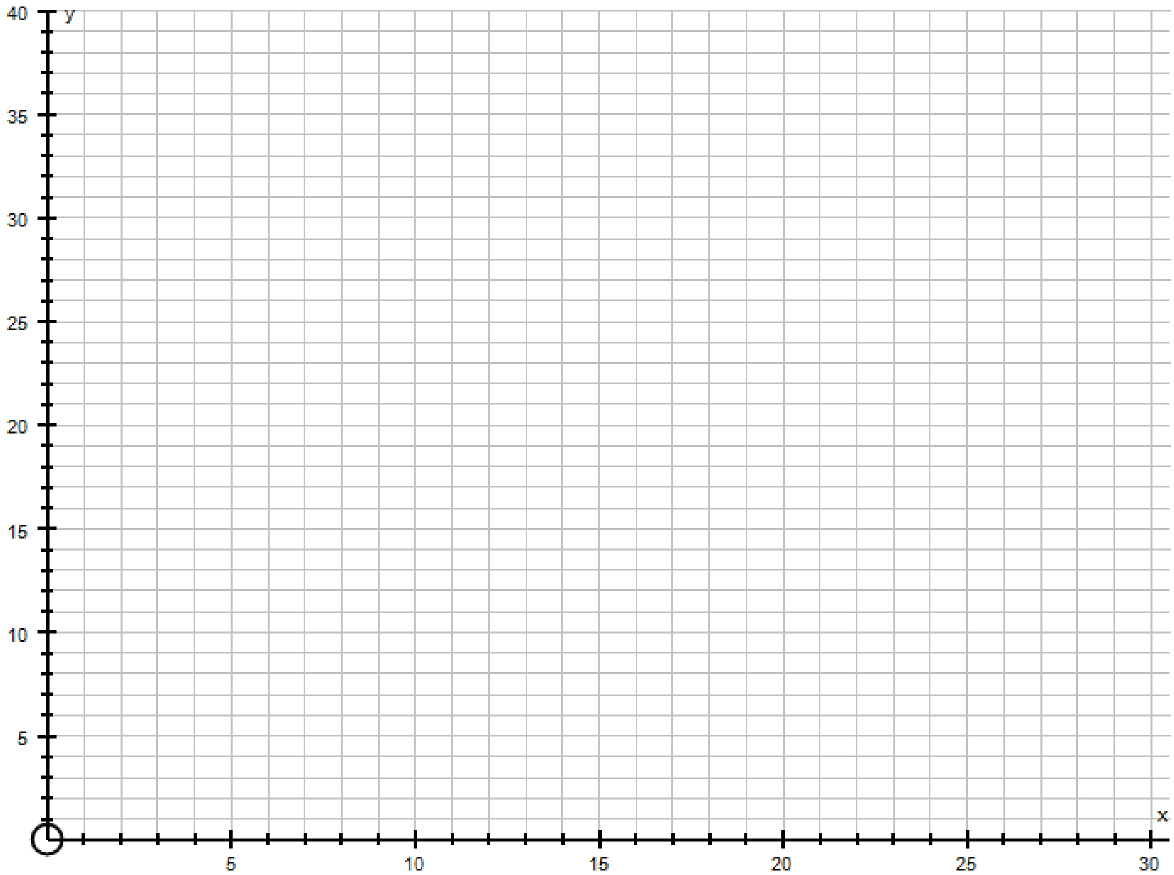
Mode 5 - 10

(c) Use the histogram to complete the given table. (3)

Intervals	Frequency	Cumulative Frequency
$0 \leq x < 5$	7	7
$5 \leq x < 10$	9	16
$10 \leq x < 15$	8	24
$15 \leq x < 20$	3	27

$20 \leq x < 25$	2	29
$25 \leq x < 30$	1	30

(d) Use your frequency table to draw an ogive below. Label your axes. (3)



(e) Determine the following using your ogive:

(1) the interquartile range (2)

14,1- 5,6 = 8,5

(2) the percentage of these staff members that stayed between 4km and 14km from the school? (2)

$$\frac{22-4}{30}\times 100 = 60\%$$

[12]

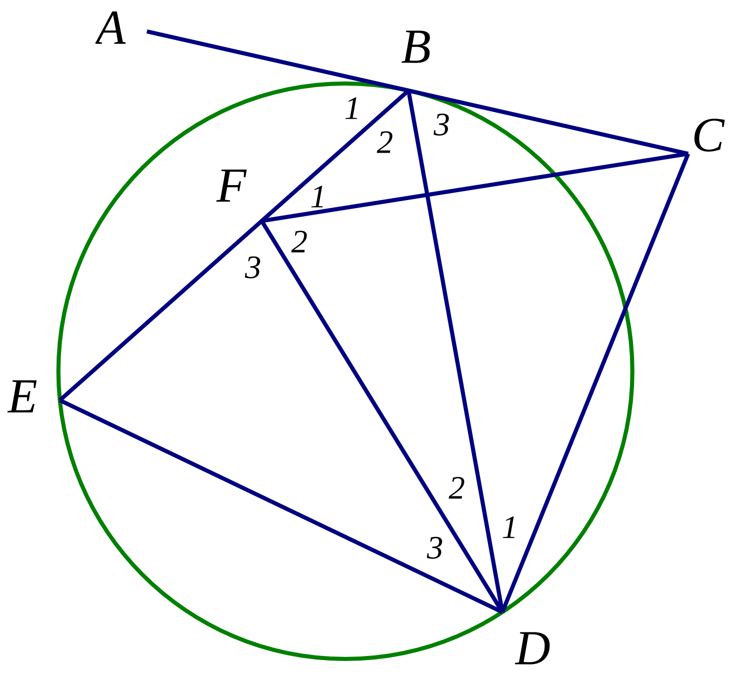
75 marks

SECTION B

QUESTION 8

(a) In the figure:

- ABC is a tangent to the circle at B.
- $\hat{D}_1 = \hat{D}_3$



Prove that:

(1) DCBF is a cyclic quadrilateral. (4)

$\hat{B}_1 = \hat{D}_{2+3}$ (tan AC $chord$ EB)

$= \hat{D}_{2+1}$ ($\hat{D}_3 = \hat{D}_1$)

$\therefore DCBF\ cyclic$ ($ext\ \angle = opp$ int $\angle$)

(2) FC is a tangent to the circle FED. (4)

$\hat{E} = \hat{B}_3$ (tan $AC\ chord \cdots BD$)

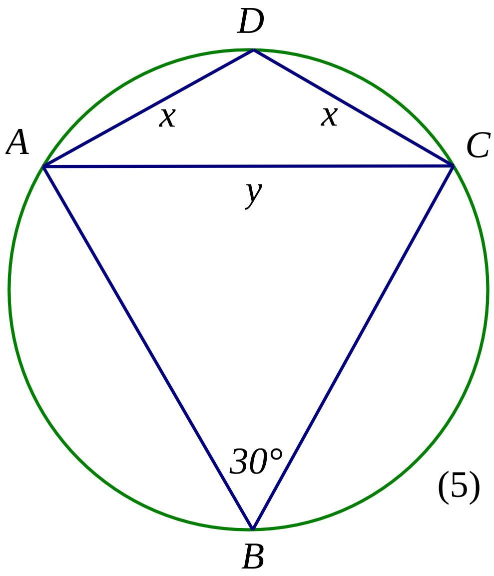
$\hat{B}_3 = \hat{F}_2$ ($\angle\ in\ same\ seg$)

$\therefore \hat{F}_2 = \hat{E}$

FC tan $to\ circle\ FED$ (tan $chord\ converse$)

(b) In the figure:

- ABCD is a cyclic quadrilateral.
- AD = DC = x
- AC = y
- $\hat{B} = 30°$



Show that $y = x\sqrt{2+\sqrt{3}}$ (5)

$\hat{D} = 150°$ $(opp \angle of\ cyclic\ quad)$

$\therefore y^2 = x^2 + x^2 - 2x.x\cos 150°$

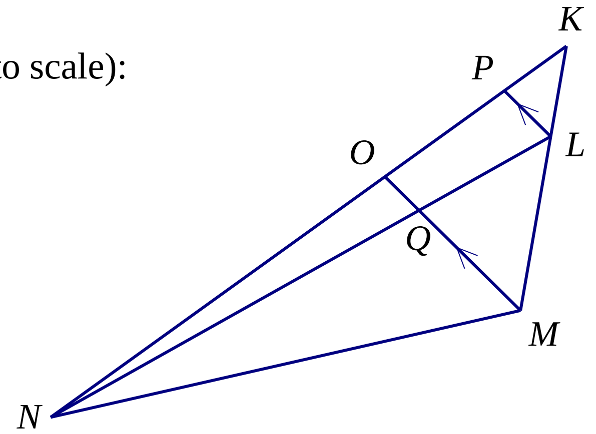
$y^2 = 2x^2 + \sqrt{3}x^2$

$y^2 = x^2(2+\sqrt{3})$

$\therefore y = x\sqrt{2+\sqrt{3}}$

(c) In the figure (not drawn to scale):

- OM // PL
- KN : KO = 8 : 3
- LM = 3KL



Calculate:

(1) $\dfrac{Area\ of\ \Delta KMO}{Area\ of\ \Delta NMO}$ (3)

$$=\frac{\frac{1}{2}KO.h_{\perp}}{\frac{1}{2}ON.h_{\perp}}=\frac{KO}{ON}=\frac{3}{5}$$

(2) $\dfrac{LQ}{QN}$ (3)

$$\frac{PO}{OK}=\frac{ML}{MK}\quad (PL//OM)$$

$$PO=\frac{3p.3k}{4k}=\frac{9}{4}p$$

$$\frac{LQ}{QN}=\frac{PO}{ON}\quad (PL//OQ)$$

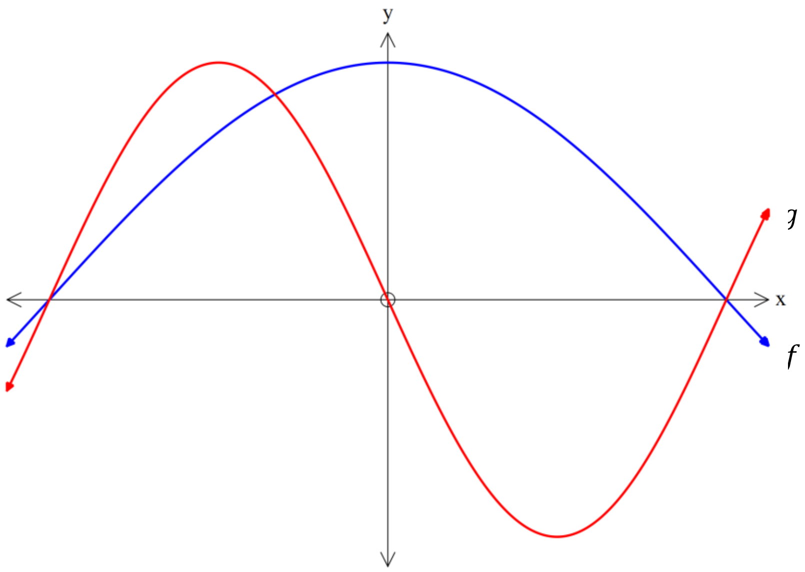
$$\frac{LQ}{QN}=\frac{\frac{9}{4}p}{5p}=\frac{9}{20}$$

[19]

QUESTION 9

The sketch below shows the graphs of $f(x) = -\sin x$ and $g(x) = \cos\frac{x}{2}$ for

$x \in [-180°; 180°]$. The coordinates of A, the point of intersection of the two graphs, are $\left(t; \frac{\sqrt{3}}{2}\right)$.



(a) Determine the value of t. (2)

$\frac{\sqrt{3}}{2} = -\sin t$

$t = -60°$

(b) Use the sketch to solve the following equation: $\cos\frac{x}{2} = -2\sin\frac{x}{2}\cos\frac{x}{2}$

for $x \in [-180°; 180°]$. (3)

$\cos\frac{x}{2} = -2\sin\frac{x}{2}.\cos\frac{x}{2} = -\sin x$

$\therefore x = -180°\,; -60°\,; 180°$

(c) If f is translated $10°$ to the left and 2 units up, what will the new equation of f be? (2)

$y = -\sin(x + 10°) + 2$

[7]

QUESTION 10

(a) Simplify: $\dfrac{\sin 170°.\cos 10°}{\sin(20° - x).\cos x + \cos(20° - x).\sin x}$ (4)

$= \dfrac{\sin 10°.\cos 10°}{\sin(20° - x + x)}$

$= \dfrac{\frac{1}{2}\sin 20°}{\sin 20°}$

$= \dfrac{1}{2}$

(b) Prove the following identity:

$\dfrac{\cos x - \cos 2x - 1}{\sin x - \sin 2x} = \dfrac{1}{\tan x}$ (5)

$LHS = \dfrac{\cos x - (2\cos^2 x - 1) - 1}{\sin x - 2\sin x \cos x}$

$= \dfrac{\cos x - 2\cos^2 x}{\sin x(1 - 2\cos x)}$

$= \dfrac{\cos x(1 - 2\cos x)}{\sin x(1 - 2\cos x)}$

$= \dfrac{\cos x}{\sin x}$

$= \dfrac{1}{\tan x}$

$= RHS$

(c) Determine the general solution for θ :

$$7\sin\theta - 2\cos^2\theta + 5 = 0$$ (7)

$$7\sin\theta - 2(1-\sin^2\theta) + 5 = 0$$
$$7\sin\theta - 2 + 2\sin^2\theta + 5 = 0$$
$$2\sin^2\theta + 7\sin\theta + 3 = 0$$
$$(2\sin\theta + 1)(\sin\theta + 3) = 0$$
$$\therefore \sin\theta = \frac{-1}{2} \quad or \quad \sin\theta \neq -3$$
$$\therefore \theta = 180° + 30° + k360°$$
$$\theta = 210° + k360°$$
or
$$\theta = 360° - 30° + k360°$$
$$\theta = 330° + k360°$$

(d) If $\frac{1}{\sin x} - \sin x = a^3$ and $\frac{1}{\cos x} - \cos x = b^3$,

Prove that $\tan x = \frac{b}{a}$. (5)

$$1-\sin^2 x = a^3 \sin x \qquad 1-\cos^2 x = b^3\cos x$$
$$\sin x = \frac{1-\sin^2 x}{a^3} \qquad \cos x = \frac{1-\cos^2 x}{b^3}$$
$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1-\sin^2 x}{a^3}}{\frac{1-\cos^2 x}{b^3}} = \frac{\cos^2 x}{a^3} \times \frac{b^3}{\sin^2 x}$$
$$\therefore \tan x = \frac{b^3}{a^3\tan^2 x}$$
$$\tan^3 x = \frac{b^3}{a^3}$$
$$\therefore \tan x = \frac{b}{a}$$

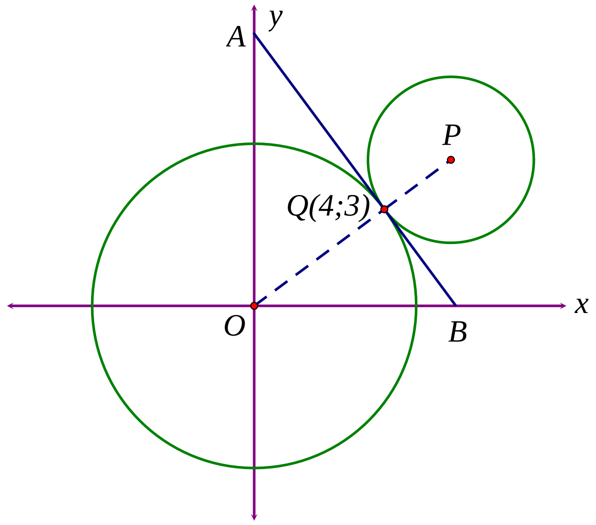
[21]

QUESTION 11

The two circles in the diagram represent two interlocking gears, which touch at point Q (4 ; 3).

The circles have the following equations:

$$x^2 + y^2 - 25 = 0 \text{ and } x^2 - 12x + y^2 - 9y + 50 = 0$$



(a) Show that the co-ordinates of P are (6; 4½). (3)

$$x^2 - 12x + 36 + y^2 - 9y + \left(4\frac{1}{2}\right)^2 = -50 + 36 + \left(4\frac{1}{2}\right)^2$$

$$(x-6)^2 + \left(y - 4\frac{1}{2}\right)^2 = \frac{25}{4}$$

$$\therefore P\left(6;4\frac{1}{2}\right)$$

(b) Determine the equation of common tangent AB. (4)

$$m_{OB} = \frac{3}{4} \quad \therefore m_{AB} = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + c \quad sub(4;3)$$

$$3 = -\frac{4}{3}.4 + c$$

$$c = \frac{25}{3} \quad \therefore y = -\frac{4}{3}x + \frac{25}{3}$$

(c) If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4)

$C_{small} = 2\pi \frac{5}{2}$ $\qquad C_{big} = 2\pi 5$

$C = 5\pi$ $\qquad C = 10\pi$

$\therefore twice$

(d) Find the area of ΔAOB. (3)

$A\left(0;\frac{25}{3}\right)$ $\qquad B\left(\frac{25}{4};0\right)$

$\therefore Area = \frac{1}{2} \times \frac{25}{4} \times \frac{25}{3} = \frac{625}{24} units^2$

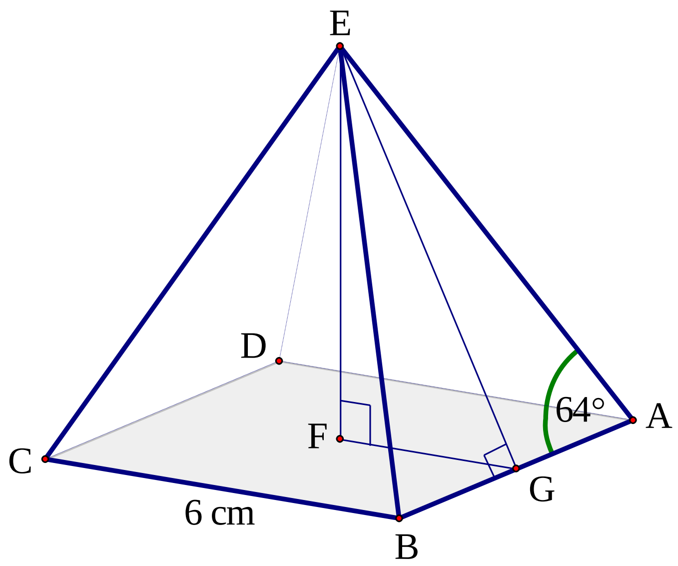
(e) Another tangent to circle O, drawn from A touches the circle at C. Determine the length of CQ. (2)

$Q(4;3)$ $\qquad \therefore C(-4;3)$

$\therefore CQ = 8units$

[16]

QUESTION 12



The diagram shows a pyramid shaped ‘cone’.
Each face is an isosceles triangle with base angles of 64^0. The base is a square of side 6*cm*.
EG is the slant height of the pyramid.
EF is the perpendicular height of the pyramid.

$$Volume\ of\ a\ pyramid = \frac{1}{3} \times area\ of\ base \times perpendicular\ height$$

(a) Determine the length of edge AE. (3)

$BG = GA = 3cm \quad (EG \perp BA\ in\ isos\Delta)$

$\cos 64° = \frac{3}{AE}$

$AE = 6,8cm$

(b) Calculate the height EF. (4)

$\tan 64° = \frac{EG}{3}$

$EG = 6,2cm$

$\therefore EF^2 = EG^2 - FG^2 \quad (pythag)$

$EF^2 = 6,2^2 - 3^2$

$EF = 5,4cm$

(c) Determine the volume of the pyramid. (2)

$$V = \frac{1}{3}.(6 \times 6).EF$$

$$V = \frac{1}{3} \times 36 \times 5,4$$

$$V = 64,8cm^3$$

(d) The pyramid is to be wrapped in a single layer of gold foil, with no overlaps.

Calculate the total area of foil that would be needed. (3)

$$TSA = 4 \times \frac{1}{2} \times 6 \times EG + 6 \times 6$$

$$TSA = 4 \times \frac{1}{2} \times 6 \times 6,2 + 6 \times 6$$

[12]

$$TSA = 110,4cm^2$$

75 marks

Total: 150 marks