QUESTION 1

Consider the points $A\ (-1;1)$; $B\ (p;p^2)$ and $C\ (1;2p-1)$.
Show that these 3 points are collinear. **(3)**

$m_{AB} = \frac{p^2-1}{p+1}$ ✓M

$= \frac{(p+1)(p-1)}{(p+1)}$

$= p-1$ ✓a

$m_{BC} = \frac{p^2-2p+1}{p-1}$

$= \frac{(p-1)^2}{(p-1)}$

$= p-1$ ✓a

$\therefore$ A, B and C are collinear

QUESTION 2

A circle with equation $x^2+y^2-4x+6y-23=0$ is given.
Determine the centre of the circle and radius. **(4)**

$(x^2-4x+4)+(y^2+6y+9) = 23+13$ ✓M

$\therefore (x-2)^2+(y+3)^2 = 36$

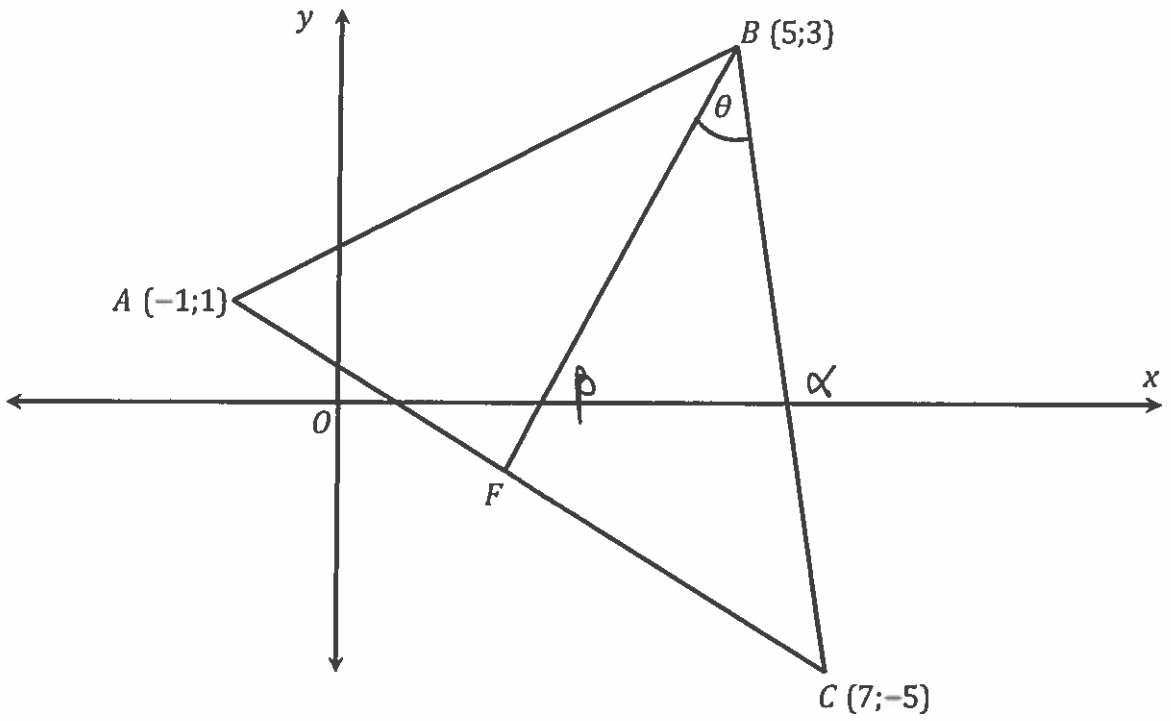
C $(2;-3)$ ✓a ✓a

$r = 6$ ✓a

QUESTION 3

Refer to the figure below:

A (−1;1); B (5;3) and C (7;−5) are points on the Cartesian plane.



(a) Calculate the coordinates of F, the midpoint of AC. **(2)**

$$F\left(\frac{-1+7}{2}\ ;\ \frac{1-5}{2}\right)$$

$$= (3\ ;\ -2)$$

(b) Determine the equation of the median BF of ΔABC. (The median is a line from the vertex to the midpoint of the opposite side.) **(4)**

$$m_{BF} = \frac{3+2}{5-3}$$

$$= \frac{5}{2} \quad \checkmark a$$

$$y - 3 = \frac{5}{2}(x - 5) \quad \checkmark M$$

$$\therefore y - 3 = \frac{5}{2}x - \frac{25}{2}$$

$$\therefore y = \frac{5}{2}x - \frac{19}{2} \quad \checkmark ca \quad \checkmark ca$$

(c) Calculate the size of the acute angle $F\hat{B}C$, shown by θ on the figure. Give your answer rounded off to the first decimal place. **(6)**

$$\tan \beta = \frac{5}{2}$$

$$\therefore \beta = 68{,}2^\circ \quad \checkmark ca$$

$$m_{BC} = \frac{3+5}{5-7} \quad \checkmark M$$

$$= -4$$

$$\tan \alpha = -4 \quad \checkmark M$$

$$\therefore \alpha = 104^\circ \quad \checkmark ca$$

$$\alpha = \theta + \beta$$

$$\therefore 104^\circ = \theta + 68{,}2^\circ \quad \checkmark M$$

$$\therefore 35{,}8^\circ = \theta \quad \checkmark ca$$

QUESTION 4

Simplify without using a calculator: $\dfrac{\sin 140^\circ \sin 120^\circ}{\sin 110^\circ \sin 340^\circ}$ **(7)**

$= \dfrac{\sin 40^\circ \times \sin 60^\circ}{\cos 20^\circ \times (-\sin 20^\circ)}$ ✓a ✓a ✓a ✓a

$= \dfrac{2\sin 20^\circ \cos 20^\circ \times \frac{\sqrt{3}}{2}}{-\sin 20^\circ \cos 20^\circ}$ ✓a ✓a

$= -\sqrt{3}$ ✓ca

QUESTION 5

(a) Complete the following: **(1)**

$\hat{A}+\hat{B}=180^\circ$

$\hat{B}-\hat{C}=90^\circ$

$\therefore \hat{A}+\hat{C}=?$

$(\hat{A}+\hat{B})-(\hat{B}-\hat{C}) = 180^\circ - 90^\circ$

$\therefore \quad \hat{A}+\hat{C} = 90^\circ$ ✓a

(b) Complete the following: **(1)**

$\hat{A}=2\hat{B}$

$\hat{B}+\dfrac{\hat{C}}{2}=45^\circ$

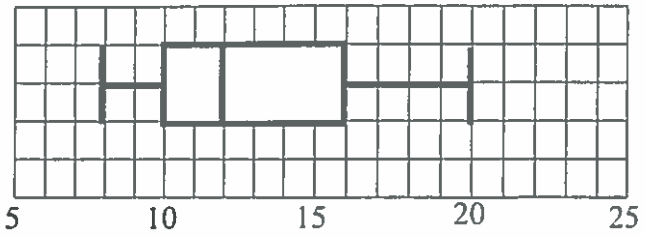
$\therefore \hat{A}+\hat{C}=?$

$\dfrac{\hat{A}}{2}+\dfrac{\hat{C}}{2} = 45^\circ$

$\therefore \quad \hat{A}+\hat{C} = 90^\circ$ ✓a

QUESTION 6

A set of data consisting of 5 numbers is represented by the box and whisker plot below.



(a) Write down the 5 numbers. **(5)**

min = 8 ✓a

Q_1 = 10 ✓a

Q_2 = 12 ✓a

Q_3 = 16 ✓a

max = 20 ✓a

(b) If 5 more numbers, 9; 11; 12; 15; 27 are added to the 5 original numbers, determine the five number summary for the new set of 10 numbers. **(3)**

8 ; 9 ; 10 ; 11 ; 12 ; 12 ; 15 ; 16 ; 20 ; 27 ✓m

min = 8

Q_1 = 10 ✓a

Q_2 = 12

Q_3 = 16

max = 27 ✓a

(c) Complete the following statements, using either MEAN, MEDIAN or MODE:

(i) The mean ✓a is more sensitive to extreme outlier data values. **(1)**

(ii) When data is negatively skewed, the median ✓a > mean ✓a. **(2)**

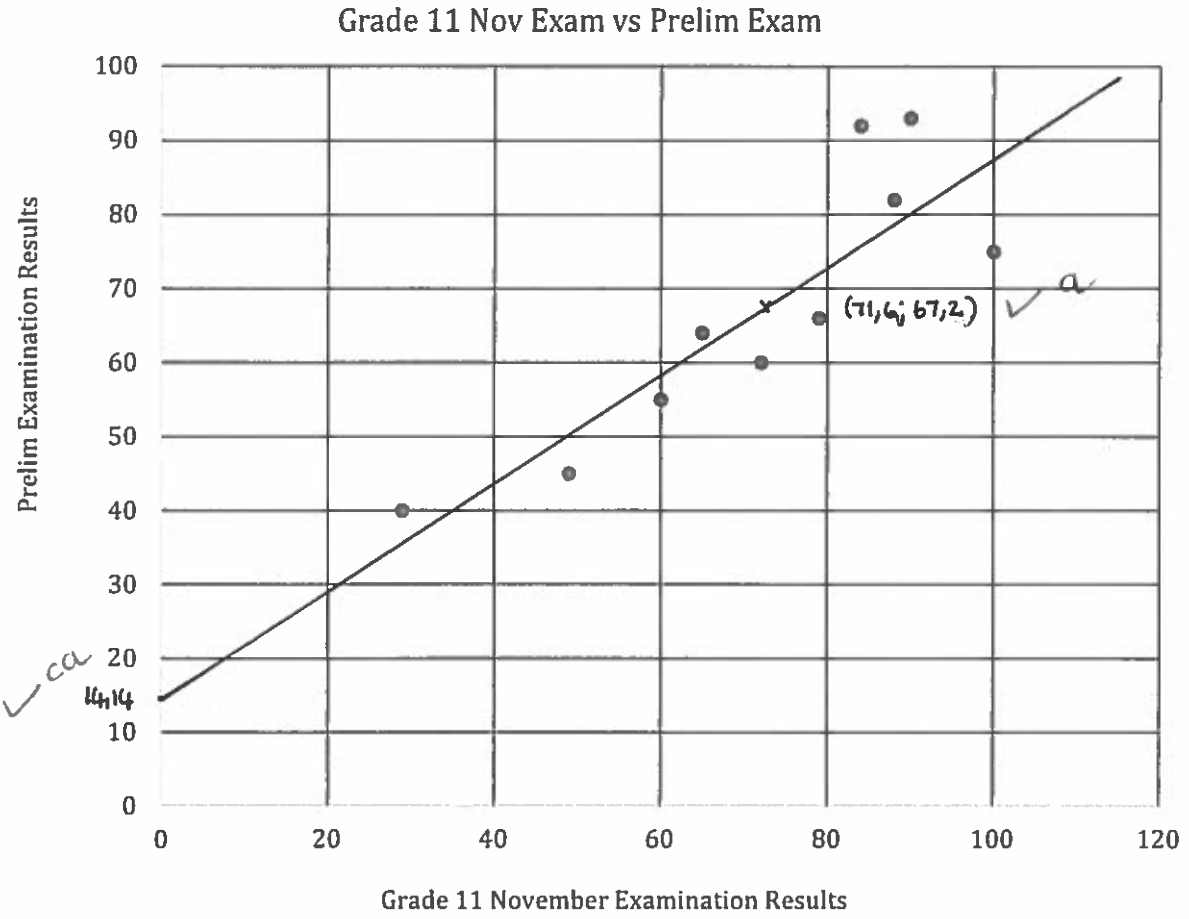
(iii) In a group of data exactly 50% of the data lie above the median ✓a. **(1)**

QUESTION 7

Sophia was unable to write the Preliminary Mathematics Paper 2 Examination. The universities required a mark that reflected her ability. The university asked the school to forecast what she would have gotten for her Preliminary Mathematics Paper 2 Examination based on what she achieved in her Grade 11 November Mathematics Paper 2 Examination. Unfortunately, the Mathematics department only has the data from a random sample of 10 students, so they know that Sophia's results are going to be unreliable (due to the small sample size).

These results are shown in the table below and plotted as a scatter plot:

Grade 11 Nov Paper 2	29	49	60	65	72	79	84	88	90	100
Prelim Paper 2	40	45	55	64	60	66	92	82	93	75



(a) Determine the equation of the regression line which models the linear relationship between the Grade 11 Exam and the Prelim Exam. You may use your calculator. Give all answers to 2 decimal places. **(2)**

$\hat{y} = 14,14 + 0,74\hat{x}$

(b) Draw the regression line for the data on the scatterplot, showing $(\overline{x};\overline{y})$. **(2)**

(c) Determine the correlation coefficient between the Grade 11 Exam and the Prelim exam results. **(1)**

$r = 0,87$ ✓a

(d) Use the correlation coefficient and your scatter plot to comment on the validity of using the Grade 11 results to predict the prelim exam results. **(2)**

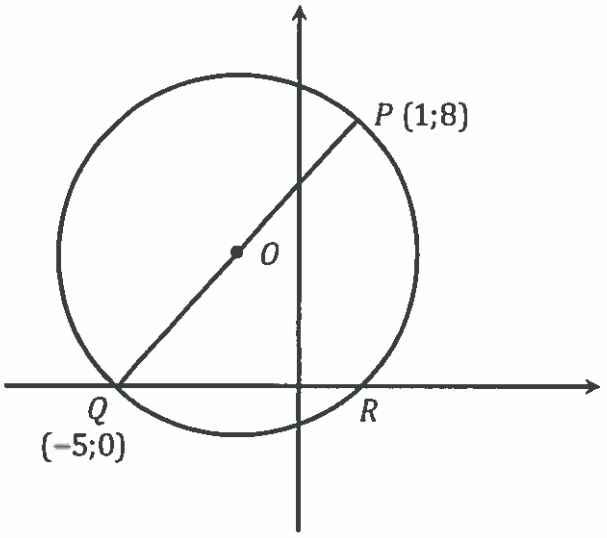
Strong positive ✓a correlation - 87 % chance to predict correctly ✓ca

(e) Use the equation of the regression line to estimate Sophia's Prelim Paper 2 mark, if she got 75% in the Grade 11 Paper 2 Exam. **(2)**

$\hat{y} = 14,14 + 0,74(75)$ ✓m

$= 69,64$ ✓ca

QUESTION 8



Determine:

(a) The equation of circle *PQR*. **(5)**

$O(-2\ ;\ 4)$ ✓a ✓a

$PO^2 = (1+2)^2 + (8-4)^2$ ✓m

$= 9 + 16$

$= 25$ ✓ca

$(x+2)^2 + (y-4)^2 = 25$ ✓ca

(b) The co-ordinates of *R*. **(3)**

X-intercept: $(y=0)$ ✓m

$(x+2)^2 + (0-4)^2 = 25$

$\therefore (x+2)^2 = 9$

$\therefore x+2 = \pm 3$

$\therefore x = -2 \pm 3$ ✓a

$R(1\ ;\ 0)$ ✓a

(c) The equation of the tangent to the circle at R. **(4)**

$m_{OR} = \frac{4-0}{-2-1} = -\frac{4}{3}$ ✓ a

$m_{\perp} = \frac{3}{4}$ ✓ ca

$y - 0 = \frac{3}{4}(x-1)$ ✓ m

$\therefore \quad y = \frac{3}{4}x - \frac{3}{4}$ ✓ ca

(d) Another circle $(x-10)^2+(y-4)^2=49$ is given. Determine whether the circles touch or intersect each other. **(3)**

$C_2\ (10;4)$ $\qquad r_2 = 7$

$C_1\ (-2;4)$ $\qquad r_1 = 5$

$d_{C_1C_2} = 12$ ✓ a $\qquad r_1 + r_2 = 12$ ✓ a

$\therefore$ Circles touch each other ✓ a

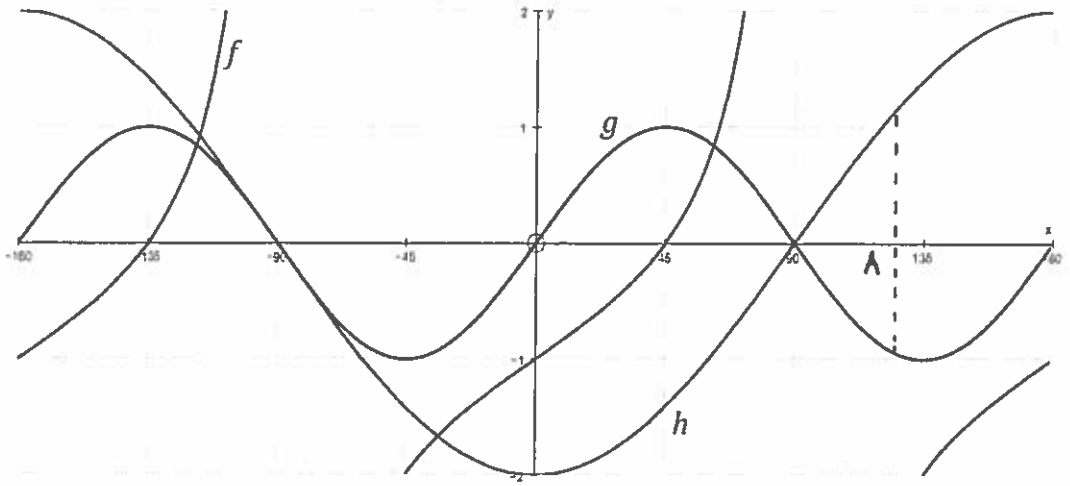
QUESTION 9

The below figure illustrates the graphs of the following functions, where $-180^\circ < x < 180^\circ$:

$f: x \to \tan x + a$

$g: x \to \sin bx$

$h: x \to c\cos x$



(a) Determine the values of a, b and c. **(3)**

$a = -1$ ✓a

$b = 2$ ✓a

$c = -2$ ✓a

(b) Solve the following equation using your graph.

$c\cos x - \sin bx = 2$ where $-180^\circ \leq x \leq 180^\circ$ **(2)**

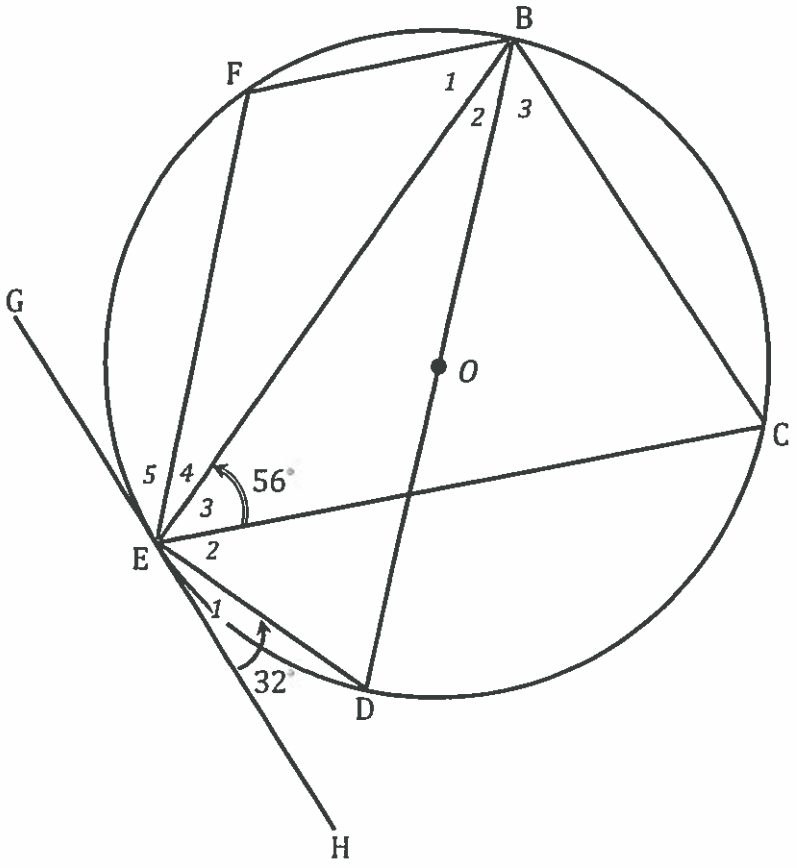
✓a ✓a

$x \in \{-180^\circ ; 180^\circ\}$

See A on graph

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn. $\hat{E}_1 = 32^\circ$ and $\hat{E}_3 = 56^\circ$



Calculate, with reasons, the value of:

(a) $\hat{E}_2$ = $90^\circ - 56^\circ$ ✓a ∠ in semi-O **(2)**

$\therefore \hat{E}_2 = 34^\circ$ ✓a

(b) $E\hat{B}C$ = $34^\circ + 32^\circ$ tan-chord ✓a **(2)**

$= 66^\circ$ ✓a

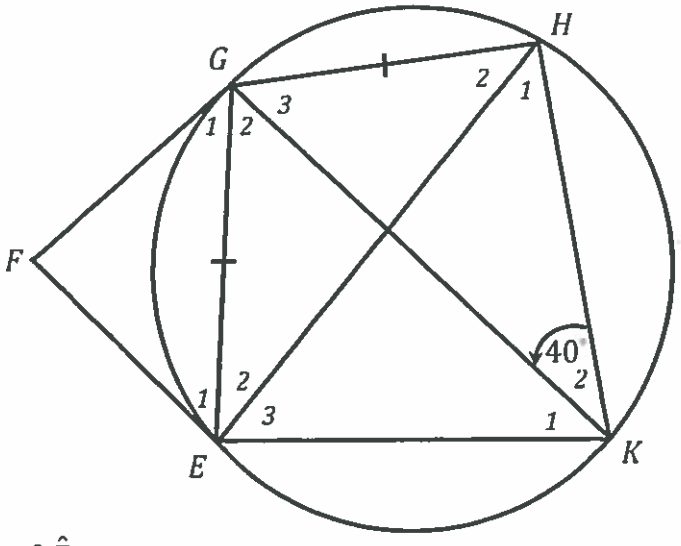
(c) $\hat{F}$ $\hat{C} = 58^\circ$ ✓a ∠'s of a Δ ✓a **(4)**

$\hat{F} = 122^\circ$ ✓a opp ∠'s, cyclic quad FECB ✓a

QUESTION 11

In the diagram:

$EG = GH$, the tangents at E and G meet at F, and $\hat{K}_2 = 40^\circ$.



Determine the size of $\hat{F}$. **(6)**

$\hat{E}_2 = 40^\circ$ L's in same segment ✓a

$\hat{H}_2 = 40^\circ$ L's opp equal sides ✓a

$\hat{E}_1 = 40^\circ$ tan_chord ✓a

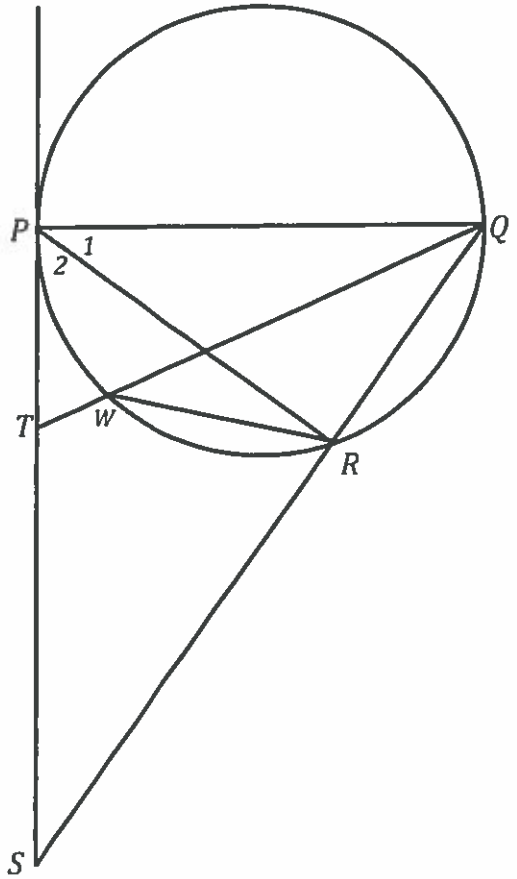
$EF = FG$ tans from the same pt. ✓a

$\hat{G}_1 = 40^\circ$ L's opp equal sides ✓a

$\hat{F} = 100^\circ$ L's of a Δ ✓a

QUESTION 12

In the figure, PQ is a diameter of circle $PWRQ$. SP is a tangent to the circle at P. QWT is a straight line with T on PS.



(a) Prove $\hat{P}_1 = \hat{S}$ **(4)**

$\hat{P}_1 + \hat{P}_2 = 90°$ rad $\perp$ tan ✓a

$P\hat{R}Q = 90°$ $\angle$ in semi-$\odot$ ✓a

$\hat{P}_2 + \hat{S} = 90°$ ext $\angle$ of $\triangle$ ✓a

$\therefore \hat{P}_1 = \hat{S}$ ✓a

(b) Prove that $SRWT$ is a cyclic quadrilateral. **(3)**

$\hat{P}_1 = Q\hat{W}R$ $\angle$'s in same segment ✓a

$\therefore \hat{S} = Q\hat{W}R$ ✓a

SRWT is a cyclic quad ext $\angle$ = opp int $\angle$ ✓a

(c) Prove that $\Delta QWR \;///\; \Delta QST$ **(3)**

In $\triangle QWR$ and $\triangle QST$:

(i) $\hat{Q} = \hat{Q}$ common ✓a

(ii) $Q\hat{W}R = \hat{S}$ proven above ✓a

(iii) $W\hat{R}Q = Q\hat{T}S$ ∠'s of a △ ✓a

$\triangle QWR \;|||\; \triangle QST$ AAA

(d) If $QW = 5cm$; $TW = 3cm$; $QR = 4cm$ and $WR = 2cm$, determine the length of TS. **(3)**

$$\frac{TS}{WR} = \frac{QT}{QR}$$ ✓a similar △'s

$$\therefore TS = \frac{8 \times 2}{4}$$ ✓a

$$= 4 \text{ cm}$$ ✓ca

QUESTION 13

(a) Prove the identity:

$$\frac{\cos x \tan^2 x}{\frac{1}{\cos x}+1} = 1-\cos x$$ **(5)**

$$\text{LHS} = \frac{\frac{\cos x}{1} \times \frac{\sin^2 x}{\cos^2 x}}{\frac{1+\cos x}{\cos x}} \quad \checkmark a \quad \checkmark a$$

$$= \frac{\frac{\sin^2 x}{\cos x}}{\frac{1+\cos x}{\cos x}} \times \frac{\cos x}{1+\cos x} \quad \checkmark m$$

$$= \frac{1-\cos^2 x}{(1+\cos x)} \quad \checkmark a$$

$$= \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)} \quad \checkmark a$$

$$= 1-\cos x$$

$$= \text{RHS}$$

(b) Hence, determine the maximum value of $\frac{\cos x \tan^2 x}{\frac{1}{\cos x}+1}$ **(1)**

$$\text{MAX value} = 1-(-1)$$

$$= 2 \quad \checkmark a$$

but $\frac{1}{\cos x}+1$

$$= \frac{1}{-1}+1$$

$$= 0$$

$\therefore$ NO maximum value

QUESTION 14

(a) Show that $\cos 6x + \cos 2x = 2\cos 4x \cos 2x$ **(4)**

M

$$\text{LHS} = \cos(4x + 2x) + \cos(4x - 2x)$$ a

$$= \cos 4x \cdot \cos 2x - \sin 4x \sin 2x + \cos 4x \cos 2x + \sin 4x \sin 2x$$ a

$$= 2\cos 4x \cos 2x$$ a

$$= \text{RHS}$$

(b) Hence, determine the general solution of: $\cos 6x + \cos 2x + \cos 4x = 0$ **(7)**

$$\therefore 2\cos 4x \cos 2x + \cos 4x = 0$$ ma

$$\therefore \cos 4x (2\cos 2x + 1) = 0$$ m

$\cos 4x = 0$ a OR $\cos 2x = -\frac{1}{2}$ a

$\therefore 4x = \pm 90^\circ + k360^\circ$ $\quad$ $\therefore 2x = \pm 120^\circ + k360^\circ$

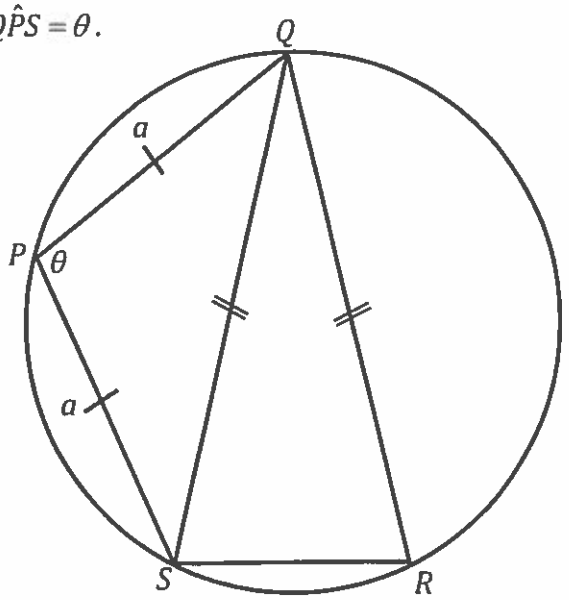
$\therefore x = \pm 22{,}5^\circ + k.90^\circ$ ca $\quad$ $\therefore x = \pm 60^\circ + k.180^\circ$, ca

$k \in \mathbb{Z}$ a

QUESTION 15

In the given figure, PQ and PS are equal chords of length a.

$QS = QR$ and $Q\hat{P}S = \theta$.



Show that:

(a) $QS = a\sqrt{2(1-\cos\theta)}$ **(3)**

$QS^2 = a^2 + a^2 - 2a^2\cos\theta$ ✓M

$= 2a^2 - 2a^2\cos\theta$

$= 2a^2(1 - \cos\theta)$ ✓a

$\therefore QS = a\sqrt{2(1-\cos\theta)}$ ✓a

(b) Area of $\Delta QRS = -a^2 \sin 2\theta(1-\cos\theta)$ **(8)**

$\hat{R} = 180° - \theta$ opp ∠'s, cyclic quad PQRS ✓a

$Q\hat{S}R = 180° - \theta$ ∠'s opp equal sides ✓a

$S\hat{Q}R = 180° - 2(180° - \theta)$ ∠'s of a Δ ✓a

$\therefore \quad SQR = 2\theta - 180°$ ✓a

Area $\Delta QRS = \frac{1}{2} \times QS \times QR \times \sin S\hat{Q}R$ ✓m

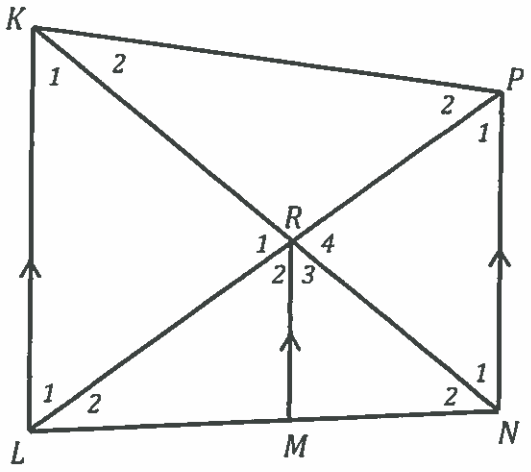
$= \frac{1}{2} \times 2a^2 (1 - \cos\theta) \times \sin(2\theta - 180°)$

$= a^2 (1 - \cos\theta) \times \sin(180° + 2\theta)$ ✓a ✓ca

$= -a^2 \sin 2\theta (1 - \cos\theta)$ ✓ca

QUESTION 16

In the figure below, MR bisects $L\hat{R}N$ with M on LN. LRP and NRK are straight lines and $LK \,//\, MR \,//\, NP$.



Prove that:

(a) $\hat{K}_1 = \hat{L}_1$ **(3)**

$\hat{K}_1 = \hat{R}_3$ corr $\angle$'s, $MR \,||\, LK$ ✓a

$\hat{L}_1 = \hat{R}_2$ alt $\angle$'s, $MR \,||\, LK$ ✓a

$\hat{R}_2 = \hat{R}_3$ given ✓a

$\therefore \hat{K}_1 = \hat{L}_1$

(b) $\frac{LR}{RN} = \frac{LM}{MN}$ **(4)**

$\frac{KR}{RN} = \frac{LM}{MN}$ ✓a line $||$ one side of $\triangle$ ✓a

but $KR = LR$ ✓a sides opp equal $\angle$'s ✓a

$\therefore \frac{LR}{RN} = \frac{LM}{MN}$

(b) *KLNP* is cyclic **(3)**

$\hat{L}_1 = \hat{P}_1$ ✓a alt L's, KL ∥ PN ✓a

$\therefore \hat{K}_1 = \hat{P}_1$

KLNP is cyclic ✓a L's in same segment equal

(c) ΔKLP /// ΔMRN **(3)**

In △ KLP and △ MRN :

(i) $\hat{P}_2 = \hat{N}_2$ ✓a L's in same segment

(ii) $\hat{N}_1 = \hat{R}_2$ alt L's, KL ∥ RM

$\hat{R}_2 = \hat{R}_3$ given

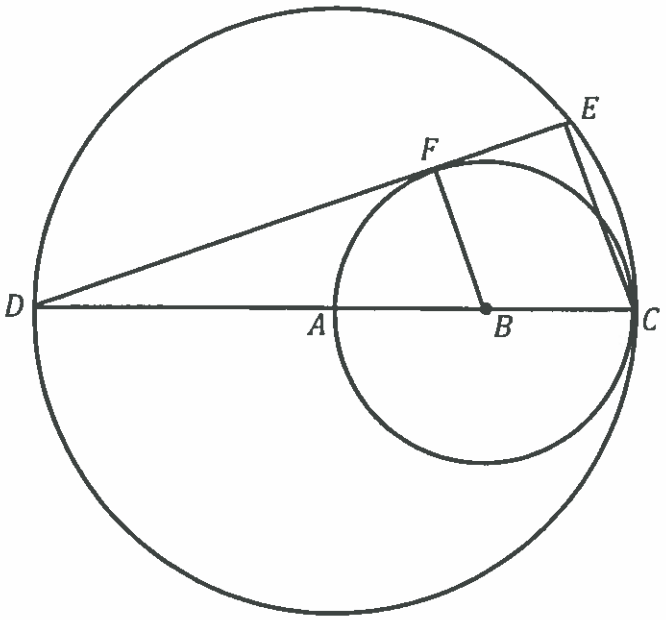
$\therefore \hat{L}_1 = \hat{R}_3$ ✓a

(iii) $L\hat{K}P = R\hat{M}N$ ✓a L's in a △

$\therefore$ △ KLP ||| △ MRN AAA

QUESTION 17

In the diagram DC is the diameter of the larger circle centred at A and AC is a diameter of the smaller circle centred at B. $DC = 12\,cm$. DE is a tangent to the smaller circle at F.



Determine the length of DE. **(7)**

$\hat{E} = 90°$ ∠ in semi-⊙ ✓a

$D\hat{F}B = 90°$ radius ⊥ tangent ✓a

$BF \parallel CE$ corr ∠'s equal. ✓a

$DF^2 = 9^2 - 3^2$ Pythagoras ✓a

$\therefore DF = 6\sqrt{2}$

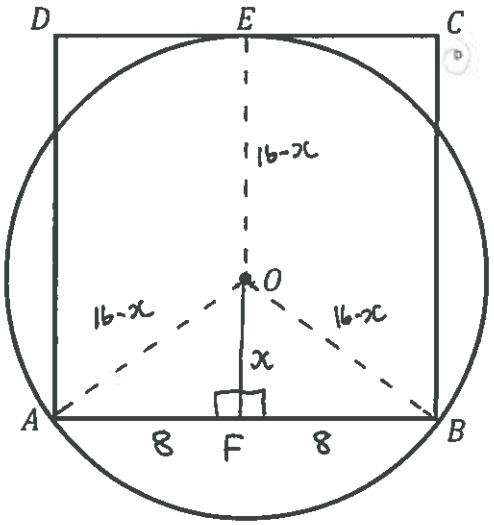
$\frac{DE}{DF} = \frac{DC}{DB}$ ✓a ✓a line ∥ one side of △

$\therefore DE = \frac{12 \times 6\sqrt{2}}{9}$

$= 8\sqrt{2}$ ✓ca

QUESTION 18

ABCD is a square with each side 16*cm* in length. Circle centre *O* passes through *A* and *B*, and touches *DC* at *E*.



Calculate the radius of the circle. **(6)**

Construction: Join O with A, B and E ✓M

Draw line OF ⊥ AB

AF = FB = 8 line ⊥ from centre to chord ✓M

Let OF = x

OE ⊥ DC radius ⊥ tangent ✓a

∴ AO = OB = OE = $16 - x$ radii ✓a

$(16 - x)^2 = x^2 + 8^2$ Pythagoras ✓M

∴ $256 - 32x + x^2 = x^2 + 64$

∴ $192 = 32x$

∴ $6 = x$

∴ Radius = 10 cm ✓ca