



MICHAELHOUSE

Mathematics Department
A BLOCK: CORE MATHEMATICS
PAPER 2
SEPTEMBER 2016

Examiner: Mr S.B. Coxon

Moderator: Mr P. Stevens

Time: 3 hours

Marks: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 22 page(s) and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated. Ensure your calculator is in **DEGREE** mode.
6. All the necessary working details must be clearly shown. Answers only, without relevant calculations, may incur penalties.
7. It is in your own interest to write legibly and to present your work neatly.
8. Round all answers to **ONE decimal place** unless told to do otherwise.

Do not write here:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOT
24	11	9	6	7	6	12	19	7	21	16	12	150

NAME: _____

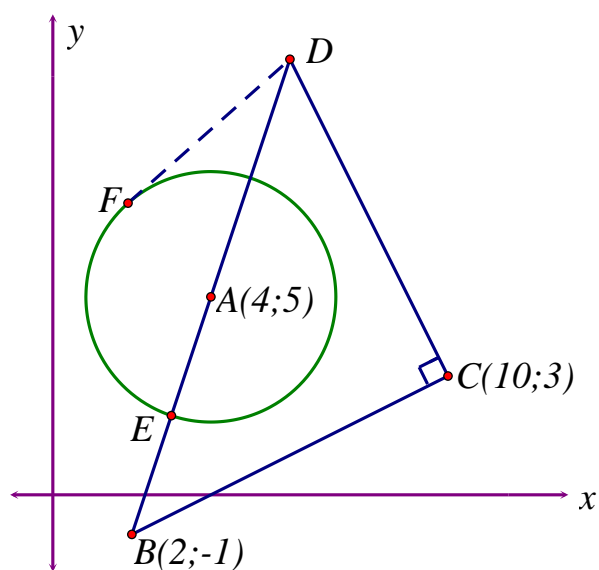
TEACHER: _____

SECTION A

QUESTION 1

In the diagram below:

- $DC \perp BC$
- A is the centre of the circle.
- E is the midpoint of AB.
- The equation of line BA is: $y = 3x - 7$
- DF is a tangent to the circle at F.



(a) Find the co-ordinates of E. (2)

(b) Determine the equation of the circle, centre A, passing through point E.

Give the equation on the form $(x - a)^2 + (y - b)^2 = r^2$ (3)

(c) Find the gradient of line BC. (2)

(d) Hence, or otherwise, determine the equation of line DC. (3)

(e) Show, by calculation, that the co-ordinates of D are (6; 11) . (3)

(f) Find the size of \hat{EBC} . (5)

(g) Find the length of DF.

(6)

[24]

QUESTION 2

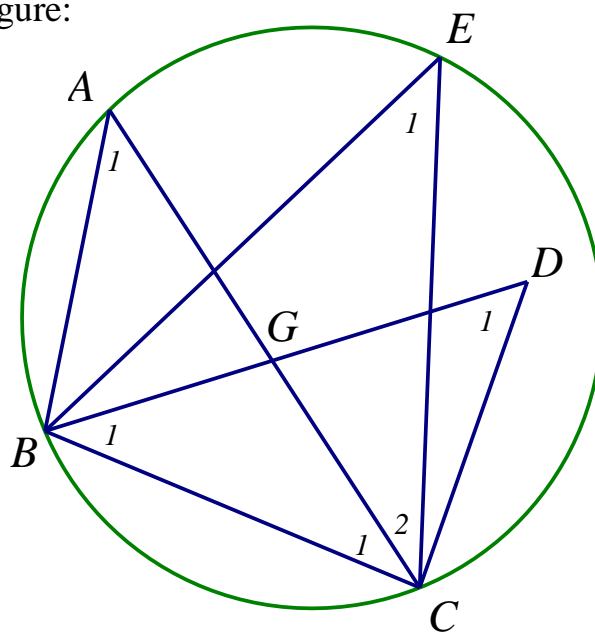
(a) Simplify:
$$\frac{(\sin x - \cos x)^2 - 1}{\sin^2 x - 1} \tag{4}$$

(b) Solve for x in the interval $[-180^\circ ; 180^\circ]$:
 $\cos(x - 30^\circ) = \sin 3x . \tag{7}$

[11]

QUESTION 3

(a) Refer to the figure:



Below are three statements that refer to the above figure. What additional information would be required, if any, to make each individual statement true?

(1) $\hat{A}_1 = \hat{E}_1 = \hat{D}_1$

(2) $\hat{B}_1 = \hat{C}_1$

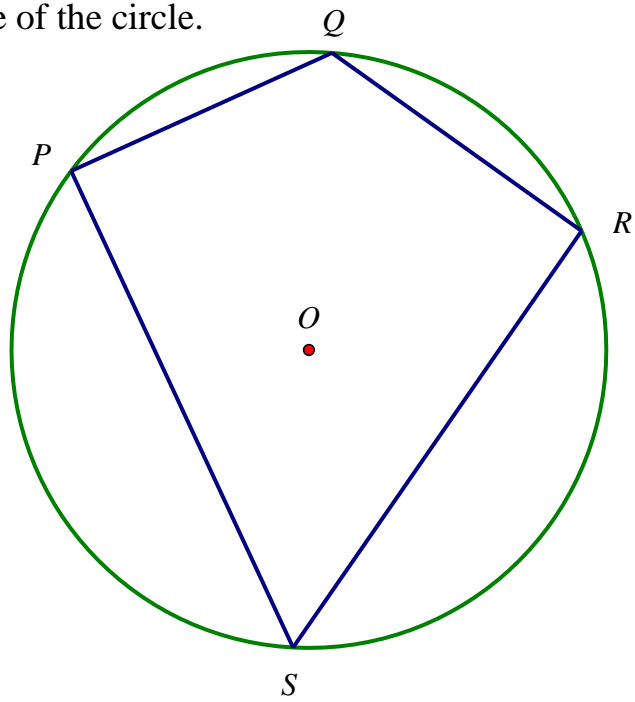
(3) $\hat{C}_1 + \hat{C}_2 = 90^\circ$

(3)

(b) Use the figure below to prove that $\hat{S} + \hat{Q} = 180^\circ$.

\hat{O} is the centre of the circle.

(6)

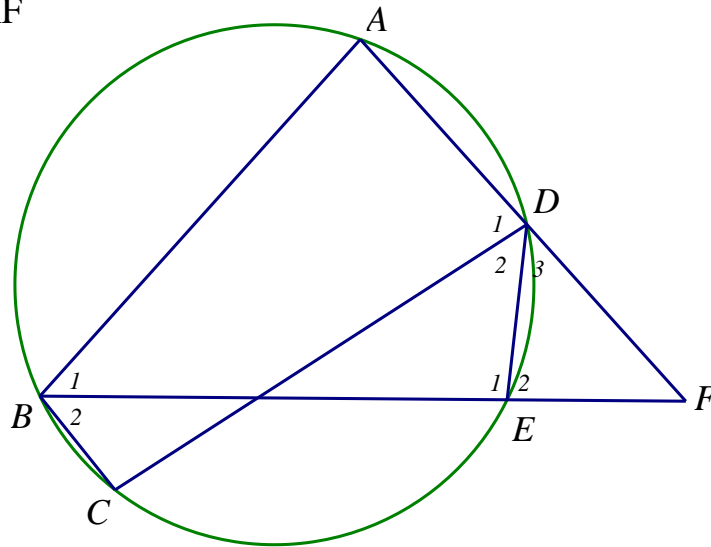


[9]

QUESTION 4

In the figure:

- ABCD is a cyclic quadrilateral.
- $AB = AF$



(a) Prove that $DE = EF$. (3)

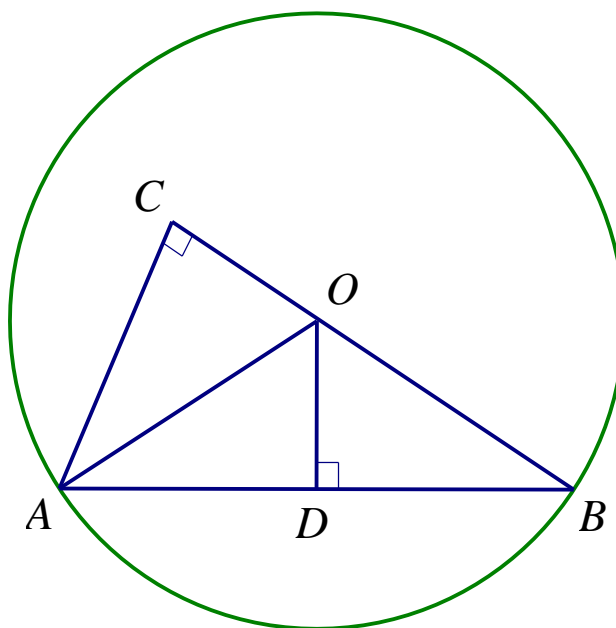
(b) If it is further given that ED bisects \hat{CDF} , prove that FB bisects \hat{ABC} . (3)

[6]

QUESTION 5

In the figure:

- O is the centre of the circle.
- $AC \perp CB$
- $OD \perp BA$



Prove that:

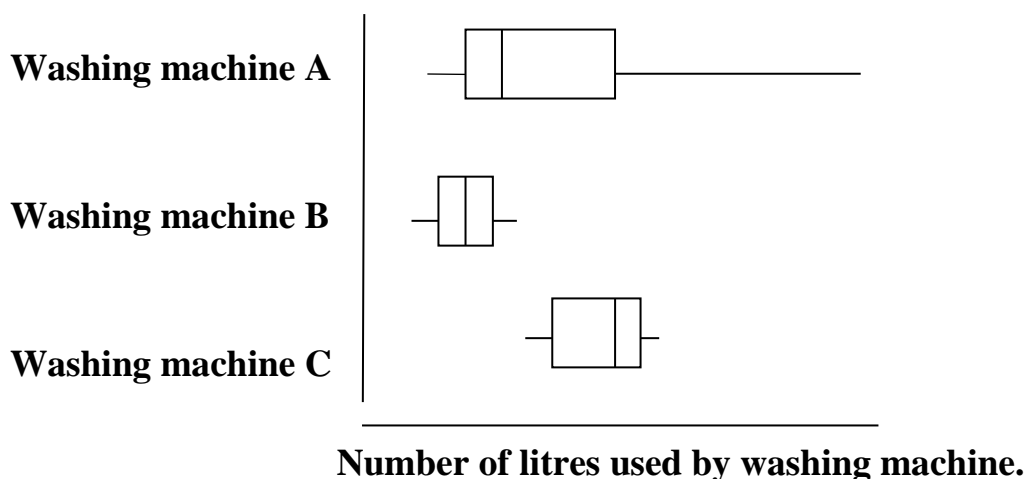
(a) $\triangle ODB \sim \triangle ACB$ (3)

(b) $2AD^2 = OA \cdot BC$ (4)

[7]

QUESTION 6

A consumer testing company studied three different brands of washing machines to see how much water was used during each wash. Each washing machine was tested 15 times. The box and whisker plots below show the results of this study.



(a) Which brand of machine (A, B or C) used up most water on average? (1)

(b) Which brand of machine (A, B or C) is the most predictable? (1)

(c) The results of Washing Machine C are shown below:

81	85	85	88	89	90	92	101	104	105	106	106	108	112	112
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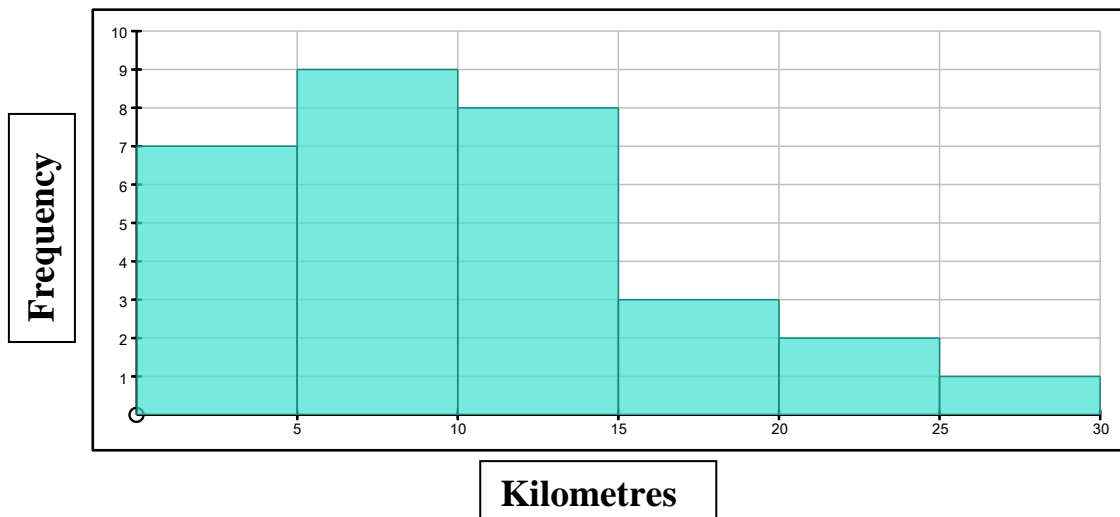
(1) Determine the standard deviation of the litres of water used. (2)

(2) Based on this data, how many litres of water would be used in 67% of the washing loads? (2)

[6]

QUESTION 7

The distance (x) in kilometres that the staff at a certain school in Durban travel to work each day is summarised in the histogram below:



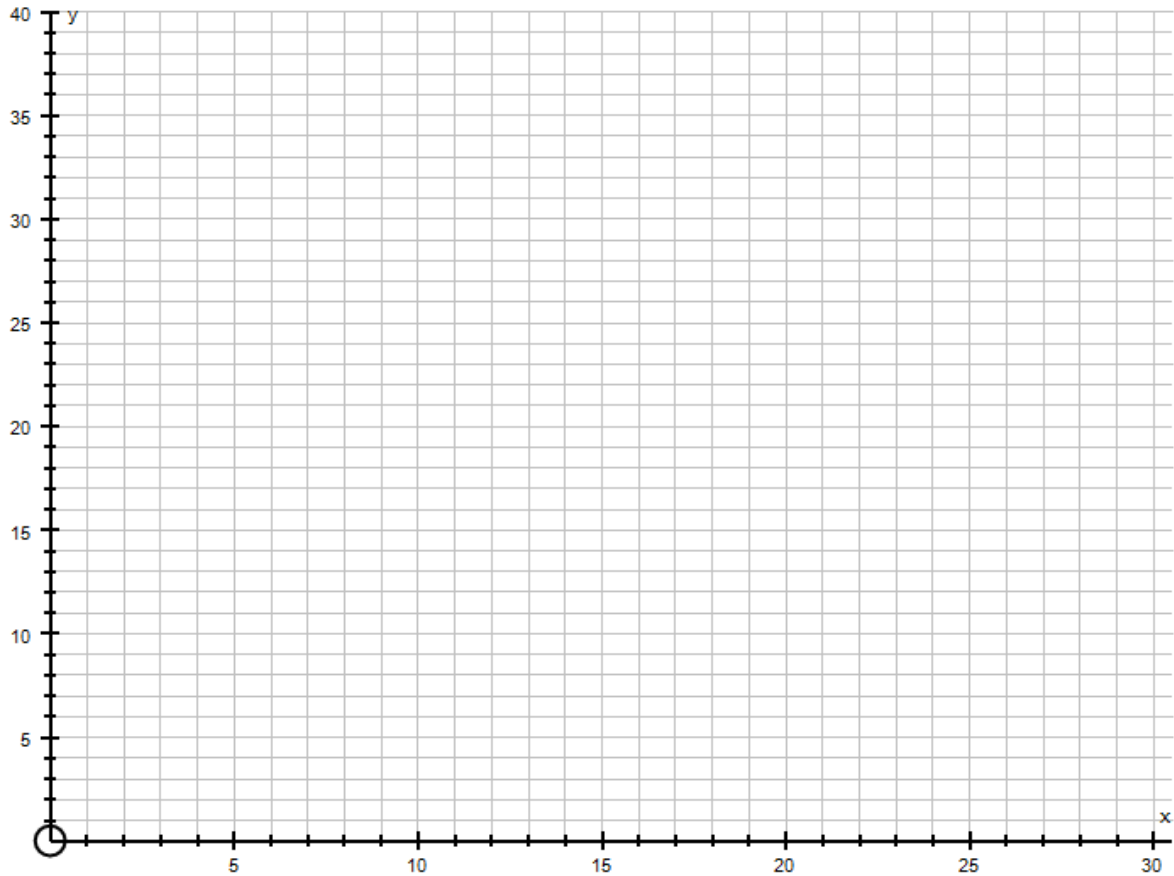
- (a) Is the data positively or negatively skewed? (1)

- (b) Which of the intervals is the modal interval? (1)

- (c) Use the histogram to complete the given table. (3)

Intervals	Frequency	Cumulative Frequency
$0 \leq x < 5$		
$5 \leq x < 10$		
$10 \leq x < 15$		
$15 \leq x < 20$		
$20 \leq x < 25$		
$25 \leq x < 30$		

- (d) Use your frequency table to draw an ogive below. Label your axes. (3)



- (e) Determine the following using your ogive:

- (1) the interquartile range (2)

- (2) the percentage of these staff members that stayed between 4km and 14km from the school? (2)

[12]

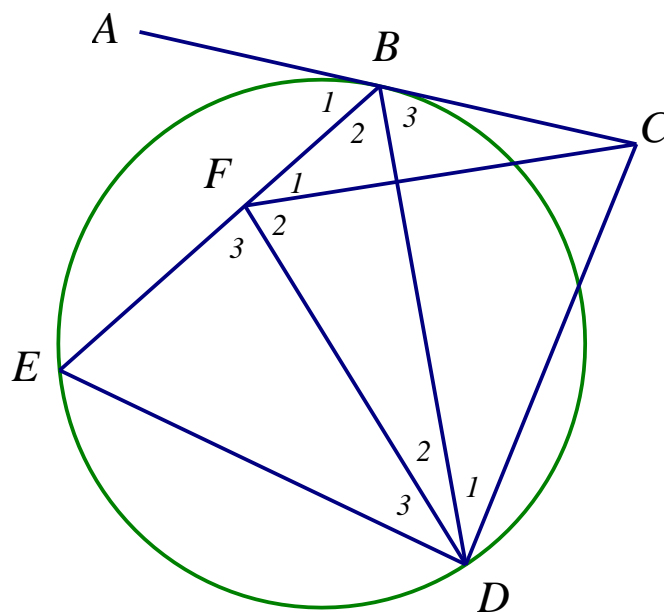
75 marks

SECTION B

QUESTION 8

(a) In the figure:

- ABC is a tangent to the circle at B .
- $\hat{D}_1 = \hat{D}_3$



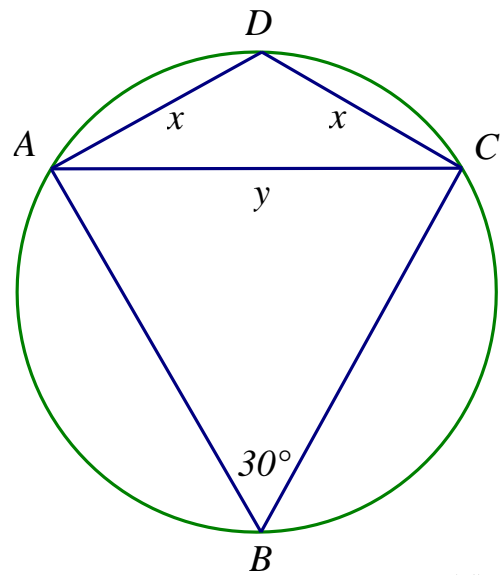
Prove that:

- (1) $DCBF$ is a cyclic quadrilateral. (4)

- (2) FC is a tangent to the circle FED . (4)

(b) In the figure:

- ABCD is a cyclic quadrilateral.
- $AD = DC = x$
- $AC = y$
- $\hat{B} = 30^\circ$

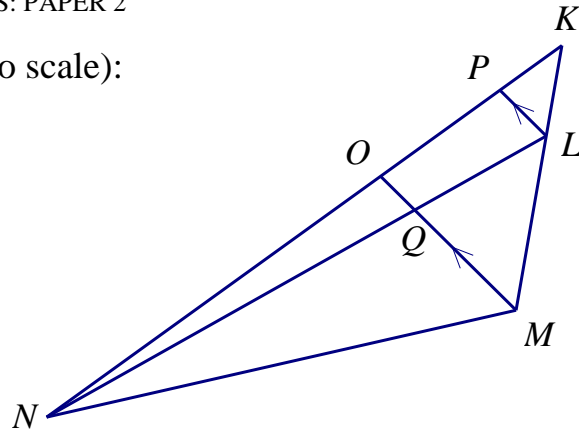


Show that $y = x\sqrt{2 + \sqrt{3}}$

(5)

(c) In the figure (not drawn to scale):

- $OM \parallel PL$
- $KN : KO = 8 : 3$
- $LM = 3KL$



Calculate:

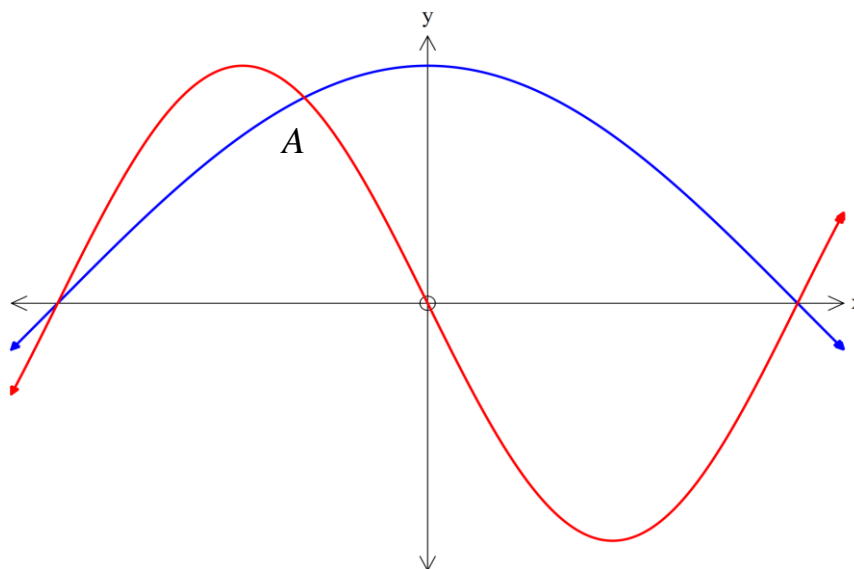
(1) $\frac{\text{Area of } \triangle KMO}{\text{Area of } \triangle NMO}$ (3)

(2) $\frac{LQ}{QN}$ (3)

[19]

QUESTION 9

The sketch below shows the graphs of $f(x) = -\sin x$ and $g(x) = \cos \frac{x}{2}$ for $x \in [-180^\circ; 180^\circ]$. The coordinates of A , the point of intersection of the two graphs, are $(t; \frac{\sqrt{3}}{2})$.



- (a) Determine the value of t . (2)

- (b) Use the sketch to solve the following equation: $\cos \frac{x}{2} = -2 \sin \frac{x}{2} \cos \frac{x}{2}$
for $x \in [-180^\circ; 180^\circ]$. (3)

- (c) If f is translated 10° to the left and 2 units up, what will the new equation of f be? (2)

[7]

QUESTION 10

(a) Simplify:
$$\frac{\sin 170^\circ \cdot \cos 10^\circ}{\sin(20^\circ - x) \cdot \cos x + \cos(20^\circ - x) \cdot \sin x} \quad (4)$$

(b) Prove the following identity:

$$\frac{\cos x - \cos 2x - 1}{\sin x - \sin 2x} = \frac{1}{\tan x} \quad (5)$$

(c) Determine the general solution for θ :

$$7 \sin \theta - 2 \cos^2 \theta + 5 = 0 \quad (7)$$

(d) If $\frac{1}{\sin x} - \sin x = a^3$ and $\frac{1}{\cos x} - \cos x = b^3$,

Prove that $\tan x = \frac{b}{a}$. (5)

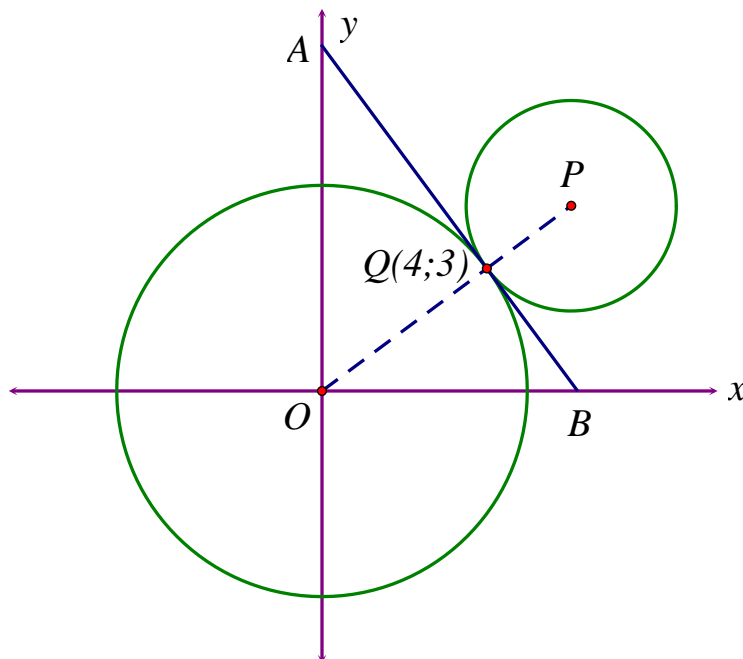
[21]

QUESTION 11

The two circles in the diagram represent two interlocking gears, which touch at point Q (4 ; 3).

The circles have the following equations:

$$x^2 + y^2 - 25 = 0 \text{ and } x^2 - 12x + y^2 - 9y + 50 = 0$$



- (a) Show that the co-ordinates of P are (6; 4½). (3)

- (b) Determine the equation of common tangent AB. (4)

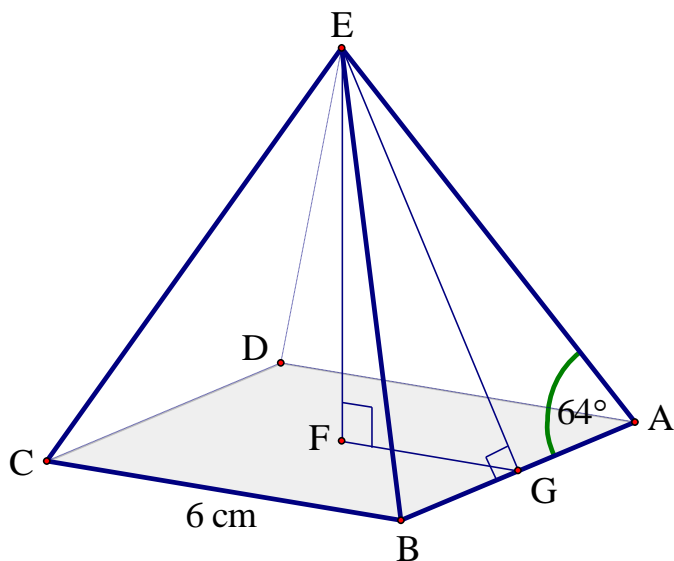
- (c) If the larger gear makes one full revolution, how many times will the smaller gear turn completely? (4)

- (d) Find the area of $\triangle AOB$. (3)

- (e) Another tangent to circle O, drawn from A touches the circle at C. Determine the length of CQ. (2)

[16]

QUESTION 12



The diagram shows a pyramid shaped 'cone'.
 Each face is an isosceles triangle with base angles of 64° . The base is a square of side 6 cm .
 EG is the slant height of the pyramid.
 EF is the perpendicular height of the pyramid.

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$

- (a) Determine the length of edge AE. (3)

- (b) Calculate the height EF. (4)

- (c) Determine the volume of the pyramid. (2)

- (d) The pyramid is to be wrapped in a single layer of gold foil, with no overlaps.

Calculate the total area of foil that would be needed. (3)

[12]

75 marks

Total: 150 marks