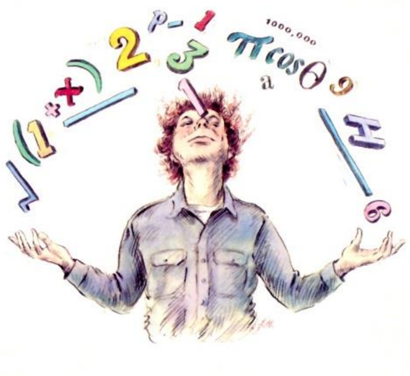
ST ANDREW'S COLLEGE | THE DIOCESAN SCHOOL FOR GIRLS | ST ANDREW'S PREPARATORY



Department of Mathematics

TIME: 3 hours	DATE: 25 July 2016	Total marks: 150
Setter: GP	**Grade 12 Mathematics Paper 1**	**Moderator: DS**

Name of student:

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists 8 pages and an Information Sheet of 2 pages (i-ii). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be shown clearly.
8. It is in your own interest to write legibly and to present your work neatly.
9. Please hand in the question paper.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
21	18	19	15	5	14	16	17	14	11	150

SECTION A

QUESTION 1

(a) Solve for *x*:

(1) $(x-3)(2^x-1)=0$ (2)

(2) $7-(x+2)(x+3)=0$ (4)

(3) $\log_3(2x-1)=\log_2 8$ (4)

(b) Solve for *x* in the following inequality: $3x^2-16x\leq -5$ (4)

(c) Given: $(3m-5)(n+3)=0$

Solve for:

(1) n if $m=2$ (1)

(2) m if $n\neq -3$ (1)

(3) m if $n=-3$ (2)

(d) If $f(x)=3(x-1)^2-5$ and $g(x)=3$, determine the value(s) of k for which $f(x)=g(x)+k$ has TWO unequal real roots. (3)

[21]

QUESTION 2

(a) Write down the nth term of the sequence $\frac{1}{4};\frac{2}{9};\frac{3}{16};\frac{4}{25};\ldots$ (2)

(b) Given the arithmetic series 20 + 18 + 16 + … , determine:

(1) the 100th term (2)

(2) the value(s) of *n* if $S_n=80$ (5)

(c) The sum of the first *n* terms of a series is given by the formula $S_n=3^{n+1}-3$.

(1) Determine the sum of the first 6 terms. (1)

(2) Determine the first 3 terms of the sequence. (4)

(d) Calculate: $\sum_{n=3}^{6} 5.2^{-n}$ (4)

[18]

QUESTION 3

(a) Given $f(x) = 4x^2 - 2$, determine $f'(x)$ from first principles. (5)

(b) Determine $\frac{dy}{dx}$, leaving your answer with positive exponents, if:

(1) $y = 3x^5 - 14\sqrt{x} + \frac{3}{x}$ (4)

(2) $y = \frac{5x-2}{10x^2}$ (5)

(c) Given $f(x) = \sqrt{x}(x+2)$, calculate $f'\left(\frac{1}{4}\right)$. (5)

[19]

QUESTION 4

(a) Given: $f(x) = \frac{-3}{x+2}$

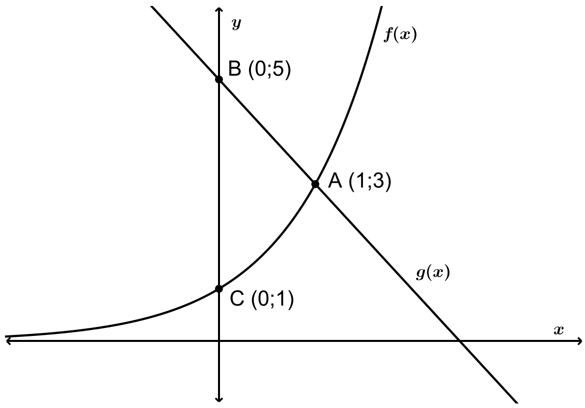
(1) Write down the domain of $f(x)$. (2)

(2) Write down the equations of the asymptotes of $f(x) - 2$. (2)

(3) Write down the equation of the graph $h(x)$ formed if f is shifted 3 units up and 2 units to the left. (2)

(4) Determine the equation of $f^{-1}(x)$ in the form $y = \ldots$ (2)

(b) The graph of $f(x) = 3^x$ and $g(x) = ax + q$ are drawn. A$(1; 3)$ is the point of intersection of the two graphs. B$(0; 5)$ is the *y*-intercept of g and C$(0; 1)$ is the *y*-intercept of f.



(1) Find the values of *a* and *q*. (2)

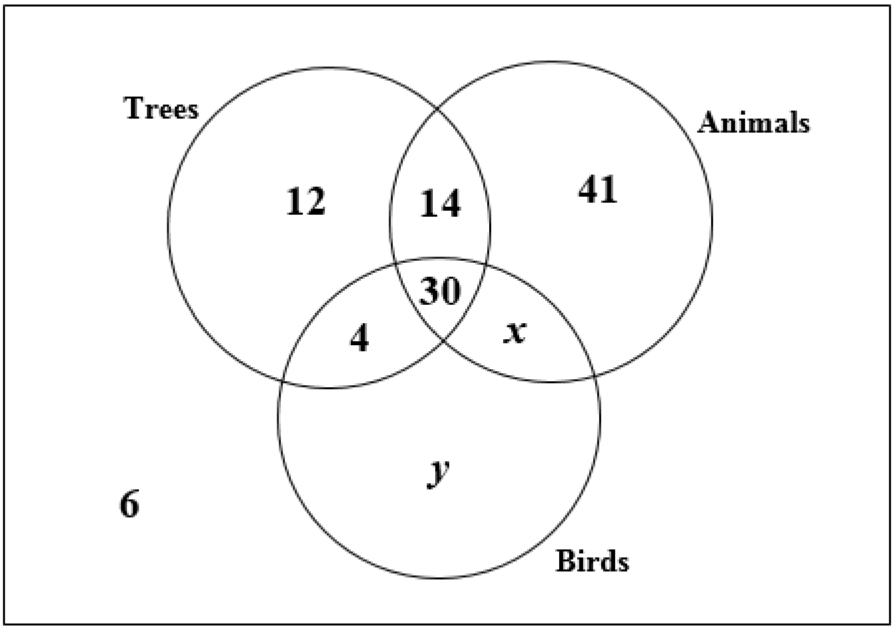
(2) Write down the domain of $f^{-1}(x)$, the inverse of f. (2)

(3) For which value(s) of *x* is $f^{-1}(x).\, g(x) \leq 0$? (3)

[15]

QUESTION 5

A survey was conducted of 200 visitors to the Kruger Park Game Reserve. The visitors were asked whether it was the trees, animals or birds that interested them. The following Venn diagram summarises the results of the survey:



(a) If 156 visitors said they were interested in Animals, what are the values of *x* and *y*? (3)

(b) What is the probability that a visitor was only interested in **one** of the sights of the Kruger Park? (2)

[5]

78 marks

SECTION B

QUESTION 6

(a) In order to determine whether people aged between 20 and 25 years were employed or not, a survey was undertaken. The table below summarizes the results.

	MALES	**FEMALES**	**TOTAL**
Employed	30	130	160
Unemployed	140	100	240
TOTAL	170	230	400

(1) Suppose that one of the people surveyed was chosen at random.
Determine the probability that the person:

(i) is a male who is employed. (1)

(ii) is unemployed. (1)

(iii) is female, given that they are employed. (2)

(2) Showing all working, determine whether it can be said that being employed is independent of gender. (3)

(b) Three boys and four girls sit in a row watching a movie.
Determine the number of arrangements in which this can be done if:

(1) they sit in any order. (1)

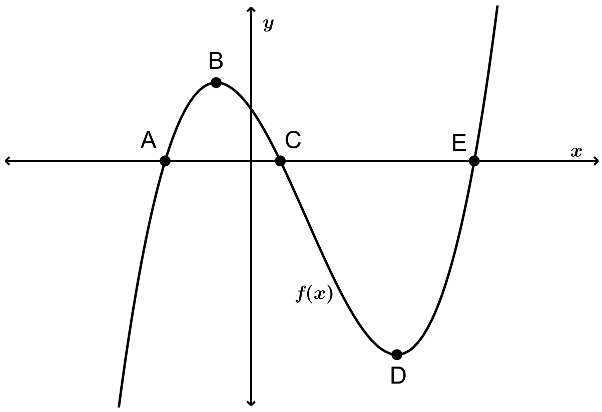
(2) a girl sits at each end. (3)

(3) the three boys do *not* sit together. (3)

[14]

QUESTION 7

The sketch below represents the function f with equation $f(x) = x^3 + bx^2 + cx + 12$, where b and c are constants. $f'(-1) = 5$ and $f''(-1) = -16$.



(a) Show that the equation of the curve is $f(x) = x^3 - 5x^2 - 8x + 12$. (7)

(b) Give the values of x for which the graph is concave up. (3)

(c) $y = -11x + 3$ is a tangent to $f(x)$ at $(k; t)$. Determine the values of k and t if k is a whole number. (6)

[16]

QUESTION 8

(a) The 5th term of an arithmetic series is 14. The sum of the first 10 terms is 160. Determine the first term and the common difference of the series. (5)

(b) Water being pumped into Grahamstown is passed through a new filtration system. Sarah measures the amount of silt, in mg, that the new filtration system removes from a fixed volume of water after each of a series of filtrations:

GRAHAMSTOWN FILTRATION SYSTEM		
After 1 filtration	After 2 filtrations	After 3 filtrations
100 mg	70 mg	49 mg

(1) Sarah plots a graph to represent the total amount of silt, $S(n)$, removed after n filtrations through the new system. Determine a formula for $S(n)$. (4)

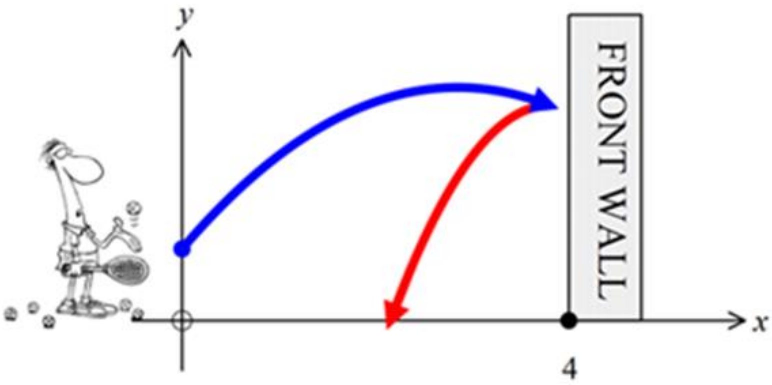
(2) Assuming this trend continues, what is the maximum amount of silt that this new system could potentially remove from the fixed volume of water? (2)

(c) Determine the value of p if: $\sum_{k=1}^{\infty} 27p^k = \sum_{t=1}^{12}(24-3t)$ (6)

[17]

QUESTION 9

(a) A squash player hits the ball against a wall which is 4 metres away.
The ball rebounds as shown in the diagram below.



The initial path of the ball as it is struck by the player is given by the equation:

$$y = -\frac{1}{4}x^2 + px + 1$$

where *y* is the height that the ball is above the ground and *x* is the distance the ball is away from the player (in metres).

(1) From what height is the ball struck? (1)

(2) Given that the ball strikes the wall at a **gradient** of $-\frac{1}{2}$, calculate the value of *p*. (4)

(3) The ball rebounds off the wall along the curve defined by $y = -3x^2 + 24x - 45$. Determine the value of *x* at which the ball hits the ground. (3)

(b) $f(x)$ is a **parabola**. It is further given that:

$$f(0) = -3\ ;\ f(2) = f(6) = 0\ ;\ f'(a) = 0\ ;\ f(a) = 5$$

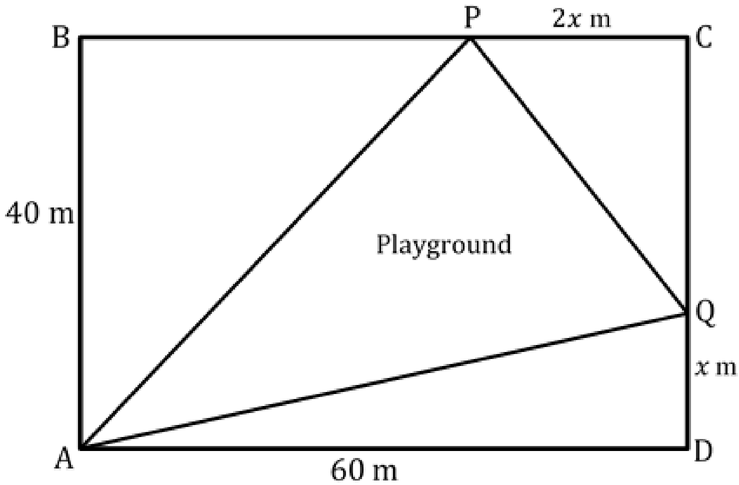
(1) Draw a sketch graph of $f(x)$, clearly indicating all important points. (4)

(2) If $g'(x) = -f(x)$, state the value(s) for which $g(x)$ is increasing. (2)

[14]

QUESTION 10

The diagram shows a plan for a rectangular park, ABCD, in which AB = 40 metres and AD = 60 metres. Points P and Q lie on BC and CD respectively and AP, PQ and QA are paths that surround a triangular playground. The length DQ is x metres and the length of PC is $2x$ metres.



(a) Show that the area, in m^2, of the triangular playground APQ is given by:

$$A = x^2 - 30x + 1200$$ (7)

(b) Given that x can vary, find the minimum area of the playground. (4)

[11]

72 marks

Total: 150 marks