

ST. DAVID'S MARIST INANDA



GRADE 12 MATHEMATICS
PRELIMINARY EXAMINATION
PAPER I
7 SEPTEMBER 2016

EXAMINER: Mrs C. Kennedy
MODERATOR: Mrs L. Nagy

MARKS: 150
TIME: 3 hours

NAME: _____

HIGHLIGHT YOUR TEACHER'S NAME:

C. KENNEDY	L. NAGY	L. BLACK	S. RICHARD
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INSTRUCTIONS:

- ✓ This paper consists of 24 pages and a separate Formula sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper and read each question carefully.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ Answers must be rounded off to ONE decimal place unless otherwise stated.
- ✓ It is in your interest to show all necessary working details.
- ✓ Work neatly. Do **NOT** answer in pencil.
- ✓ Diagrams are not drawn to scale.

SECTION A	Q1	Q2	Q3	Q4	Q5	Q6	Q7	TOTAL
MARKS	17	14	11	5	5	10	13	75
SECTION B	Q8	Q9	Q10	Q11	Q12	Q13		TOTAL
MARKS	15	14	13	7	9	17		75

SECTION A**QUESTION 1 [17 marks]**a) Solve for x in each of the following:

i) $(x-3)^2 = 4x^2$ (4)

ii) $\log_{\frac{1}{2}}(x+3) = 0$ (3)

iii) $5^x(x-5) \leq 0$ (2)

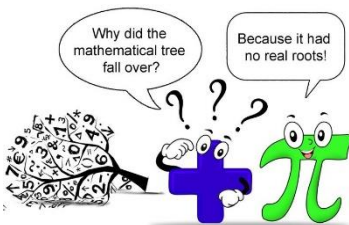
b) Evaluate:

$$\sqrt{\left(2 - \frac{\sqrt{7}}{2}\right)} \cdot \sqrt{\left(2 + \frac{\sqrt{7}}{2}\right)}, \text{ without using a calculator.} \quad (4)$$

c) Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$

i) Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

ii) Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2)



QUESTION 2 [14 marks]

a) Consider the series $\sum_{n=1}^{\infty} 2\left(\frac{1}{2}x\right)^n$

For which values of x will the series converge? (5)

b) The sum of the first n terms of a series is given by the formula $S_n = 3^{n-1} + 9$.
Determine the value of the 6th term. (4)

- c) A polygon has 25 sides. The lengths of these sides form an arithmetic sequence. If the polygon has a perimeter of 1100cm, and the longest side is ten times the shortest side, determine the lengths of the shortest and longest sides. (5)

QUESTION 3 [11 marks]

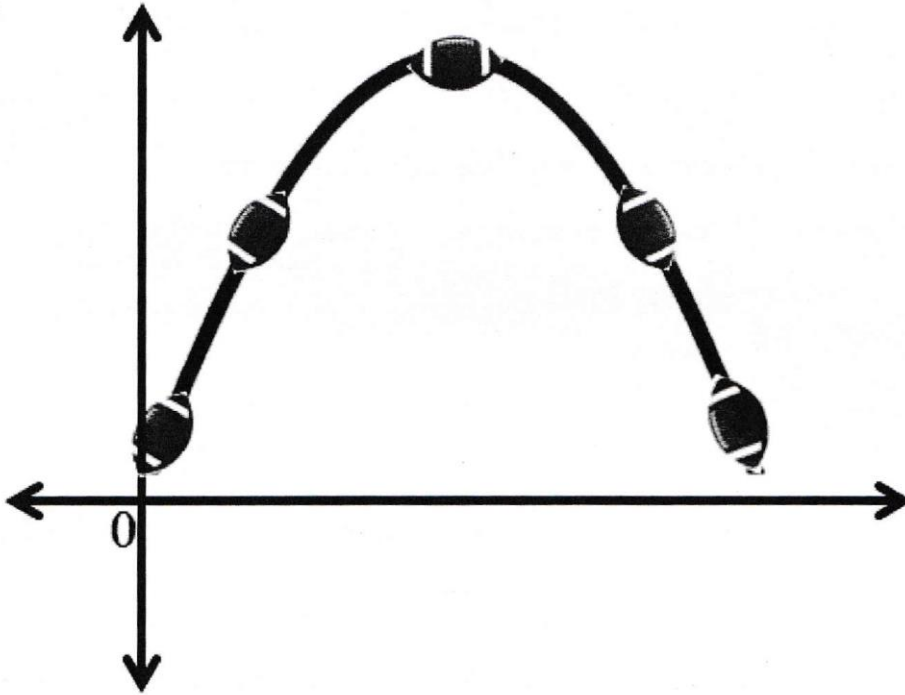
a) If $f(x) = \frac{4x+16}{2}$, show that $f^{-1}(x) \neq f(x^{-1})$. (3)

b) If $f(x) = 4^x$ and $f(x+2) - f(x+1) + f(x) = k \times f(x)$, determine the value of k . (4)

- c) The trajectory of a kicked rugby ball can be modelled by the equation

$$h = -4t^2 + 18t + 2, \text{ for } t \geq 0,$$

where h is the height in metres above the ground and t is the time in seconds after it is kicked.



Determine the range of values of t for which the height of the ball above the ground is higher than 10 metres.

(4)

QUESTION 4 [5 marks]

The inverse of $f(x) = \frac{3}{x-2} - 5$ is given by the equation $f^{-1}(x) = \frac{k}{x+p} + q$.

a) i) Write down the equations of the asymptotes of f . (2)

ii) Hence, write down the values of p and q . (2)

b) Hence, write down the value of k . (1)

QUESTION 5 [5 marks]

The probability that a certain rugby team has all its players fit to play is 70%.

The probability that they will win a game if all their players are fit is 90%.

When they are not fit the probability of them winning becomes 40%.

Calculate the probability of them winning their next game.

(5)



QUESTION 6 [10 marks]

a) Determine:

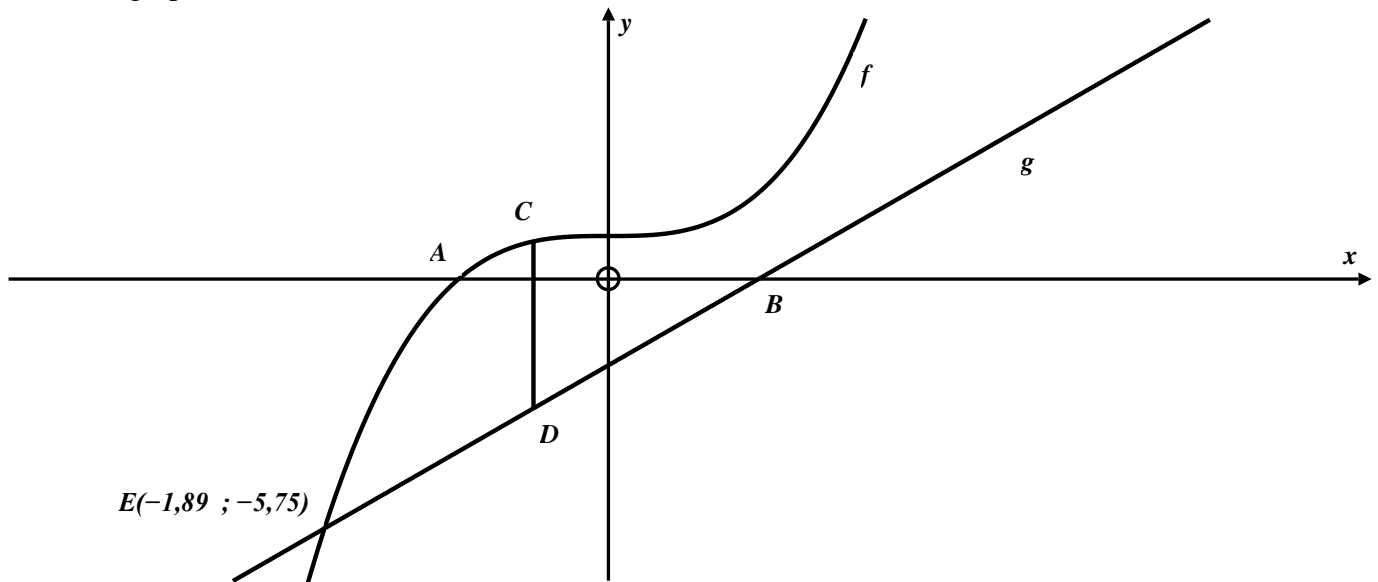
$$D_t \left[\frac{3t^4 + 7t^2 - 5t}{2t^2} \right], \text{ leaving your answer with positive exponents.} \quad (4)$$

b) The gradient of the curve $y = 2x^3 + \frac{a}{\sqrt{x}}$ at $x=1$ is 8.Determine the value of a . (3)

- c) For what value of the constant m will the curve of $y = mx - 2x^3$ have a local minimum at $x = -\frac{1}{2}$. (3)

QUESTION 7 [13 marks]

The graphs of f and g with $f(x) = x^3 + 1$ and $g(x) = 2x - 2$ are sketched below. A and B are the x -intercepts of f and g respectively. C is a point on f and D is a point on g such that CD is parallel to the y -axis. CD lies to the left of the y -axis. The graphs intersect at $E(-1, 89 ; -5, 75)$.



Determine:

- a) the coordinates of A . (2)
- b) the equation of the tangent of f at point A . (3)

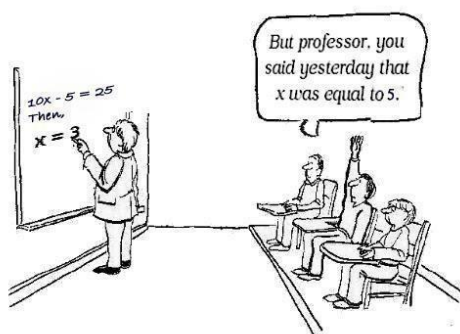
- c) the TWO possible x -coordinates for C so that $CD = 4$.
SHOW ALL YOUR WORKING.

(6)

- d) the values of x for which:

$$g'(x)f(x) > 0$$

(2)

**SUB-TOTAL: 75**

SECTION B**QUESTION 8 [15 marks]**

- a) Rory deposits R8600 into a bank account, offering an interest rate of 7,5% per annum compounded semi-annually.

Calculate the number of years it takes for Rory's investment to be worth R15500. (4)

- b) Adam inherits R800 000. He invests all of his inheritance in a fund which earns an effective interest rate of 14% per annum compounded. He wishes to withdraw a monthly income from the fund for a period of 20 years, starting with the first withdrawal exactly one month after his initial investment.

i) Convert the given effective interest rate to an equivalent nominal interest rate compounded monthly. (2)

ii) Calculate Adam's monthly income from the fund. (4)

iii) Exactly four years after his initial investment Adam decides to withdraw all the remaining money in his account and to use it as a deposit towards a house.

1) Calculate the balance in Adam's account, to the nearest rand? (4)

2) Adam's deposit is exactly 30% of the purchase price of the house. Determine the purchase price of the house to the nearest rand. (1)




QUESTION 9 [14 marks]

- a) Each passenger on a certain  Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.


	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	<i>b</i>	<i>c</i>	80
Fruit juice	<i>d</i>	<i>e</i>	20
TOTAL	60	100	<i>a</i>

- i) Write down the value of *a*. (1)
- ii) What is the probability that a randomly selected passenger is male? (2)
- iii) Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of *b*. (4)

b) A  Airways aeroplane has 6 seats in each row.

i) How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)



ii) Joshua and Bogosi and 4 other passengers sit in a certain row on a certain  Airways flight. In how many ways can these 6 passengers be seated if Joshua and Bogosi must sit next to each other. (2)

iii) Mrs Richard and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mrs Richard will sit at the end of the row? (3)

QUESTION 10 [13 marks]

a) A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$

$$T_3 - T_2 = 13$$

$$T_4 - T_3 = 19$$

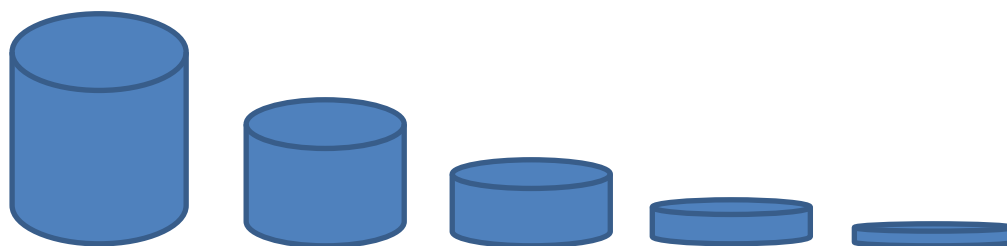
i) Write down the value of:

1) $T_5 - T_4$ (1)

2) $T_{80} - T_{79}$ (3)

ii) Calculate the value of T_{69} if $T_{89} = 23594$. (5)

- b) Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Is it possible for the first water tank to hold all the water from the other 19 tanks?
Motivate your answer. (4)

QUESTION 11 [7 marks]

a) Given $-4 \leq a \leq 3$ and $-5 \leq b \leq 4$

Determine:

i) The largest possible value of a^2 . (1)

ii) The smallest possible value of ab . (1)

iii) The value of b if $b^2 = 25$ (1)

b) $f(x)$ and $g(x)$ are both parabolas, such that:

$$f(x) = -g(x); f(1) = f(4) = 0; f'(a) = 0; f(a) = 8$$

Determine the co-ordinates of the turning point of $g(x)$. (4)

QUESTION 12 [9 marks]

The graph of g is defined by the equation $g(x) = \sqrt{ax}$. The point $(8 ; 4)$ lies on g .

- a) Show that $a = 2$. (2)
- b) Write down the range of g . (1)
- c) Determine the equation of g^{-1} , the inverse of g , in the form $g^{-1}(x) = \dots\dots$ (2)
- d) If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g . (4)

QUESTION 13 [17 marks]

- a) If $f(x) = ax^3 + bx^2 + cx - 5$ and the gradient at any point $(x; f(x))$ is given by $6x^2 - 24$, determine the values of a , b and c . (4)

- b) i) The cubic $g(x) = ax^3 + bx^2 + cx$ has a point of inflection at $x = p$.
Show that $p = -\frac{b}{3a}$. (4)

- ii) Hence, determine the coordinates of the point of inflection of f , in terms of k if $f(x) = x^3 - 2kx^2 + k^2x$ and $k < 0$. (3)

- c) A cinema owner noticed that by raising the temperature inside his cinema he could increase the sales of ice cream during the interval.

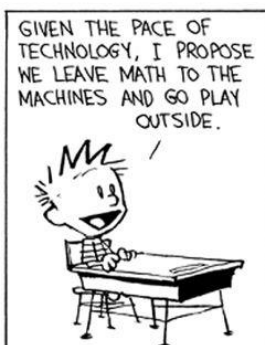


With an audience of 100, the owner made a profit of $\left(200 - \frac{625}{T}\right)$ rands on ice cream sales when the temperature was T degrees, but he had to pay out $(20 + T)$ rands for heating.



- i) Determine an expression for the overall profit that the cinema owner makes, in terms of T . (2)

- ii) Hence, calculate the temperature that leads to the maximum overall profit. (4)



SUB-TOTAL: 75

TOTAL: 150