ST BENEDICT'S COLLEGE

VERITAS IN CARITATE

SUBJECT	**Mathematics Paper 2**	**DATE**	22 July 2016
GRADE	12	**MARKS**	150
EXAMINER	Mr Benecke	**MODERATOR**	Gr 12 Educators
NAME	Memo	**DURATION**	3 Hours
CLASS			

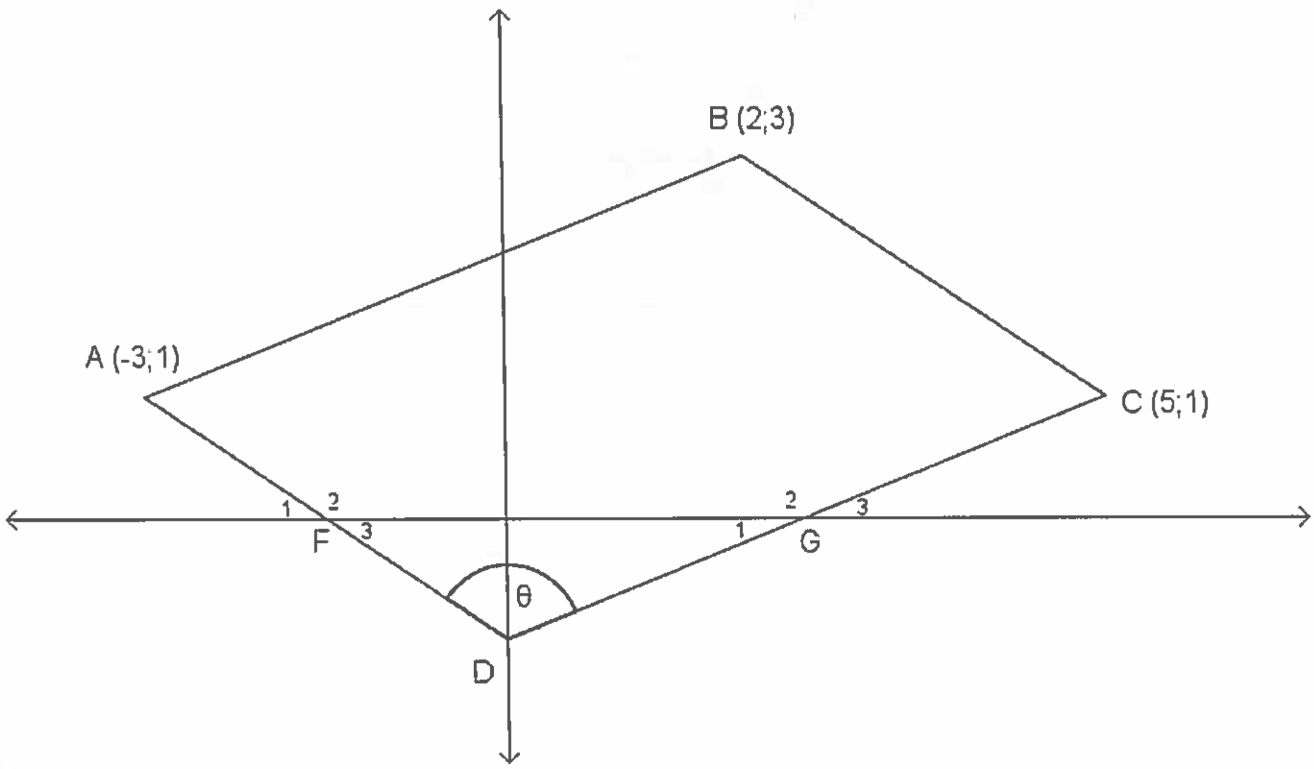
QUESTION NO.	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1 ; 2	Analytical Geometry	21	
3	Trigonometry	13	
4; 5; 6	Euclidean Geometry	20	
7; 8	Statistics	27	
9	Analytical Geometry	16	
10; 11	Trigonometry	36	Nathan 11
12	Euclidean Geometry	7	
13	Measurement	10	Nathan
	Total	150	

1. This paper consists of 13 questions and 25 pages.
2. Read the questions carefully.
3. Answer all questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated
5. Round off your answers to two decimal digits where necessary.
6. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks.
7. It is in your interest to write legibly and present your work neatly.

SECTION A

QUESTION 1 **14 MARKS**

In the diagram below, ABCD is a parallelogram given points A (3;1), B(2;3), and C(5;1) .



a) Prove that the equation of line AB is $y = \frac{2}{5}x + \frac{11}{5}$ 4

$y = mx + c$ $\quad m = \frac{3-1}{2+3} = \frac{2}{5}$

$y = \frac{2}{5}x + c$

$3 = \frac{2}{5}(2) + c$

$\frac{11}{5} = c$

$y = \frac{2}{5}x + \frac{11}{5}$

b) Find the coordinates of point D 2

D (0;-1)

c) Determine the coordinates of point M, the intersection of the diagonals of $ABCD$ 3

$M\left(\frac{-3+5}{2};\frac{1+1}{2}\right)$ ✓ ✓

$= M\left(\frac{2}{2};\frac{2}{2}\right)$

$= M(1;1)$ ✓

d) Calculate the value of θ if $D(0:-1)$ 5

$\tan \hat{F}_2 = grad_{AD}$ ✓

$\tan \hat{F}_2 = -\frac{2}{3}$

$\hat{F}_2 = -33,69° + 180°$

$\hat{F}_2 = 146,31°$

$\hat{F}_3 = 180 - 146,31° = 33,69°$ ✓

$\tan \hat{G}_3 = grad_{CD}$

$\tan \hat{G}_3 = \frac{2}{3}$

$\hat{G}_3 = 21,80°$ ✓

$\hat{G}_3 = \hat{G}_1 = 21,80°$ (Vert opp ∠'s)

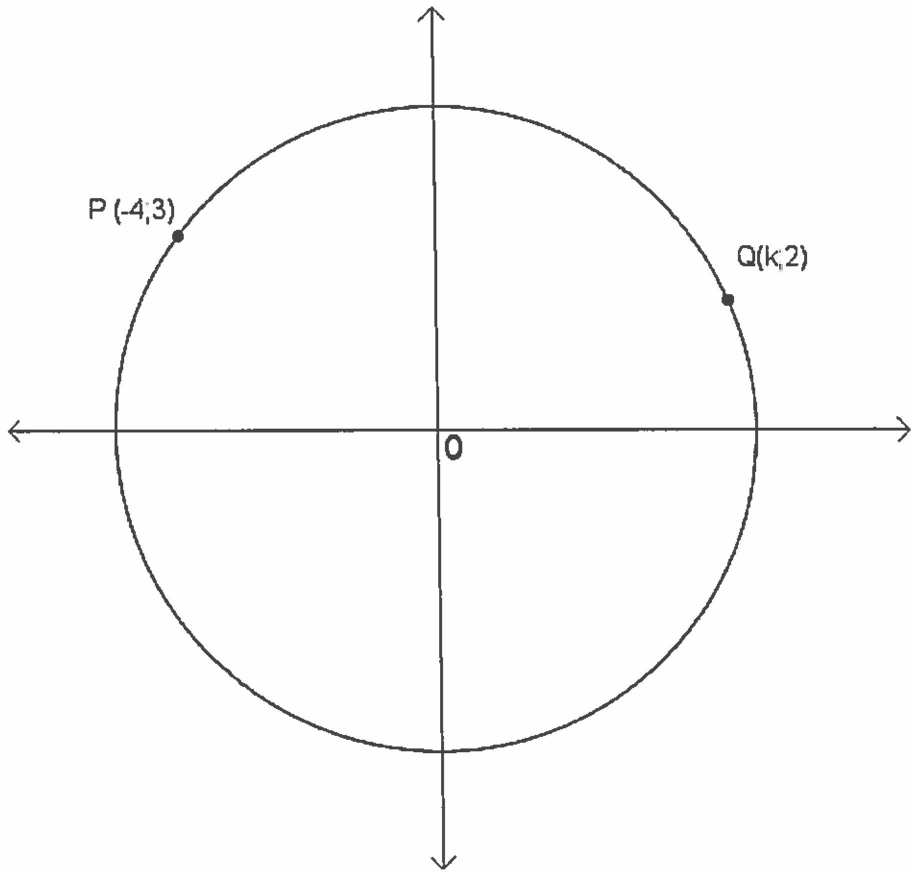
$\hat{F}_3 + \hat{G}_1 + \theta = 180°$ ✓ (∠'s in Δ add up to 180°)

$33,69° + 21,80° + \theta = 180°$

$\theta = 124,51°$ ✓

QUESTION 2 **7 MARKS**

In the diagram below, a circle centred at the origin is drawn. Points P(-4;3) and Q(k;2) lie on the circumference of the circle



a) Determine the equation of the circle. 2

$x^2+y^2=r^2$

$(3)^2+(-4)^2=r^2$

$9+16=r^2$

$r^2=25$ ✓

$x^2+y^2=25$ ✓

b) Determine the value of k 1

$x^2+y^2=25$

$k^2+2^2=25$

$k^2=21$

$k=\sqrt{21}$ ✓

c) If a tangent is drawn to the circle at point P, determine the equation of the tangent. 4

$m_{op} = -\frac{3}{4}$ ✓ so $m_{tangent} = \frac{4}{3}$ ✓

$y = \frac{4}{3}x + c$

$3 = \frac{4}{3}(-4) + c$ ✓

$3 = -\frac{16}{3} + c$

$c = \frac{25}{3}$

$y = \frac{4}{3}x + \frac{25}{3}$ ✓

QUESTION 3 – DO THIS QUESTION WITHOUT A CALCULATOR **13 MARKS**

a) Simplify the following equation as far as possible: 5

$$\frac{\sin(-\theta)\cos(\theta+180°)-\cos(90°+\theta)}{\sin(540°-\theta)}$$

$$= \frac{(-\sin\theta)(-\cos\theta) + \sin\theta}{\sin\theta}$$

$$= \frac{\sin\theta(\cos\theta+1)}{\sin\theta}$$

$$= \cos\theta + 1$$

b) Hence determine the value of $\frac{\sin(-\theta)\cos(\theta+180°)-\cos(90°+\theta)}{\sin(540°-\theta)}$ if $\theta = 60°$ 2

$\cos\theta + 1$

$= \cos 60 + 1$

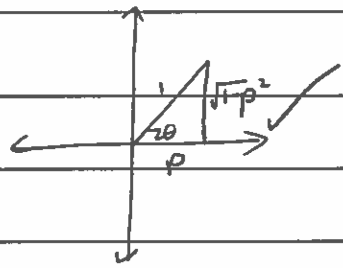
$= \frac{1}{2} + 1 = \frac{3}{2}$

c) If $\cos 2\theta = p$, determine the value of $2\sin\theta\cos\theta - \tan 2\theta$. (Hint: Draw a diagram to help you) 3

$2\sin\theta\cos\theta - \tan 2\theta$

$= \sin 2\theta - \tan 2\theta$

$= \sqrt{1-p^2} - \frac{\sqrt{1-p^2}}{p}$



d) Prove the following identity: $\frac{1}{\cos^2\theta} - \tan^2\theta = 1$ 3

$$LHS = \frac{1}{\cos^2\theta} - \tan^2\theta$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} \checkmark$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} \checkmark$$

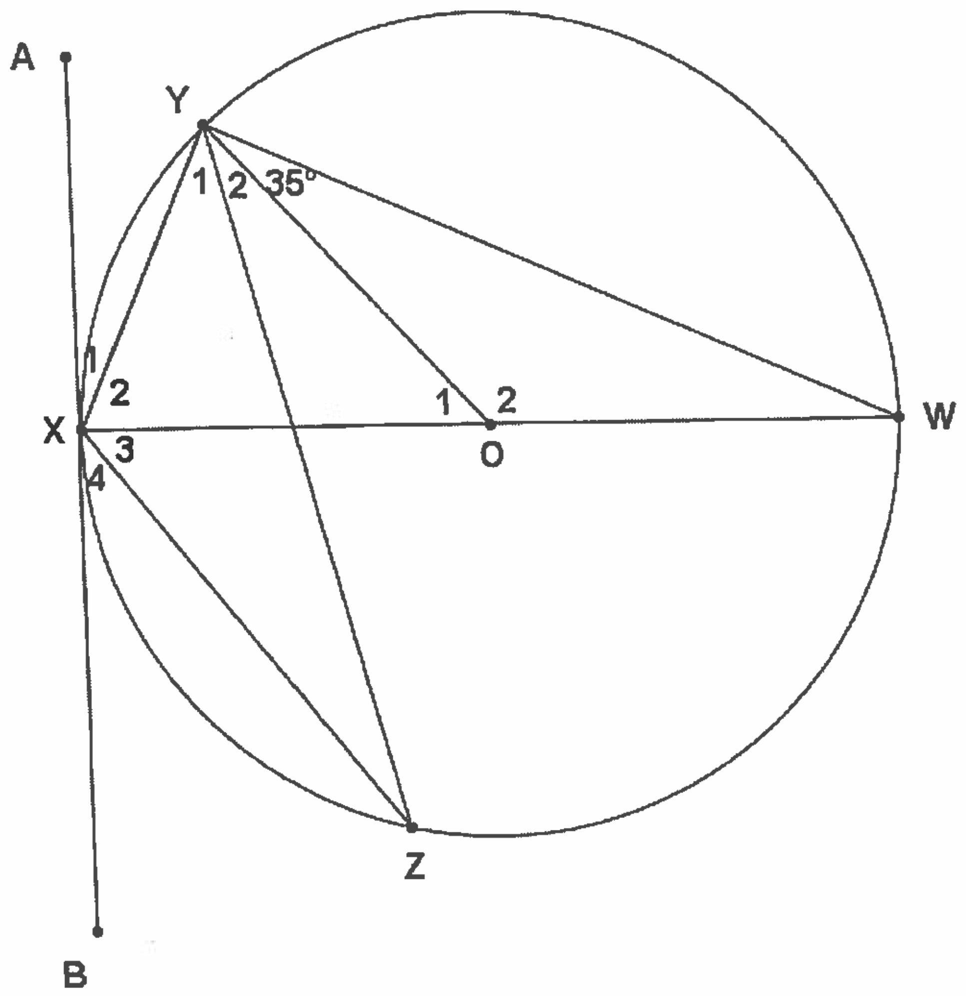
$$= \frac{\cos^2\theta}{\cos^2\theta} \checkmark$$

$$= 1$$

QUESTION 4 **9 MARKS**

In the diagram below, AB is a tangent to the circle (with centre O) at X.

W, X, Y and Z are points on the circle. $O\hat{Y}W = 35°$



a) Prove (with reasons) that $X\hat{Z}Y = 35°$ 3

OY = OW ✓	radii
$O\hat{Y}W = \hat{W} = 35°$ ✓	Isos Δ
$X\hat{Z}Y = \hat{W} = 35°$ ✓	∠'s in the same segment

b) If $\hat{X}_4 = 30°$, calculate (with reasons) the value of $\hat{Y}_2$ 3

$\hat{Y}_1 = 30°$ ✓	tan-chord
$\hat{Y}_1 + \hat{Y}_2 + 35° = 90°$ ✓	∠ in semi-circle
$\hat{Y}_2 = 25°$ ✓	

c) Hence calculate (with reasons) the value of $\hat{X}_3$ 3

$\hat{X}_2 + X\hat{Y}W + \hat{W} = 180°$ ✓	∠'s in △ = 180°
$\hat{X}_2 = 55°$	
$\hat{X}_1 = 35°$ ✓	tan-chord
$\hat{Y}_1 + \hat{X}_2 + \hat{X}_3 + \hat{X}_4 = 180°$ ✓	adj ∠'s on a str line
$\hat{X}_3 = 60°$	

Alternative solution:

$\hat{X}_3 = \hat{Y}_2 + 35°$ ✓ ∠'s in same segment

$\hat{X}_3 = 25° + 35°$ ✓

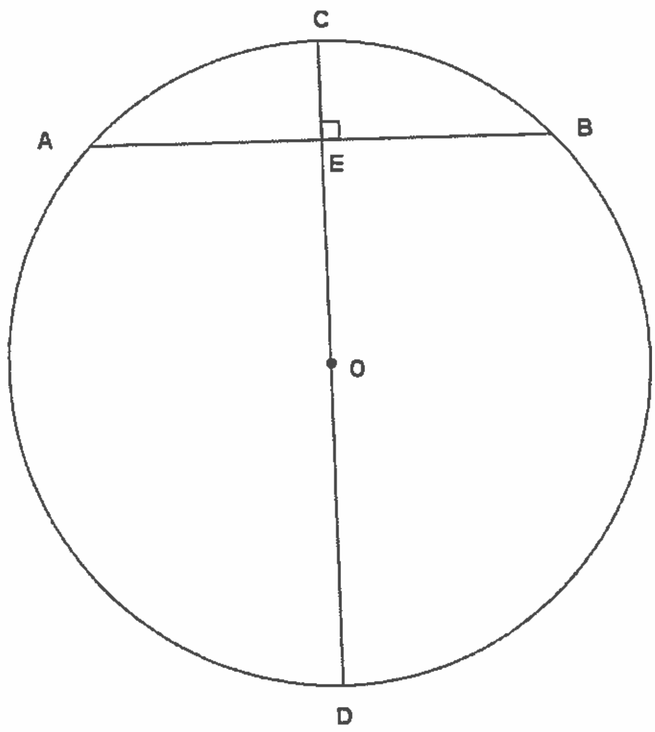
$= 60°$ ✓

QUESTION 5 **6 MARKS**

In the diagram below, O is the centre of the circle.

AB is perpendicular to CD.

$AB = 6\ units$ and $OE = 4\ units$



Determine the length of the following sides **without** giving reasons:

a) OB 2

EB = 3 ✓

∴ OB = 5 units ✓

b) OD 1

5 units ✓

c) CE 1

1 unit ✓

d) BD 2

$9^2 + 3^2 = BD^2$ ✓

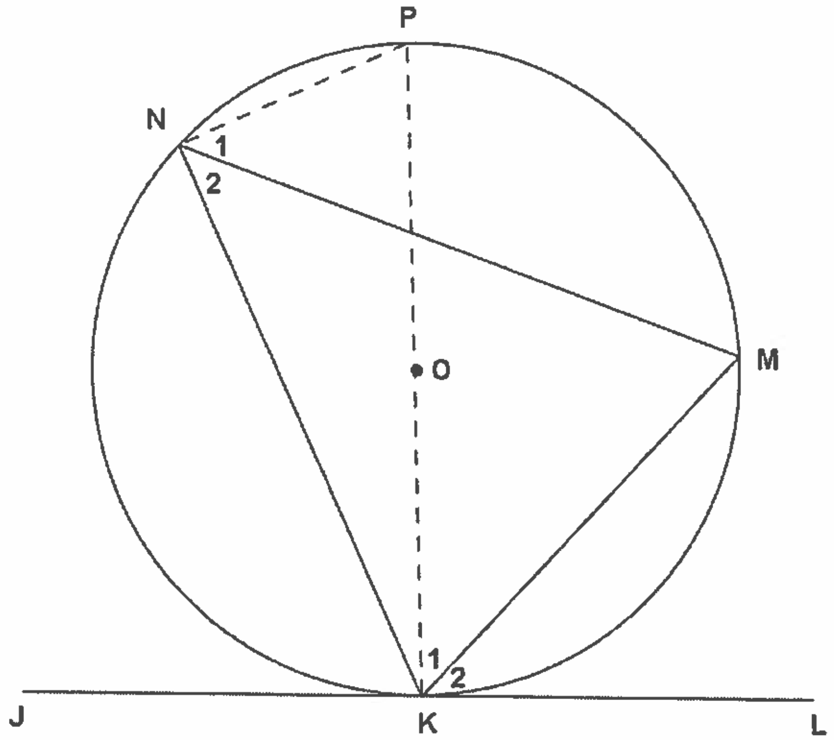
$81 + 9 = BD^2$

$BD = \sqrt{90}$ ✓

QUESTION 6 **5 MARKS**

Use the diagram below to prove the theorem that states that the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment

5



FILL IN THE BLANKS

Given: Tangent JKL

RTP: $\hat{K}_2 = \hat{N}_2$ (1) ✓

Statement		Reason
Draw diameter POK and join NP		Contruction
$\hat{K}_1 + \hat{K}_2 = 90°$ ✓	(1)	Tan ⊥ Radius
$\hat{N}_1 + \hat{N}_2 = 90°$ ✓	(1)	Angle in semi-circle = 90°
$\hat{K}_1 = \hat{N}_1$ ✓	(1)	Chord PM subtends equal angles
$\therefore \hat{K}_2 = \hat{N}_2$ ✓	(1)	
	~~(1)~~	

QUESTION 7 **18 MARKS**

The data in the table below shows the score (in percentage) of 10 Mathematics learners in their Grade 11 and Grade 12 final exam:

Grade 11	45	62	85	29	67	71	59	67	66	92
Grade 12	50	66	82	33	69	66	60	78	57	97

a) Determine the equation of the least squares regression line for this data. 2

$y = 0{,}94x + 5{,}69$ ✓ ✓

b) Calculate the correlation coefficient, what does this tell us about the relationship between Grade 11 final exam marks and Grade 12 final exam marks? 3

$r = 0{,}95$ ✓

Very strong positive linear correlation ✓✓

c) John achieved a mark of 25% for his grade 11 exam, predict his grade 12 mark. 2

$y = 0{,}94(25) + 5{,}69$ ✓

$y = 29{,}19$ ✓

d) Comment on the reliability of your answer in (c) 3

Eventhough we have a strong correlation ✓ the data needs to be extrapolated ✓ and therefore unreliable ✓

e) Determine the standard deviation of the grade 12 marks 2

$\sigma = \cancel{17{,}74}$ ✓✓

16,83

f) If the standard deviation of the grade 11 marks is calculated at 16,98. In which set of data (grade 11 or 12) is the mean more reliable? Motivate your answer. 2

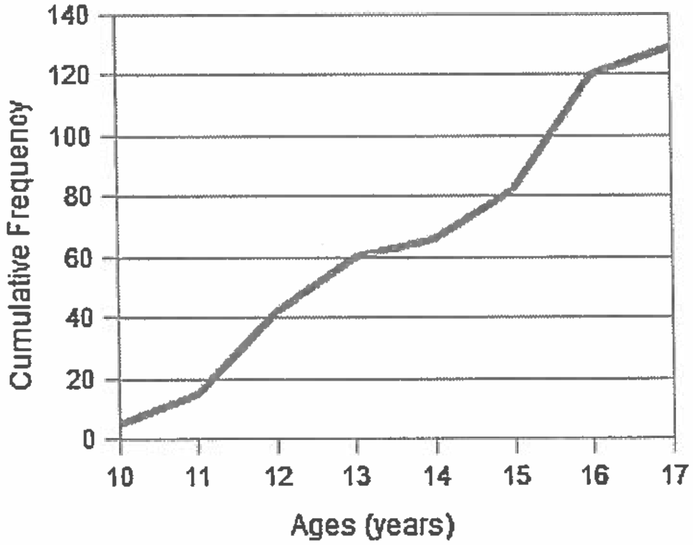
Grade ~~#~~ 12 ✓
Smaller σ means data less spread ✓

g) In the "Global Information Technology Report (2015)" South Africa scored the lowest marks in Mathematics worldwide. In light of this, if the grade 11 and 12 marks were collected from a cross-section of the population of South Africa, would you expect the data to be symmetrical or skewed? Motivate your answer. 4

Skewed to the right ✓✓
South African population is generally poor at
mathematics hence more low marks than
high marks ✓✓

QUESTION 8 **9 MARKS**

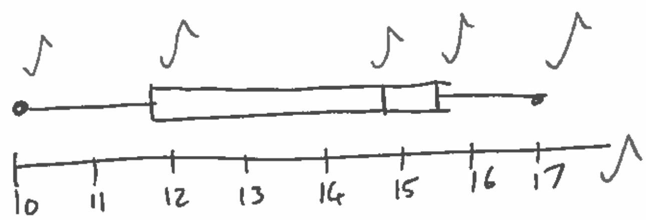
A certain statistician asked 129 boys at last year's Big Bennies Bash what their ages were. He drew the following ogive to represent the data he had collected:



a) Use the Ogive to estimate the values of Q_1, Q_2, and Q_3 3

$Q_1 = \pm 11{,}6$ ✓ $Q_2 = \pm 13{,}6$ ✓ $Q_3 = \pm 15{,}6$ ✓

b) Draw a box-and–whisker plot to represent the ogive above 3



c) How many boys were older than 15? 1

49 ✓

d) How many boys were between 12 and 16 years old 1

80 ✓

e) 20% of the boys were younger than x years old. Estimate the value of x. 1

$x = \pm 11{,}4$ ✓

SECTION B

QUESTION 9 **16 MARKS**

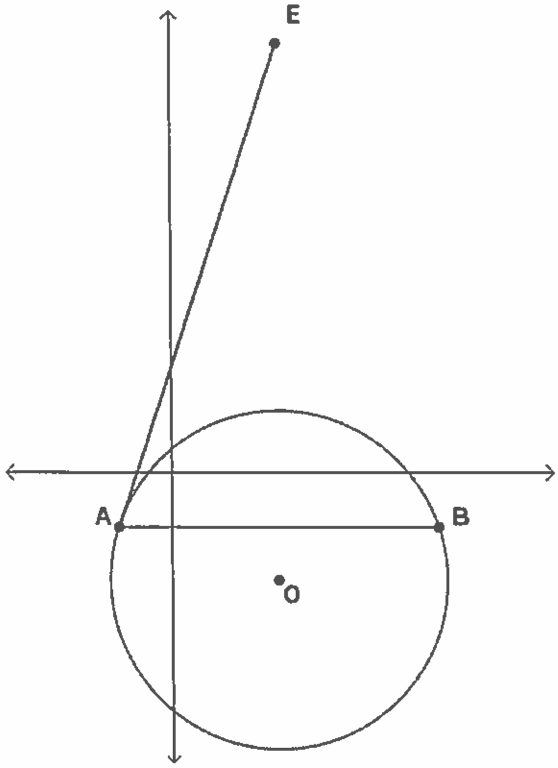
In the figure to the right, the circle with centre O is defined by:

$x^2 + y^2 = 9 + 4(x - y)$.

A and B are point on the circle, and the line AB is defined by:

$y = -1$.

AE is a tangent to the circle at A



a) Prove that the coordinates of the centre O are $(2; -2)$ **5**

$x^2 + y^2 = 9 + 4x - 4y$ ✓

$x^2 - 4x + 4 + y^2 + 4y + 4 = 9 + 4 + 4$ ✓ ✓

$(x-2)^2 + (y+2)^2 = 17$ ✓

$\therefore O(2; -2)$ ✓

b) Determine the coordinates of A and B **4**

$(x-2)^2 + (y+2)^2 = 17$

$(x-2)^2 + (-1+2)^2 = 17$ ✓

$(x-2)^2 + 1 = 17$

$(x-2)^2 = 16$ ✓

$x - 2 = \pm 4$

$x = 6$ or $x = -2$

$\therefore A(-2; -1)$ ✓ and $B(6; -1)$ ✓

c) Determine the equation of the tangent AE. 4

$y = mx + c$ $\qquad$ $grad_{AO} = -\frac{1}{4}$ ✓

$y = 4x + c$ $\qquad$ $grad_{AE} = 4$ ✓

$-1 = 4(-2) + c$ ✓

$-1 = -8 + c$

$c = 7$

$y = 4x + 7$ ✓

d) Determine coordinates of E if BE is a tangent drawn through point B and is defined by: $y = -4x + 23$ 3

$4x + 7 = -4x + 23$ ✓

$8x = 16$

$x = 2$ ✓

$y = 4(2) + 7$

$y = 15$

$E\ (2; 15)$ ✓

QUESTION 10 **21 MARKS**

a) Prove the following identity: $\frac{\cos 2\theta}{(\cos\theta+\sin\theta)^3}=\frac{\cos\theta-\sin\theta}{1+\sin 2\theta}$ 4

$$LHS=\frac{\cos^2\theta-\sin^2\theta}{(\cos\theta+\sin\theta)^3}$$ ✓

$$=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)^3}$$ ✓

$$=\frac{\cos\theta-\sin\theta}{(\cos\theta+\sin\theta)^2}$$

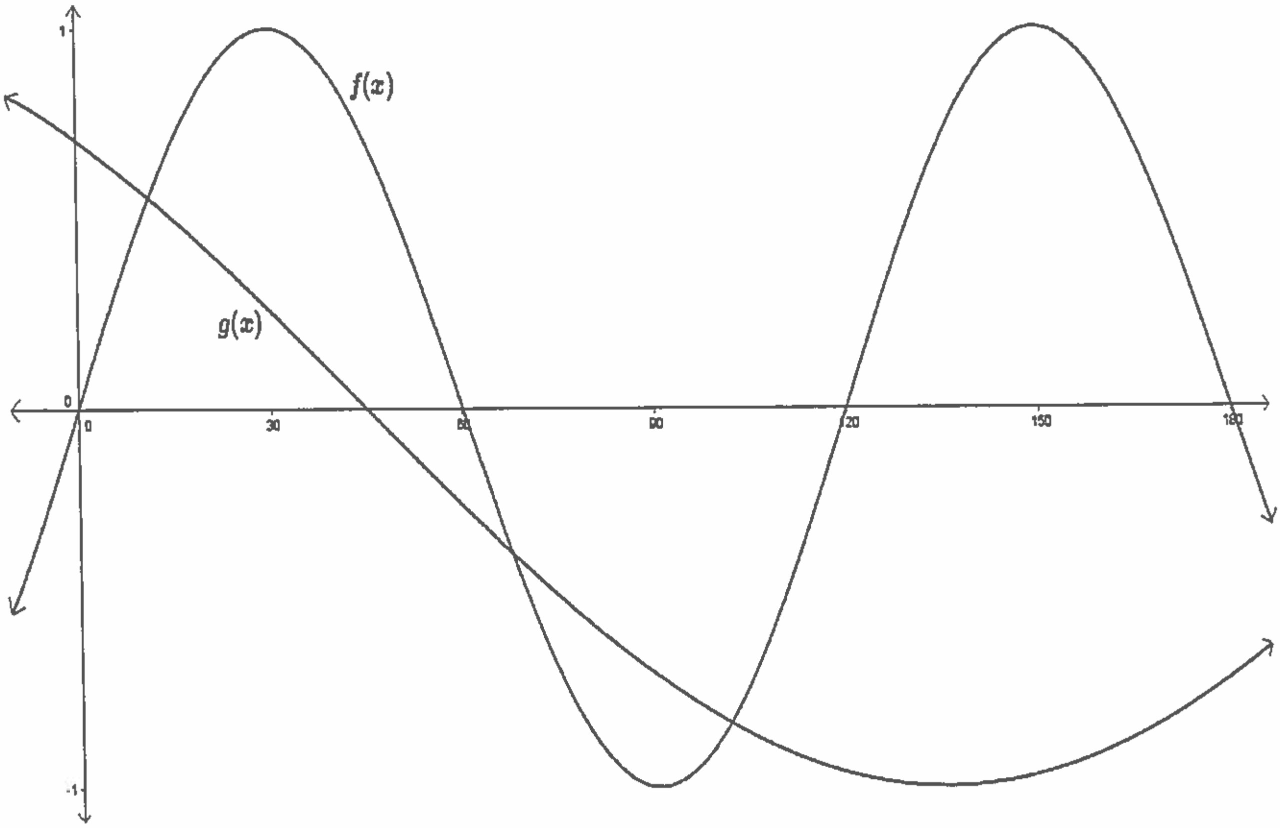
$$RHS=\frac{\cos\theta-\sin\theta}{1+2\sin\theta\cos\theta}$$ ✓

$$=\frac{\cos\theta-\sin\theta}{(\cos\theta+\sin\theta)^2}$$ ✓

LHS=RHS

b) The diagram below is a sketch of $f(x)$ and $g(x)$ for $x \in (0°; 180°)$:

$f(x) = \sin 3x$ and $g(x) = \cos(x + 45°)$



i) Determine the values of x for which $g(x) = f(x)$, for $x \in (0°; 180°)$ 9

$\sin 3x = \cos(x + 45°)$ ✓

$\sin 3x = \sin(90° - (x + 45°))$

$\sin 3x = \sin(45° - x)$ ✓

$3x = 45° - x + k.360°$ ✓ or $3x = 180° - (45° - x) + k.360°$ ✓ for $k \in \mathbb{Z}$

$4x = 45 + k.360°$ or $3x = 135° + x + k.360°$

$x = 11,25° + k.90$ ✓ or $2x = 135° + k.360°$

$x = 67,5° + k.180°$ ✓

$x \in \{11,25°; 67,5°; 101,25°\}$ ✓ ✓ ✓

ii) Determine the values of x for which $g(x) > f(x)$, for $x \in (0°; 180°)$ 2

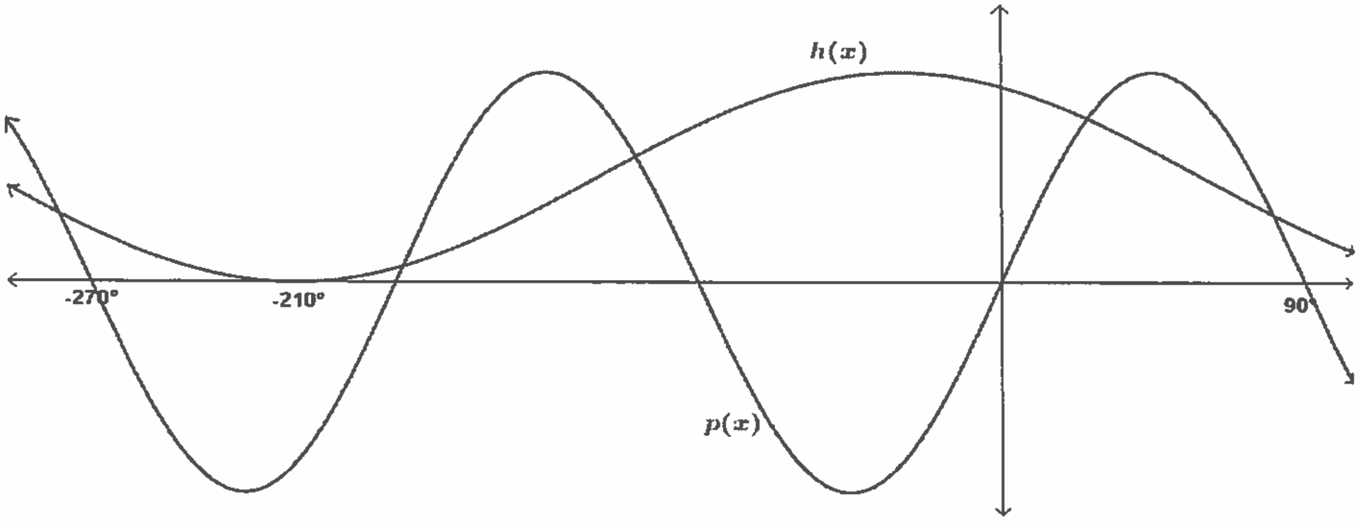
$0° < x < 11,25°$ ✓

$67,5° < x < 101,25°$ ✓

c) The diagram below is a sketch of $h(x)$ and $p(x)$:

$h(x) = \cos(x° + a) + 1$ and $p(x) = b\sin(cx°)$

(Hint: $h(x)$ intersects the x-axis at $-210°$)

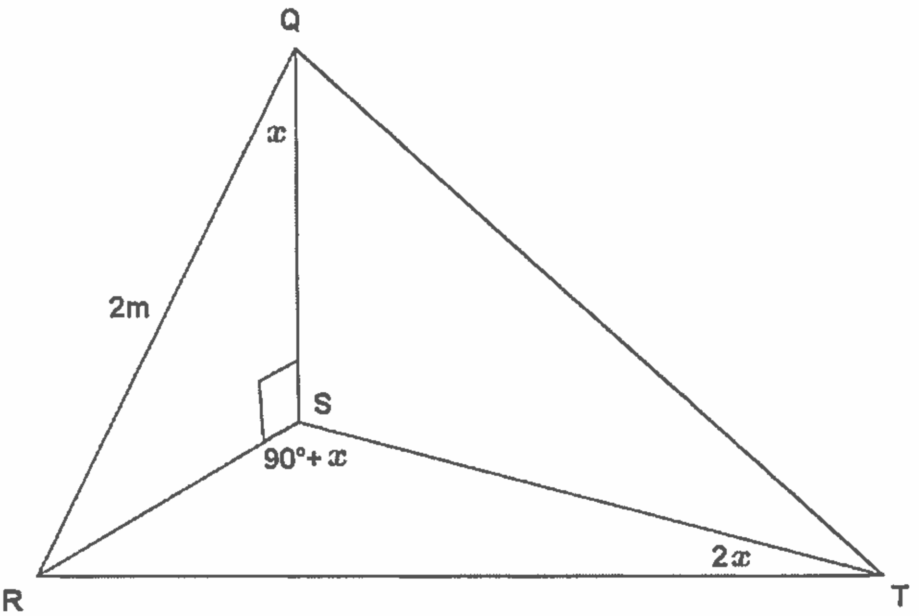


Determine the values of a, b, and c 6

$a = 30°$ ✓✓
$b = 2$ ✓✓
$c = 2$ ✓✓

QUESTION 11 **15 MARKS**

Rory and Trent are standing at points R and T respectively. As shown below, they are looking at point Q, the top of the vertical pole SQ. Rory is standing 2 meters from the top of the pole. S, R, and T are in the same horizontal plane. $R\hat{Q}S = x$, $R\hat{S}T = 90° + x$, and $S\hat{T}R = 2x$



a) Prove that Rory and Trent are standing 1 meter apart. 6

$\sin x = \frac{RS}{2}$ ✓

$RS = 2\sin x$ ✓

$\frac{RT}{\sin(90°+x)} = \frac{RS}{\sin 2x}$ ✓

$\frac{RT}{\sin(90°+x)} = \frac{2\sin x}{2\sin x\cos x}$ ✓

$\frac{RT}{\cos x}$ ✓ $\frac{1}{\cos x}$

$RT = 1$ ✓

b) Find $S\hat{R}T$ in terms of x. 1

$S\hat{R}T = 180° - (90° - x) - 2x$

$= 90° - 3x$

c) Prove that $ST = 2\cos 2x - 1$ 8

$\dfrac{ST}{\sin(90° - 3x)} = \dfrac{RT}{\sin(90° + x)}$

$ST = \dfrac{1 \times \sin(90° - 3x)}{\sin(90° + x)}$

$ST = \dfrac{\cos 3x}{\cos x}$

$= \dfrac{\cos(2x + x)}{\cos x}$

$= \dfrac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$

$= \dfrac{\cos 2x \cos x - 2\sin x \cos x \sin x}{\cos x}$

$= \cos 2x - 2\sin^2 x$

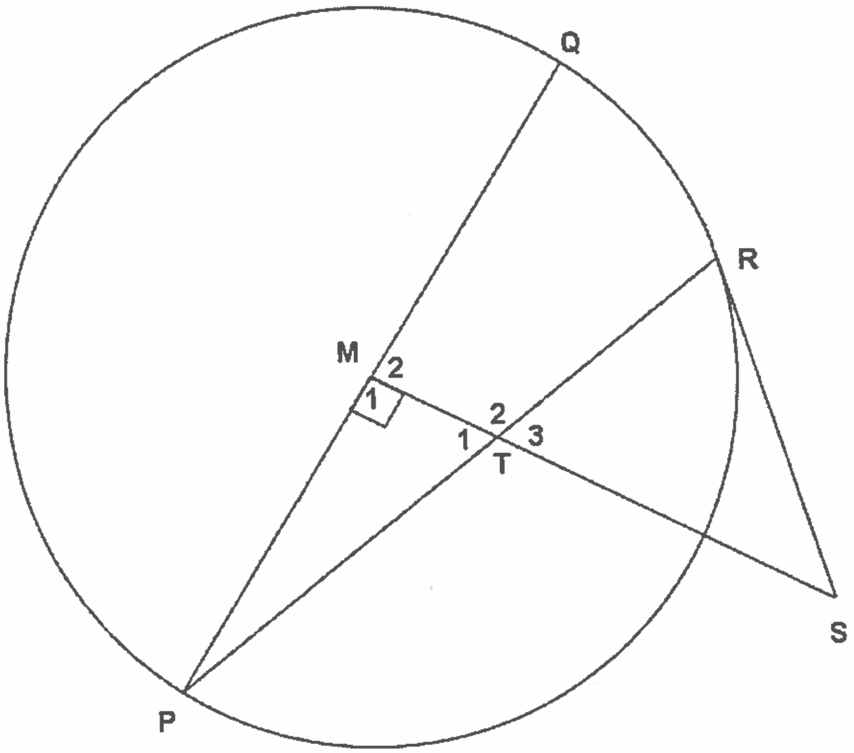
$= \cos 2x - (1 - \cos 2x)$

$= 2\cos 2x - 1$

QUESTION 12 **7 MARKS**

In the figure below, M is the centre of the circle and SM is perpendicular to PQ.

PR and SM intersect at T and $ST = RS$.



a) Prove that $QMTR$ is a cyclic quadrilateral. 3

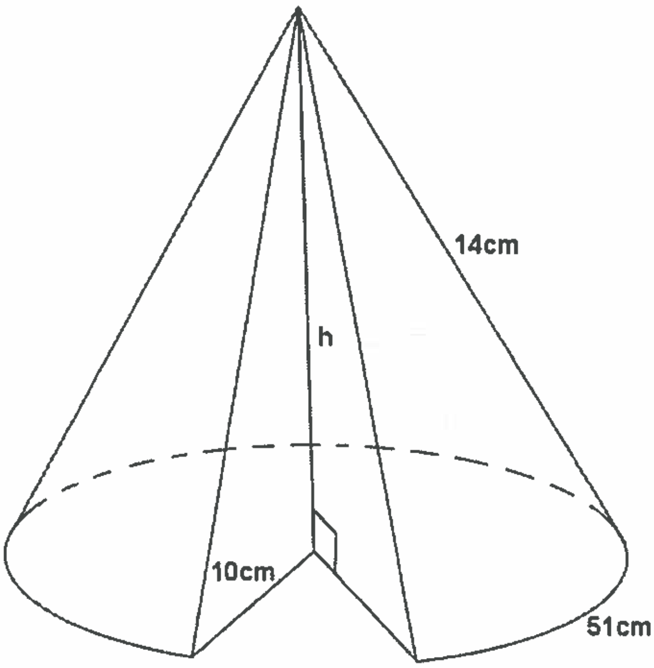
Construct line QR
$Q\hat{R}T = 90°$ ✓ ($\angle$ in semi-circle)
$\hat{M}_2 = 90°$ ✓ (given $SM \perp PQ$)
$\therefore$ MTRQ is a cyclic quad ✓ (opp $\angle$s supp)

b) Prove that RS is tangent to the circle at R 4

$S\hat{R}P = \hat{T}_3$ ✓ (Isos Δ)
$\hat{T}_3 = P\hat{Q}R$ ✓ (Ext $\angle$ of cyclic quad = opp int $\angle$)
$\therefore S\hat{R}P = P\hat{Q}R$ ✓
$\therefore$ RS is a tangent ✓ (converse of tan-chord)

QUESTION 13 **10 MARKS**

A cake in the form of a cone, has a radius of 10cm and a slant height of 14cm. After the cake has been cut the remaining circumference of the circle is measured as 51cm. You may need these formulae: $\frac{1}{3}\pi r^2 h$; $\pi r l + \pi r^2$



a) Determine the surface area of the cake before it was cut **3**

$SA = \pi r l + \pi r^2$

$SA = \pi(10)(14) + \pi(10)^2$

$SA = 753{,}98 \text{ cm}^2$

or $240\pi \text{ cm}^2$

b) Determine the volume of the cake before it was cut. **3**

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(10)^2(\cancel{9{,}8}\ 4\sqrt{6})$

$V = 1\cancel{026{,}25} \text{ cm}^3$

$1026{,}04$

$h^2 = 14^2 - 10^2$

$h = \cancel{9{,}8}$

$4\sqrt{6}$

c) Determine the volume of the cake after it was cut. **4**

$C = 2\pi r = 2\pi(10) = 62,83 \text{ cm}$

After cut $C = 51 \text{ cm}$

$\therefore \frac{51}{62,83} = 81,17\%$ of cake left

New $V = 1026,\cancel{25}^{04} \times 81,17\%$

$= \cancel{833,01 \text{ cm}^3}$ $\cancel{832,85}$

$832,84$