SAHETI SCHOOL

JULY EXAMINATION 2016

MATHEMATICS GRADE 12

PAPER 2

Time: 3 hours

Total: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 19 pages, graph paper, and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1:

A sample of 120 bolts was taken from the output of a machine. The diameter of each bolt in the sample was measured and the results are given below.

Diameter of bolt d (mm)	Number of bolts
$2.01 \le d < 2.04$	24
$2.04 \le d < 2.05$	36
$2.05 \le d < 2.06$	26
$2.06 \le d < 2.07$	22
$2.07 \le d < 2.09$	12

a) State the modal class of the distribution of the diameters of the bolts. (1)

$2.04 \le d < 2.05$ ✓ 1

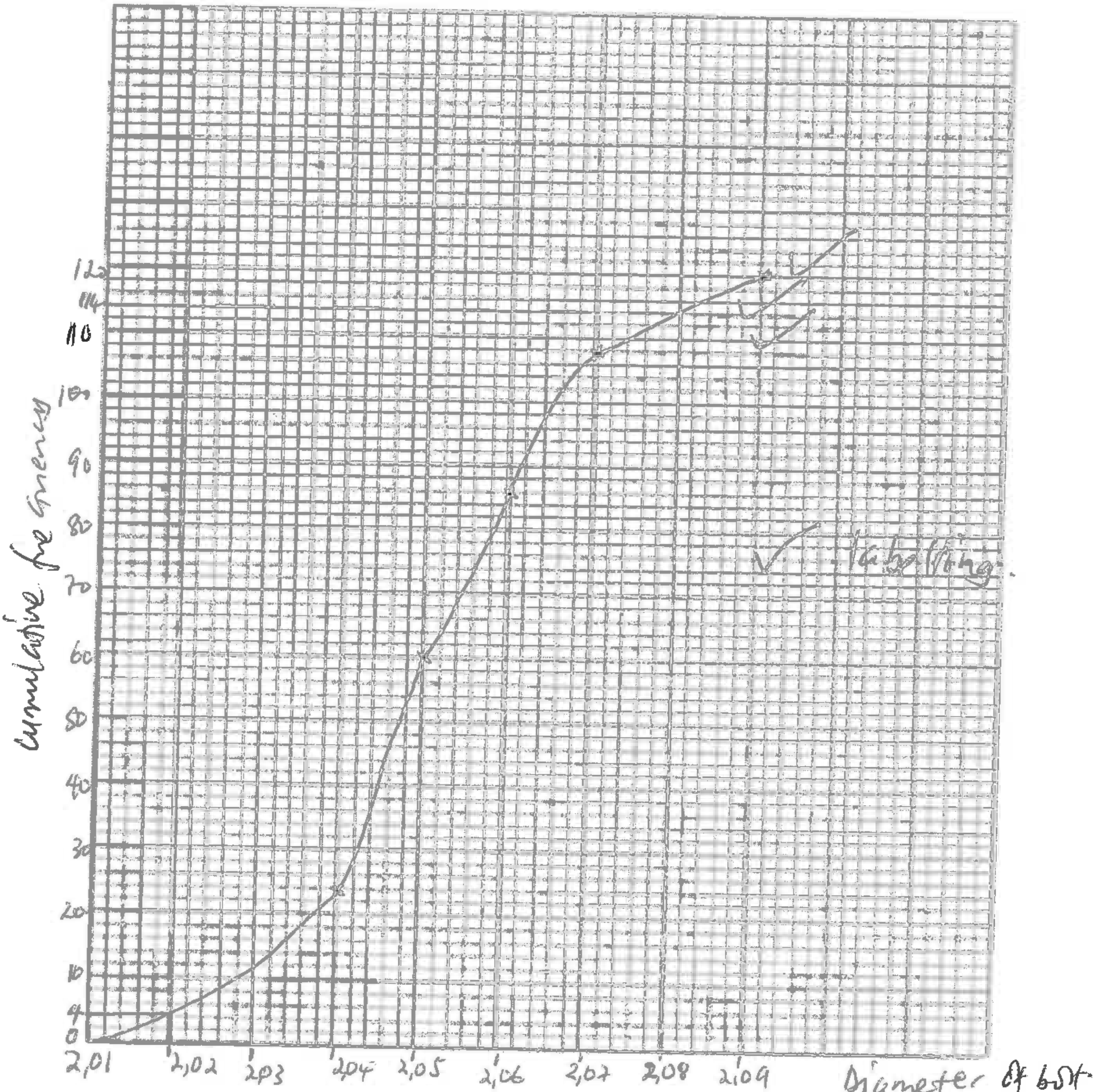
b) Calculate to 2 decimal places, an estimate of mean diameter of the bolts. (2)

$\bar{d} = 2{,}05$ ✓ 2

c) Calculate to 2 decimal places, an estimate of standard deviation of the diameters of the bolts. (2)

$\sigma = 0{,}02$ ✓ 2

d) Draw the ogive for the above data. (4)



e) Bolts are accepted by the purchaser if their diameter falls within the limits $2.02 \le d \le 2.08$. Estimate, to the nearest integer, the percentage of bolts which are not accepted by the purchaser. (3)

$\frac{4+6}{120} \times 100 = 8{,}33\% = 8\%$

from diagram.

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QUESTION 2:

a) The points O (0; 0), A (2; 2), B (-2; 4) and C (-6; 0) are points of a quadrilateral.

1) Find the midpoint of the line segment AC (2)

$M(-2; 1)$

2

2) Find the equation of the line through the points A and C. (3)

$m = \frac{2}{8} = \frac{1}{4}$

$y = \frac{1}{4}x + c$

$2 = \frac{1}{4}(2) + c$

$y = \frac{1}{4}x + \frac{3}{2}$

3

3) Find the coordinates of the point of intersection of the diagonals of the quadrilateral. (5)

Equation OB: $y = -2x$

$\therefore -2x = \frac{1}{4}x + \frac{3}{2}$

$x = -\frac{2}{3}$

5

$y = \frac{4}{3}$

4) Find the shortest distance from the vertex B to the line segment OA. (5)

$M_{OA} = 1$; Equation $OA \Rightarrow y = x$ ✓

$m_\perp = -1$ ✓ $y = -x + c$

$4 = -(-2) + c$

$y = -x + 2$ ✓

$x = -x + 2$

$2x = 2$

$\underline{x = 1}$ ✓

$\underline{y = 1}$

$\therefore$ Distance $= \sqrt{(-3)^2 + 3^2}$

$= \underline{\underline{3\sqrt{2}}}$ ✓

b) P (6; 9) and Q (a; 5) are two points. The perpendicular bisector of PQ cuts the x-axis at 2. Find two possible values of a. (6)

$M\left(\frac{a+6}{2} ; 7\right)$ ✓ Grad $PQ = \frac{4}{6-a}$ ✓ $m_\perp = \frac{a-6}{4}$ ✓

$y = \frac{a-6}{4} \cdot x + c$

$7 = \frac{a-6}{4} \cdot \frac{a+6}{2} + c$

$y = \frac{a-6}{4} \cdot x + \left(7 - \frac{a^2-36}{8}\right)$ ✓

$0 = \frac{a-6}{\cancel{4}2} \cdot \cancel{x} + 7 - \frac{a^2-36}{8}$

$0 = 4a - 24 + 56 - a^2 + 36$

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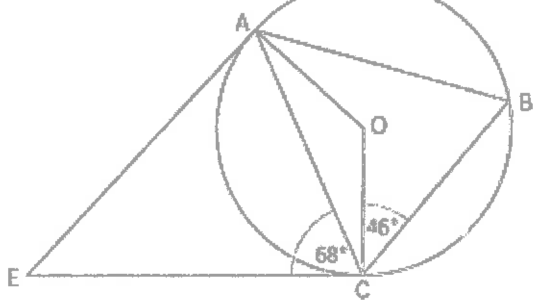
$a^2 - 4a - 68 = 0$

$a = 2 \pm 6\sqrt{2}$ ✓

$\underline{a = 10,5 \text{ or } -6,5}$ ✓

QUESTION 3:

a) In the diagram below, A, B and C are points on a circle with centre O. AE and CE are tangents to the circle. $A\hat{C}E = 68^o$ and $B\hat{C}O = 46^o$.



Calculate, with reasons:

1) $A\hat{O}C$ (4)

$\hat{B} = 68^\circ$ (tan ch ord thm)

$\therefore A\hat{O}C = 136^\circ$ ($\angle$ at centre = 2 $\angle$ at circum)

2) $A\hat{E}C$ $= 44^\circ$ (int $\angle$s of Δ) $E\hat{A}C = 68$ (tangents from same point) (2)

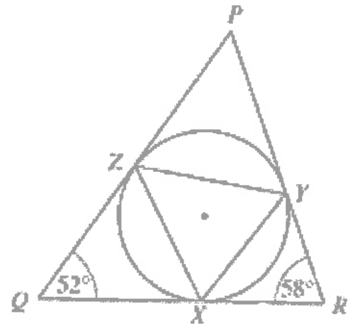
3) $B\hat{A}E$ (3)

$O\hat{A}C = 22^\circ$ (Isosc Δ)

$\therefore B\hat{A}C = 44^\circ$ (Sum of int $\angle$s of Δ)

$\therefore B\hat{A}E = 112^\circ$

b) The diagram shows a circle which passes through X, Y and Z. PZQ, QXR and RYP are tangents to the circle. Given that $P\hat{Q}R = 52^\circ$ and $Q\hat{R}P = 58^\circ$



Calculate and state the reason:

1) $Q\hat{P}R$ $= 70^\circ$ (Int $\angle$s of Δ) (2)

2) $Q\hat{Z}X$ $= 64^\circ$ (Isosc Δ – tangents from same point) (2)

3) $Z\hat{X}Y$ (3)

$R\hat{X}Y = 61^\circ$

$\therefore Z\hat{X}Y = 180 - (64 + 61)$ (supp $\angle$s)

$= \underline{55^\circ}$

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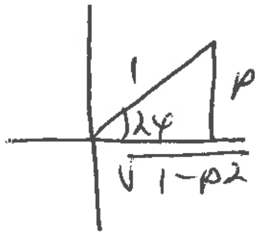
QUESTION 4:

a) If $\sin 24^{o} = p$, express the following in terms of p.

1) $\cos 24^{o} = \sqrt{1-p^2}$ (3)



2) $\sin 48^{o} = 2\sin 24 \cos 24$ (3)

$= 2p\sqrt{1-p^2}$.

3) $\sin 12^{o} \cos 12^{o} - \sin(-66^{o})\tan(204^{o})$ (5)

$\sin 12 \cos 12 + \sin 66 \cdot \tan 24$

$\sin 12 \cdot \cos 12 + \cancel{\cos 24} \cdot \dfrac{\sin 24}{\cancel{\cos 24}}$

$\dfrac{\sin 24}{2} + \sin 24$

$= \frac{3}{2} \sin 24$

$= \frac{3}{2} p$

b) Simplify without the use of a calculator.

$\sin(180° - x).\cos(90° - x) - \cos(180° + x).\cos(360° - x)$ (5)

$\sin x \cdot \sin x - (-\cos x) \cdot \cos x$

$= \sin^2 x + \cos^2 x$

$= 1$

c) 1) Prove that $\frac{\sin x}{1-\cos x} - \frac{1}{\sin x} = \frac{1}{\tan x}$ (5)

$LHS = \frac{\sin^2 x - 1 + \cos x}{(\sin x)(1-\cos x)}$

$= \frac{-\cos^2 x + \cos x}{\sin x(1-\cos x)}$

$= \frac{\cos x(-\cos x + 1)}{\sin x(1-\cos x)} = \frac{\cos x}{\sin x}$

$RHS = 1 \div \frac{\sin x}{\cos x}$

$= \frac{\cos x}{\sin x}$

2) Hence, solve the equation $\frac{\sin(2x+45)}{1-\cos(2x+45)} - \frac{1}{\sin(2x+45)} = 3$ (3)

$\frac{1}{\tan(2x+45)} = 3$

$\tan(2x+45) = \frac{1}{3}$

$2x+45 = 18,4 + 180k$

$2x = -26,6 + 180k$

$x = -13,3 + 90k \quad k \in \mathbb{Z}$

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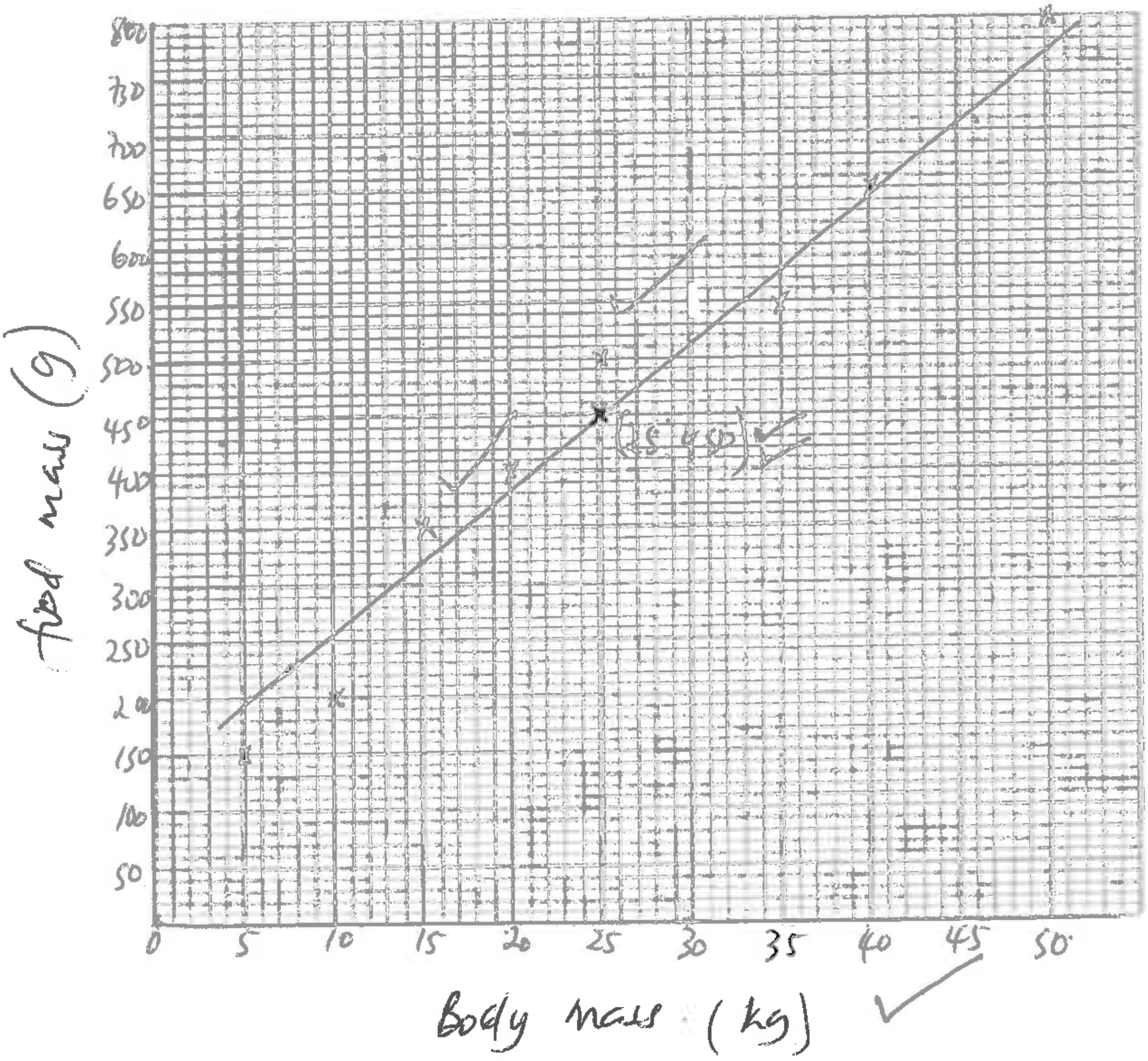
SECTION B

QUESTION 5:

The manageress of a boarding kennel in Gauteng feeds the dogs in her care on Puppygrow complete dog food. The instructions on the bags in which the dog food is supplied give the following guidance regarding the mass of food to be given.

Body mass of dog to the nearest 5 kg. (*x* kg)	5	10	15	20	25	35	40	50
Mass of food per day (*y* grams)	150	200	350	400	500	550	650	800

a) Draw the scatter diagram and line of best fit on the same set of axes. (5)



$\bar{x} = 25$
$\bar{y} = 450$

b) State, giving a reason, whether you think it would be correct to extend the line of best fit to the y-axis. (2)

No, body mass will never be zero.

c) Determine the equation of the least squares regression line of y on x to 3 decimal places (3)

$y = 100,735 + 13,971x$

d) Use your calculator to evaluate the correlation coefficient r, for this data to 3 decimal places and explain the purpose of this value. (3)

0,989 – indicates the relationship between body mass of dog and the mass of food per day. (Strong positive linear relationship).

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QUESTION 6:

a) Prove the following identity: $\dfrac{2}{1-\tan^2\theta} = \dfrac{2-2\sin^2\theta}{\cos 2\theta}$ (5)

$LHS = 2 \div \left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right)$

$= 2 \times \dfrac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{2\cos^2\theta}{\cos 2\theta}$

$= \dfrac{2(1-\sin^2\theta)}{\cos 2\theta} = RHS$

b) Calculate the value(s) of x, $x \in [-180°; 180°]$ if $\tan x = \sin 2x$ (6)

$\frac{\sin x}{\cos x} = 2\sin x \cos x$ ✓

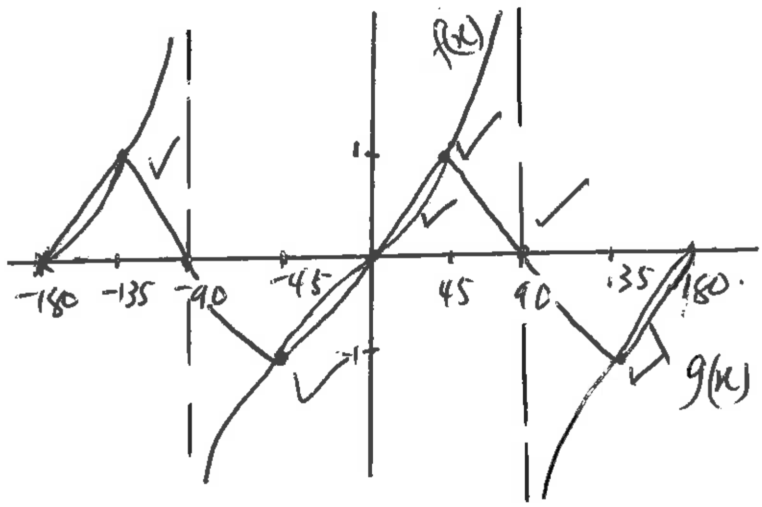
$2\sin x \cos^2 x - \sin x = 0$

$\sin x$ ✓ $(2\cos^2 x$ ✓ $-1) = 0$

$\sin x = 0$ $\qquad \cos x = \frac{1}{\sqrt{2}}$ $\qquad \cos x = -\frac{1}{\sqrt{2}}$ ✓

$x = 180k$ ✓ $\qquad x = \pm 45 + 360k$ ✓ $\qquad x = \pm 135 + 360k \quad k \in \mathbb{Z}$ ✓

c) 1) Draw on the same set of axes the graphs of $f(x) = \tan x$ and $g(x) = \sin 2x$ for $x \in [-180°; 180°]$. Indicate the intercepts with the axes as well as the co-ordinates of any turning points of the graphs. (6)



2) For which values of x is $\tan x \geq \sin 2x$ for $x \in [40°; 180°]$ (2)

$135 \leq x \leq 180$ ✓✓

3) Write down the amplitude of $g(x) - 2$ (1)

1 ✓

d) If $x+\frac{1}{x}=2\cos A$; $0^\circ \leq A \leq 180^\circ$ and $x^2+\frac{1}{x^2}=1$, Calculate $\hat{A}$ (4)

$$\left(x+\frac{1}{x}\right)^2 = (2\cos A)^2$$

$$x^2+2+\frac{1}{x^2} = 4\cos^2 A.$$

$$3 = 4\cos^2 A$$

$$\cos A = \pm\sqrt{3/4}$$

$k \in \mathbb{Z}.$

$A = \pm 30 + 360k$ $\quad$ $A = \pm 150 + 360k$

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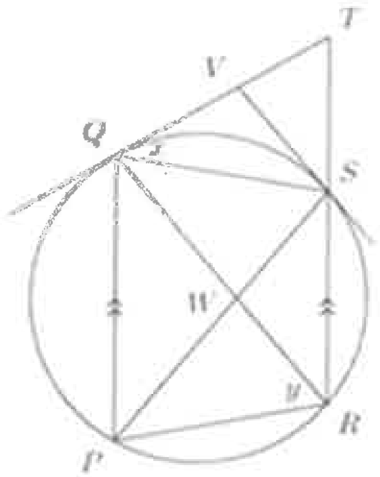
QUESTION 7:

PQ and RS are chords of the circle and $PQ \parallel RS$. The tangent to the circle at Q meets RS produced at T. the tangent TS meets QT at V. QS and PR are drawn.

Let $T\hat{Q}S = x$ and $Q\hat{R}P = y$.



Prove that:

a) $T\hat{V}S = 2Q\hat{R}S$ (4)

$V\hat{S}Q = V\hat{Q}S = x$ (∠s opp equal sides)

$\therefore T\hat{V}S = 2x$ (sum of opp int ∠s = ext ∠ of Δ)

$Q\hat{R}S = x$ (tan chord)

$\therefore TVS = 2Q\hat{R}S$ (proven)

b) QVSW is a cyclic quadrilateral (4)

$Q\hat{P}S = x$ (same segment)

$P\hat{Q}R = x$ (Alt ∠s QP∥SR)

∴ $Q\hat{W}S = 2x$ (sum of opp int ∠ = ext ∠ of △)

$Q\hat{W}S = T\hat{V}S = 2x$

∴ QVSW is a cyclic quad (ext ∠ = int opp ∠)

c) $Q\hat{P}S + \hat{T} = P\hat{R}T$ (3)

$P\hat{R}T = x + y$

$\hat{Q} = Q\hat{R}P = y$ (tan chord)

∴ $\hat{Q} = T = y$ (corres ∠s QP∥TR)

$Q\hat{P}S + T = x + y = P\hat{R}T$

d) W is the centre of the circle. (3)

$Q\hat{R}S = x$ (proven)

$Q\hat{W}S = 2x$ (proven)

W = is circle centre (∠ centre = 2 ∠ at circum)

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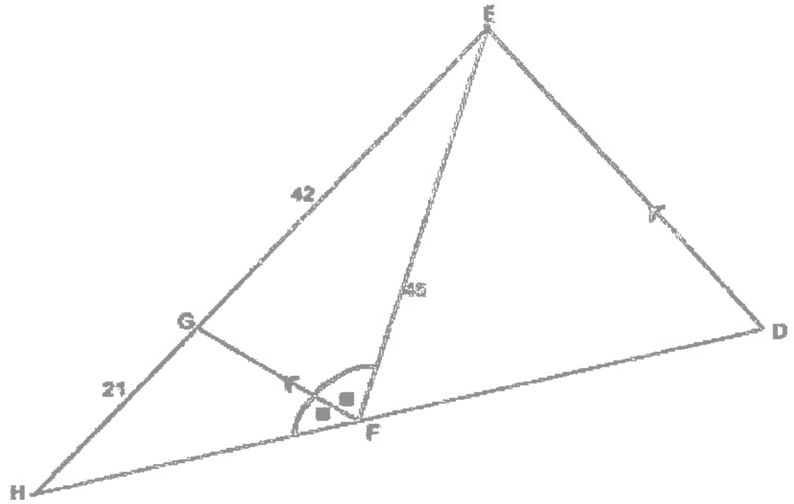
QUESTION 8:

a) In ΔHED, F is a point on HD, G is a point on HE and $FG \parallel DE$. HG = 21 units, GE = 42 units, FE = 45 units.



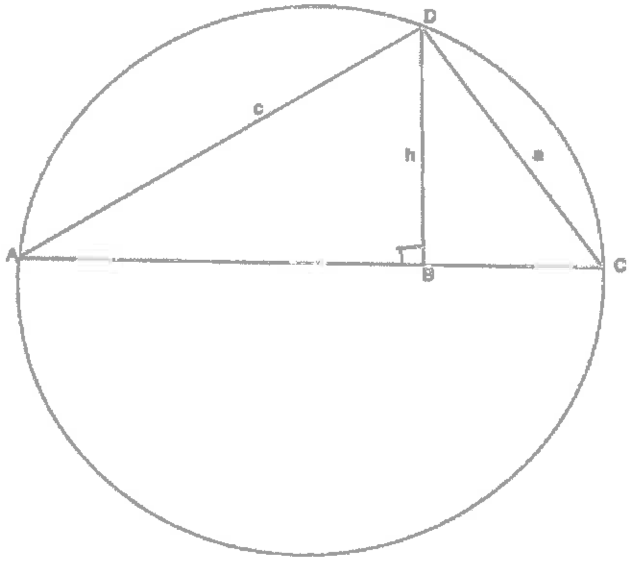
1) Calculate with reasons $\frac{FG}{DE}$ $= \frac{1}{3}$ ✓✓ (4)

A · common
$H\hat{F}G = \hat{D}$ (corr ∠s $GF \parallel DE$)
$\hat{G} = \hat{E}$ (3^{rd} ∠ of Δ) ✓✓
∴ ΔHFG ||| HDE (AAA)

2) Calculate FH (4)

$G\hat{F}E = F\hat{E}D$ (Alt ∠s $GF \parallel ED$) ✓
∴ $F\hat{E}D = \hat{D}$
$FD = 45$ ✓
$\frac{HF}{FD} = \frac{21}{42}$ ✓
∴ $HF = \frac{1}{2}(45) = 22{,}5$ ✓

b) In the figure, AC is a diameter of circle ADC. $DB \perp AC$. In the diagram below $AC = d$, $AD = c$ and $DC = a$ and $DB = h$.



Prove that $h = \dfrac{ac}{d}$ (5)

$\Delta ADB \mathbin{|||} \Delta DCB \mathbin{|||} \Delta ACD$ ✓✓

$\dfrac{DB}{AD} = \dfrac{CD}{AC}$ ✓

$\dfrac{h}{c} = \dfrac{a}{d}$ ✓

$h = \dfrac{ac}{d}$ ✓

Area $\Delta ADC = \frac{1}{2}ac$

Area $\Delta ADC = \frac{1}{2}h(AB) + \frac{1}{2}hBC$

$= \frac{1}{2}h(AB + BC)$

$= \frac{1}{2}hd$

$\therefore \frac{1}{2}ac = \frac{1}{2}hd$

$\dfrac{ac}{d} = h$

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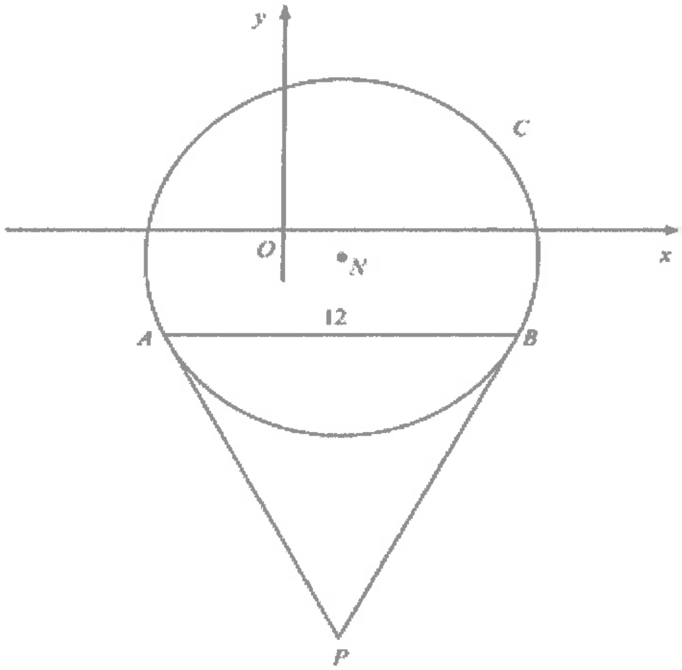
QUESTION 9:

The diagram below shows a sketch of the circle C with centre N and equation

$$(x-2)^2+(y+1)^2=\frac{169}{4}$$

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12.



a) Write down the coordinates of N and radius of circle C (3)

$N(2;-1)$ ✓✓ $r=\frac{13}{2}$ ✓

b) Find the coordinates of A and B. (5)

N to $AB = \sqrt{\left(\frac{13}{2}\right)^2-6^2} = \frac{5}{2}$ ✓

$\therefore AB:\ y=-\frac{7}{2}$ ✓

$(x-2)^2+\left(-\frac{7}{2}+1\right)^2=\frac{169}{4}$ ✓

$x-2=\pm 6$

$x=8$ or -4

$A\left(-4;-\frac{7}{2}\right)$ ✓ $B\left(8;-\frac{7}{2}\right)$ ✓

c) Calculate the angle ANB. (2)

$\cos A\hat{B}N = 6/\tfrac{13}{2}$

$A\hat{B}N = 22{,}6$

$\therefore A\hat{N}B = 134{,}8$

d) The tangents at the points A and B meet at the point P. Find the length AP. (3)

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$P\hat{A}B = 90 - 22{,}6 = 67{,}4$

$\therefore \hat{P} = 45{,}2$

$\frac{AP}{\sin 67{,}4} = \frac{12}{\sin 45{,}2}$

$AP = 15{,}6$

OR

$\frac{AP}{\sin 90} = \frac{6}{\sin 22{,}62}$

$AP = 15{,}6$

$A\hat{P}B = 45{,}24$

OR $m_{BN} = -5/12$ Equation BP : $y = 12/5\,x + c$ $(8 ; -7/2)$

$y = 12/5\,x - \frac{227}{10}$

$m_{AN} = 5/12$ $(-4 ; -7/2)$

$y = -12/5\,x + c$

$y = -12/5\,x - \frac{131}{10}$

$\therefore -12/5\,x - \frac{131}{10} = 12/5\,x - \frac{227}{10}$

$x = 2$

$\therefore y = -\frac{179}{10}$

$AP = \sqrt{(2+4)^2 + \left(-\frac{179}{10} + \frac{7}{2}\right)^2}$

$= 15{,}6$