Section A.

1a. 1. $x - 4x^3 = 0$

$x(1-4x^2) = 0$

$x(1-2x)(1+2x) = 0$

$\therefore x = 0$ or $x = \frac{1}{2}$ or $x = -\frac{1}{2}$ (3)

2. $8^{2x+3} = \frac{1}{2}$

$(2^3)^{2x+3} = 2^{-1}$

$6x + 9 = -1$

$6x = -10$

$x = -10/6$

$= -5/3$ (3)

3. $2 + \sqrt{5x-4} = x$

$\sqrt{5x-4} = x-2$

$5x - 4 = x^2 - 4x + 4$

$0 = x^2 - 9x + 8$

$= (x-1)(x-8)$

$\therefore x \cancel{=} 1$ or $x = 8$.

$x = 8$ only soln. (4)

b. $(k+3)x^2 + 6x + k = 5$

1. $a = k+3$ $b = 6$ $c = k-5$

$\Delta > 0$ for 2 unequal, real rts

$b^2 - 4ac > 0$

$(6)^2 - 4(k+3)(k-5) > 0$

$36 - 4(k^2 - 2k - 15) > 0$

$36 - 4k^2 + 8k + 60 > 0$

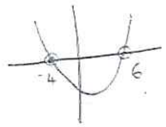
$-4k^2 + 8k + 96 > 0$

$(\div -4)$ $k^2 - 2k - 24 < 0$ (4)

2. $k^2 - 2k - 24 < 0$

$(k-6)(k+4) < 0$

crit val: $k = 6$ or $k = -4$



Soln: $-4 < x < 6$ (3)

2 a. $f(x) = \frac{2}{x-5} - 1$

1. $x = 5$; $y = -1$ (2)

2. $g(x) = \frac{2}{x-3} + 3$ (2)

3. $h(x) = -\frac{2}{x+5} - 1$

$= \frac{2}{-x-5} - 1$

$\therefore (x, y) \rightarrow (-x; y)$

$\therefore$ Reflection about y-axis $(x = 0)$. (2)

b. 1. $f(x) = k^x + q$

$(0;2)$: $f(0) = k^0 + q = 2$

$1 + q = 2$

$q = 1$

$\therefore f(x) = k^x + 1$

$(1;5)$: $f(1) = k^1 + 1 = 5$

$k = 4$ (3)

2. $g(x) = ax^2 + bx + c$

$g(x) = a(x-3)^2 + 0$

$(0;2)$: $2 = a(0-3)^2$

$2 = 9a$

$\therefore a = 2/9$ (3)

3. For some values of x there is more than 1 y-value. (1)

4. $x \geq 3$ or $x \leq 3$

$g: x \geq 3$; $y \geq 0$ | $g: x \leq 3$ $y \geq 0$

5. D: $x \geq 0$ or $x \geq 0$

R: $y \geq 3$ | $y \leq 3$ (2)

3. a. $A = P(1+i)^n$

$66611 = 45000(1 + \frac{x}{2})^{10}$

$\therefore 2\left(\sqrt[10]{\frac{66611}{45000}} - 1\right) = x$

$\therefore x = 0{,}080\ldots$

$\therefore$ IR $= 8{,}0\%$ (3)

b. $1 + i_e = \left(1 + \frac{i_{nom}}{m}\right)^m$

$1 + i_e = \left(1 + \frac{0{,}095}{4}\right)^4$

$i_e = 0{,}09843\ldots$

$\therefore$ E.I.R $= 9{,}8\%$ (2)

c. $80\,000 = 2000\left[\frac{(1+\frac{0.12}{12})^n - 1}{\frac{0.12}{12}}\right]$

$\left(\frac{80\,000}{2\,000}\right) \cdot \frac{0.12}{12} + 1 = (1{,}01)^n$

$1{,}4 = 1{,}01^n$

$\therefore \log_{1{,}01}(1{,}4) = n$

$n = 33{,}81$

$\therefore$ { 33 full payments were made and 1 payment < R2000.

for 34 payments (4)

4. a. $f(x) = 4x - x^2$

1. $f(x+h) = 4(x+h) - (x+h)^2$

$= 4x + 4h - x^2 - 2xh - h^2$

$\therefore f'(x) = \lim_{h \to 0} \frac{(4x+4h-x^2-2xh-h^2)-(4x \ldots}{h}$

$= \lim_{h \to 0} \frac{4h - 2xh - h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4 - 2x - h)}{\cancel{h}}$

$= 4 - 2x - 0$

$= 4 - 2x$ (4)

2. $f'(x) < 0$

$4 - 2x < 0$

$-2x < -4$

$x > 2$ (3)

b. 1. $y = (3x-2)^2$

$y = 9x^2 - 12x + 4$

$\therefore \frac{dy}{dx} = 18x - 12$ (3)

2. $y = \frac{x^2 + \sqrt{x}}{x}$

$= \frac{x^2}{x} + \frac{x^{\frac{1}{2}}}{x}$

$y = x + x^{-\frac{1}{2}}$

$\therefore \frac{dy}{dx} = 1 - \frac{1}{2x^{3/2}}$

or $1 - \frac{1}{2\sqrt{x^3}}$ (4)

5. a. $2 ; 12 ; x ; y ;$

1. $x = 22 ; \; y = 32$ (1)
2. $x = 72 ; \; y = 432$ (1)
3. $2 ; 12 ; x ; y$

10   12   14

2

$\therefore x = 24 ; \; y = 38$ (2)

b.

T_1 2000	T_2 2001	T_3 2002
1500	1600	1700

1. $a = 1500 \quad d = 100$

$T_n = a + (n-1)d$

$= 1500 + (n-1)(100)$

$T_n = 100n + 1400$ (2)

2. $2007 \rightarrow T_8$

$\therefore T_8 = 100(8) + 1400$

$= 2200$ (2)

3. $a = 1500 \qquad n = 14$

$S_{14} = \frac{14}{2}\left[2(1500) + 13(100)\right]$

$= 30\,100$ (3)

4.

T_1 2000	T_2 2001	T_3 2002
R9000	8800	8600

$\therefore a = 9000 \qquad d = -200$

$T_n = 9000 + (n-1)(-200)$

$= 9000 - 200n + 200$

$T_n = 9200 - 200n$ (Selling Price)

from (1) $T_n = 100n + 1400$ (No. of comps. sold)

$\therefore 9200 - 200n = 3(100n + 1400)$

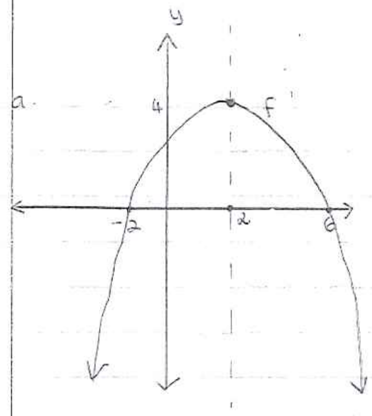
$9200 - 200n = 300n + 4200$

$5000 = 500n$

$\therefore n = 10$ (5)

$\therefore$ in 2009

6. a.



(4)

b. $\therefore y = a(x+2)(x-6)$

$(2;4)$: $4 = a(2+2)(2-6)$

$4 = -16a$

$-\frac{1}{4} = a$

$\therefore y = -\frac{1}{4}(x+2)(x-6)$

Let $x = 0$:

$y = -\frac{1}{4}(2)(-6)$

$= 3$ (4)

$\therefore$ y-int is 3

OR

$y = a(x-2)^2 + 4$

Sub: $(6;0)$: $0 = a(6-2)^2 + 4$

$0 = 16a + 4$

$-4 = 16a$

$-\frac{1}{4} = a$

$\therefore y = -\frac{1}{4}(x-2)^2 + 4$

Let $x = 0$: $y = -\frac{1}{4}(-2)^2 + 4$

$= -1 + 4$

$= 3$

$\therefore$ y-int is 3. (4)

SECTION B

7. $f(x) = 2x^3 + 5x^2 - 4x - 3$

a). $f(x) = (x-1)(2x^2 + 7x + 3)$

$f(x) = (x-1)(2x+1)(x+3)$

$\therefore$ x-ints : $(1;0)$; $(-\frac{1}{2};0)$; $(-3,0)$

y-int : $(0;-3)$ (4)

b) $f'(x) = 6x^2 + 10x - 4 = 0$

$3x^2 + 5x - 2 = 0$

$(3x-1)(x+2) = 0$

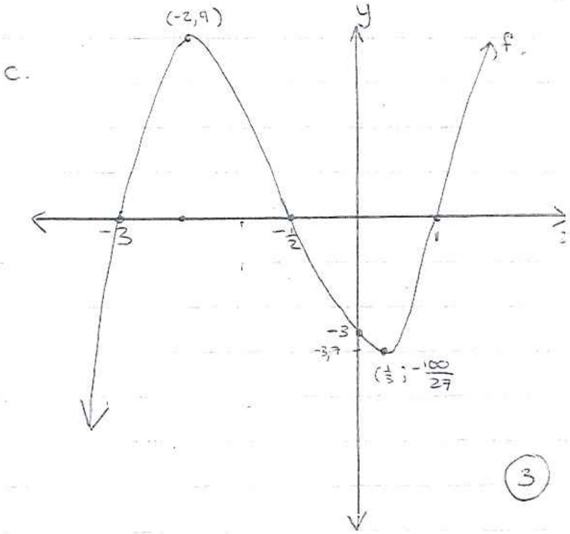
$\therefore x = \frac{1}{3}$ or $x = -2$

$f(\frac{1}{3}) = -\frac{100}{27}$ or $f(-2) = 9$

$\therefore$ S.P's: $(\frac{1}{3}, -\frac{100}{27})$ or $(-2;9)$

$\therefore$ local min.   local max (6)

c.



(3)

Section B

7d.1. $-\frac{100}{27} < k < 9$, $k \in \mathbb{R}$ (2)

2. $f'(x) \cdot f(x) < 0$ $(x \leq 0)$

grad.

slope $\oplus \times \ominus$ ← y-values

or $\ominus \times \oplus$ ← y-values

Soln: $x < -3$ or $-2 < x < -\frac{1}{2}$ (3)

e. $m = -4$

$\therefore f'(x) = -4$

$6x^2 + 10x - 4 = -4$

$6x^2 + 10x = 0$

$3x^2 + 5x = 0$

$x(3x+5) = 0$

$\therefore x = 0$ or $x = -5/3$

$\therefore$ dist $= 0 - (-5/3)$

$= 5/3$ or $1,7$ units apart (4)

8 a. $105\,000 = 0,15 \times$ S.P

SP $= 105\,000 \div 0,15$

$= R\,700\,000$ (1)

b. $595\,000 = x\left[\frac{1-\left(1+\frac{0,0975}{12}\right)^{-180}}{\frac{0.0975}{12}}\right]$ (3)

$x = R6303,21.$

OR $6303,21\left[\frac{1-\left(1+\frac{0,0975}{12}\right)^{-180}}{\frac{0.0975}{12}}\right] = $ S.P.

S.P $= R594\,980,38$

$\approx 700\,000 - 105\,000$.

c. O/B:

PV → 90 payments still to be pd.

$P_V = \frac{6303,21\left[1-\left(1+\frac{0,0975}{12}\right)^{-90}\right]}{\frac{0.0975}{12}}$

$= R401\,286,63$

or

FV:

OB $= 595\,000\left(1+\frac{0,0975}{12}\right)^{90} - \frac{6303,21\left[\left(1+\frac{0,0975}{12}\right)^{90}-1\right]}{\frac{0.0975}{12}}$

$= 1232569,996 - 831283,7906$

$= R401\,286,21$ (3)

d. Total payments $= 90 \times 6303,21$

$= R567\,288,90$ (2)

e. • After 90 months still owe R401 286,63.

$\therefore$ Loan reduced by

$595000 - 401\,286,63$

$= R193\,713,37$

Interest $= 567\,288,90 - 193\,713,37$

$= R373\,575.53$

$\therefore \% = \frac{373\,575.53}{567\,288.90} \times 100$

$= 65,9\,\%$ (3)

9.a. $a = 20$ $r = 7/8$

1.) $S_\infty = \frac{a}{1-r}$

$= \frac{20}{1-7/8}$

$= 160$ (2)

2). $S_{12} = \frac{a(1-r^n)}{1-r}$

$= \frac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-7/8}$

$= 127,8$ (2)

3) $S_\infty - S_n < 0,5$

$160 - \frac{20\left(1-\left(\frac{7}{8}\right)^n\right)}{\frac{1}{8}} < 0,5$

$20 - 20\left(1-\left(\frac{7}{8}\right)^n\right) < \frac{1}{16}$.

$20 - 20 + 20.\left(\frac{7}{8}\right)^n < \frac{1}{16}$

$\left(\frac{7}{8}\right)^n < \frac{1}{320}$

$\therefore \log\left(\frac{7}{8}\right)^n < \log\frac{1}{320}$

$n \underbrace{\log\frac{7}{8}}_{\text{negative}} < \log\frac{1}{320}$

$n > \frac{\log\frac{1}{320}}{\log\frac{7}{8}}$

$n > 43,2$

$\therefore n = 44$ (5)

b. $\sum_{t=2}^{22}(2t+5)$

$n = 22 - 2 + 1 = 21.$

$a = T_1 = 2(2)+5 = 9$

$T_2 = 2(3)+5 = 11$

$T_3 = 2(4)+5 = 13$

$\therefore d = 2.$

$T_{22} = 2(22)+5 = 49$

$\therefore S_{21} = \frac{21}{2}\left[9 + 49\right]$

$= 609$ (4)

c. $S_n = n^2 + n$

$S_6 = 6^2 + 6$ $S_5 = 5^2 + 5$

$= 42$ $= 30$

$\therefore T_6 = S_6 - S_5$

$= 42 - 30$

$= 12$ (3)

10.a)1. $P\left(x, \frac{4}{x}\right)$ (1

2. $OP^2 = (x-0)^2 + \left(\frac{4}{x}-0\right)^2$

$= x^2 + \frac{16}{x^2}$ (

3. $OP^2 = x^2 + 16x^{-2}$

$\frac{d(OP^2)}{dx} = 2x - \frac{32}{x^3} = 0$

$2x^4 - 32 = 0$

$x^4 - 16 = 0$

$x^4 = 16$

$x = \pm 2$ (4)

but $x > 0$, $\therefore x = 2$

$y = \frac{4}{2} = 2$

4. When $x=2$: $y = 4.x^{-1}$

$$\frac{dy}{dx} = -\frac{4}{x^2}$$

@ $x=2$: $m_1 = \frac{-4}{(2)^2}$

$= -1$

ie tangent to curve @ $x=2$ has a grad of -1.

The eqn of OP is $y=x$
(it passes through $(0,0)$ and $(2,2)$)

$\therefore m_2 = 1$.

$\therefore m_1 \times m_2 = -1 \times 1$

$= -1$

$\therefore$ OP $\perp$ tangent. (4)

b. $s(t) = 2t^2 - 18t + 45$

1) $s(0) = 2(0)^2 - 18(0) + 45$

$= 45$ m. (2)

2) $s'(t) = 4t - 18$ m/s

$s''(t) = 4$ m/s^2

(3)

11. a) $f(x+1) = f(x) + 3$

$f(1) = 2$

$f(2) = f(1+1) = f(1) + 3$

$= 2 + 3$

$= 5$

$f(3) = f(2+1) = f(2) + 3$

$= 5 + 3$

$= 8$

$T_1 \quad T_2 \quad T_3 \qquad T_{2016}$

$\therefore$ 2 ; 5 ; 8 ; . . .

$a = 2 \quad d = 3$

$T_{2016} = a + 2015d$

$= 2 + 2015(3)$

$= 6047$. (4)