

PRELIMINARY EXAMINATION 2016

MATHEMATICS GRADE 12

PAPER 1

Time: 3 hours

Total: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages, graph paper, and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

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SECTION A

QUESTION 1:

a) Solve for x : (show all working)

1) $2 = mx$ (2)

2) $-3(x+1)(x-3) < 0$ (3)

3) $x + 5 = \sqrt{3 - 3x}$ (5)

4) $3^{x^2-1} = \frac{27^{-x}}{3}$ (4)

b) Simplify: $\frac{4 \cdot 3^{a+2} - 3 \cdot 3^a}{5 \cdot 3^{a+1} - 3 \cdot 3^a}$ (4)

[18]

QUESTION 2:

The first term of a quadratic sequence is 4, the third term is 34 and the common second difference is 10.

a) Determine the 4th term of the sequence. (4)

b) Hence, determine an expression for the n th term of the quadratic sequence. (4)

c) Write down the minimum value of the sequence (2)

[10]

QUESTION 3:

a) Determine $f'(x)$ from first principles if $f(x) = \frac{-1}{2x}$. (5)

b) Determine $\frac{dy}{dx}$ in each of the following (leave your answer with positive exponents):

1) $y = x^{\frac{1}{2}} - \theta$ (2)

2) $y = \sqrt[3]{x} + \frac{2}{5x}$ (4)

[11]

QUESTION 4:

- a) Given the functions $f(x) = 3x - 7$ and $g(x) = \frac{12}{x-2}$
- 1) Find $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x , stating all restrictions. (5)
 - 2) Sketch the graphs of $g^{-1}(x)$ and $f^{-1}(x)$ on the same set of axis, giving points of intersection of each graph with the axes. (5)
 - 3) Write down the values of x for which $g^{-1}(x) > f^{-1}(x)$. (3)
- b) Given $t(x) = \log_{\frac{1}{2}}(2x-1)$. Write down the domain of $t(x)$. (2)

[15]

QUESTION 5:

Given: $f(x) = 2(x-2)^2 - 2$

- a) Write down the co-ordinates of the turning point of the curve of $f(x)$ (2)
- b) Determine all the intercepts of $f(x)$. (4)
- c) Draw a sketch graph of $f(x)$. Clearly indicate the co-ordinates of the turning point as well as the intercepts with the axes. (4)
- d) Use your graph to solve for k if $2x^2 - 8x + 6 = k$ has one positive and one negative root. (2)

[12]

QUESTION 6:

At an athletic event, athletes are tested for steroids using two different tests. The first test has a 93.0% probability of giving accurate results, while the second test is accurate 87% of the time. For a sample that does contain steroids, what is the probability that:

- a) neither test shows that steroids are present? (3)
- b) both tests show that steroids are present? (3)
- c) at least one of the tests detects the steroids? (3)

[9]

SECTION B

QUESTION 7:

Among the seven nominees for two vacancies on the city council are three men and four women. In how many ways may these vacancies be filled.

- a) with any two of the nominees? (3)
- b) with any two of the women? (3)
- c) with one of the men and one of the women (3)

[9]

QUESTION 8:

- a) Mark deposits R500 into a savings account, which earns interest at 6.81% per annum compounded quarterly. How long will it take for the savings account to have a balance of R749.77 (4)
- b) Nicky has been working for 5 years and gets an increase in her salary. She opens a savings account at FNB bank and begins with deposits of R350 every month. The account earns 5.53% interest per annum compounded monthly. Her plan is to continue saving on a monthly schedule until she retires. However, after 8 years she stops making the monthly payments and leaves the account to continue growing.
 - 1) How much money will Nicky have in her account 29 years after she has opened it? (6)
 - 2) Calculate the difference between total deposits made into the account and the amount of interest paid by the bank. (3)
- c) Thabo invests R8 500 in a special banking product which will pay 1% per month for 1 month, then 2% per month for the next 2 months, then 3% per month for the next 3 months, 4% per month for the next 4 months, and 0% for the rest of the year. How much can he expect to get back at the end of the year? (6)

[19]

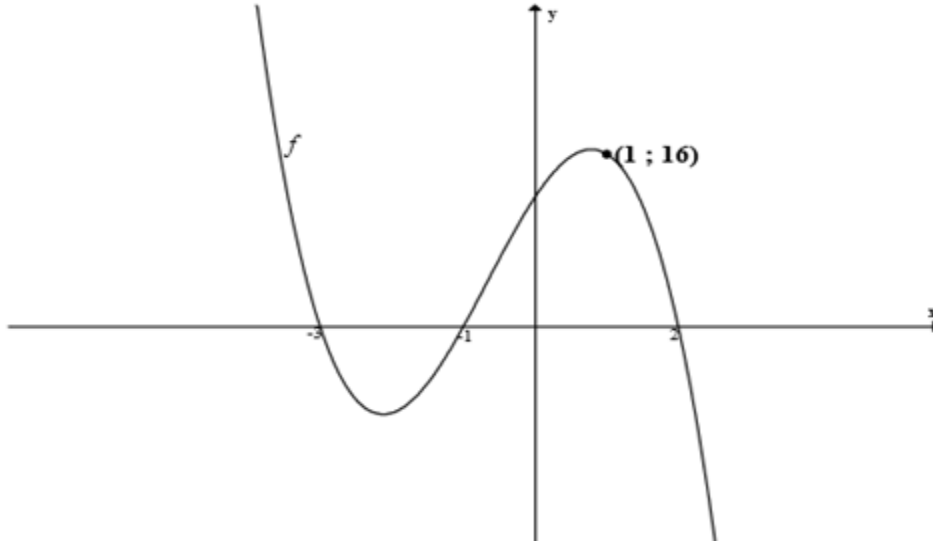
QUESTION 9:

- a) Solve for x if: $199+195+191+\dots+7+3=197+193+189+\dots+5+1+x$ (4)
- b) A geometric series has a second term of 4 and the sum to infinity of the series is 25. Show that $25r^2 - 25r + 4 = 0$. (4)
- c) The r^{th} term of an arithmetic series is $(2r - 5)$. Show that $\sum_{r=1}^n (2r - 5) = n(n - 4)$. (4)

[12]

QUESTION 10:

If $f(x) = ax^3 + bx^2 + cx + d$. $(1; 16)$ is the point on the graph of $f(x)$ and the graph cuts the x -axis at $x = -3, x = -1$ and $x = 2$.



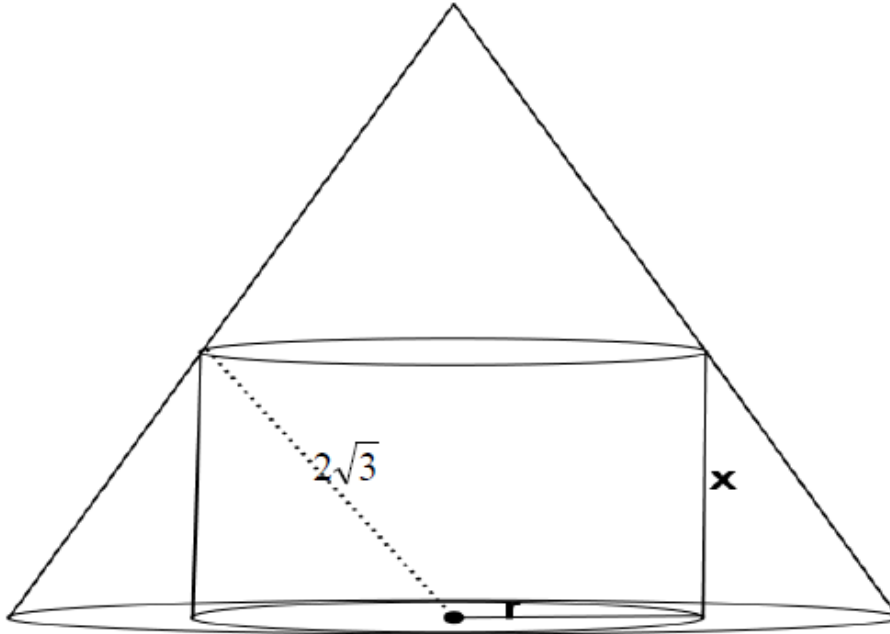
- a) Determine the values of a, b, c and d . (4)
- b) Determine the coordinates of the turning points of $f(x)$. (4)
- c) For which values of x is $f(x)$ concave up? (3)
- d) Determine the value(s) of k such that $20x + y - k = 0$ is a tangent to the graph of $f(x)$. (6)
- e) For which values of x is $x \cdot f''(x) < 0$? (2)

[19]

QUESTION 11:

A cylinder fits in a cone as shown in the diagram below. The length from the rim of the cylinder to the center of the base is $2\sqrt{3}$.

The radius of the cylinder is r and its height is x .

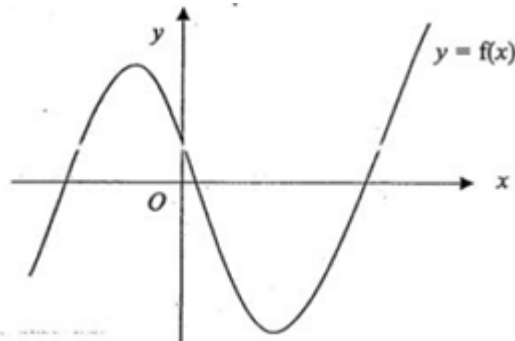


- a) Show that $r^2 = 12 - x^2$ (2)
- b) Write down the expression for the volume of the cylinder in terms of x . (3)
- c) Calculate the value of x that will maximise the volume of the cylinder. (4)
- d) Calculate the radius of the cylinder if it is of maximum volume. (3)

[12]

QUESTION 12:

The graph of $y = f(x)$ is shown in the diagram below.



It is given that the roots of the equation $f(x) = 1$ are -2 , 0 and 4 . If $f(a^2) > 1$, find the range of values of a . [4]