



ST MARY'S DSG, KLOOF

GRADE: 12

AUGUST 2016

MATHEMATICS: PAPER I

Examiner: J van Rooyen

Moderators: J. Kinsey
S. Drew

TIME: 3 HOURS

TOTAL: 150 MARKS

INSTRUCTIONS:

1. This question paper consists of 10 pages.
2. There are 3 sections. Answer each section on a new sheet of paper.
3. Write your number and Maths teacher's name on each sheet of paper.
4. A formula sheet has been provided.
5. Diagrams are not drawn to scale.
6. Please give all answers correct to 2 decimal places unless otherwise indicated.
7. Read all the questions carefully.
8. Number your answers exactly as they have been numbered.
9. An approved non-programmable and non-graphical calculator may be used, unless otherwise specified.
10. Make sure that your calculator is in degree mode.

NAME: _____

TEACHER'S NAME: _____

SECTION A [55]

QUESTION 1

(a) If $(x - \sqrt{-3})(3^x - 6)(2^x - 8) = 0$

(1) Solve for $x \in Z$ (2)

(2) Solve for $x \in Q'$ (2)

(3) Solve for $x \in R'$ (1)

(b) (1) Solve for x , in terms of h , if $2x^2 - 7x - h = 0$ (2)

(2) Give the value of h for which the roots will be equal. (2)

[9]

QUESTION 2

The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively.
 p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$ (3)

(b) Hence show that $p = 5$. (1)

(c) Find the common ratio of this series. (1)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer. (3)

[8]

QUESTION 3

- (a) Consider the series

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots$$

Give the answers to the following in the form $\frac{a}{b}$:

- (1) $S_2 =$ (1)
- (2) $S_3 =$ (1)
- (3) $S_4 =$ (1)
- (4) $S_n =$ (1)
- (5) $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{2015 \times 2016} =$ (1)
- (b) A parent is prepared to give the St Vincent's library R10 for one distinction scored by the St Mary's matriculants in English at the end of the year, R20 for two distinctions, R40 for three distinctions and so on. The St Mary's group achieves 18 distinctions. How much does the parent give the St Vincent's library? (3)
- (c) Find the value of a in the following: (4)

$$\sum_{r=0}^{\infty} \frac{a}{4^r} = 16$$

[12]

QUESTION 4

A rectangular room has a width of x m. The length of the room is 4 m longer than the width. Given that the perimeter of the room is greater than 19,2 m.

- (a) Show that $x > 2,8$ (2)

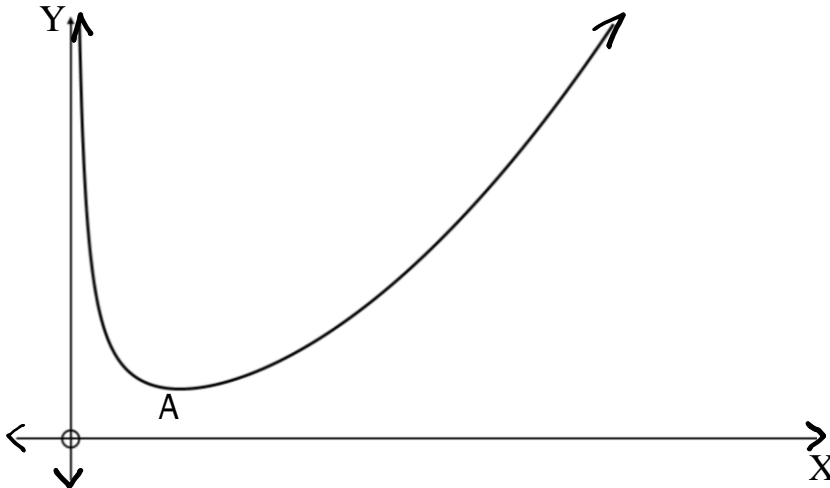
Given also that the area of the room is less than 21 m^2

- (b) Write down an inequality in terms of x for the area of the room
- (c) Solve this inequality (4)
- (d) Hence find the range of possible values for x . (1)

[7]

QUESTION 5

In the figure, part of the curve $y = f(x)$ is shown where $f(x) = x^2 + \frac{16}{x}$ for $x > 0, x \in R$. The curve has a minimum turning point at A.



- (a) Find $f'(x)$ (2)
- (b) Hence find the co-ordinates of A (4)
- (c) Give the domain of $f^{-1}(x)$ (2)
- (d) Using your answer in (b), write down the co-ordinates of the turning point of the curve with equation

(1) $y = f(x - 2)$

(2) $y = \frac{1}{2}f(x)$ (2)

[10]

QUESTION 6

- (a) Without using your calculator solve for x , where

$$x\sqrt{27} + 21 = \frac{6x}{\sqrt{3}}$$

Give your answer(s) in the form $a\sqrt{b}$, where a and b are integers. (4)

- (b) A sailing competition between two boats A and B, consists of a series of independent races. For each race the weather is either rough or fine, the probability of rough weather being 0,2. Every race is won by either A or B, and their respective probabilities of winning are influenced by the weather. In rough weather the probability that A will win is 0,9. In fine weather the probability that A will win is 0,4. Show, using a tree diagram, that the probability that A will win the first race is 0,5. (5)
- [9]

SECTION B [46]

START ON A NEW FOLIO SHEET

QUESTION 7

$$f(x) = 2x^3 - 5x^2 + ax + 18, \text{ where } a \text{ is a constant.}$$

Given that $(x - 3)$ is a factor of $f(x)$,

- (a) Show that $a = -9$ (2)
- (b) Factorise $f(x)$ completely. (4)

$$\text{Given that } g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

- (c) Find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate. (3)

[9]

QUESTION 8

A 25-year programme for building new houses started in Utopia City at the beginning of 1986 and finished at the end of 2010.

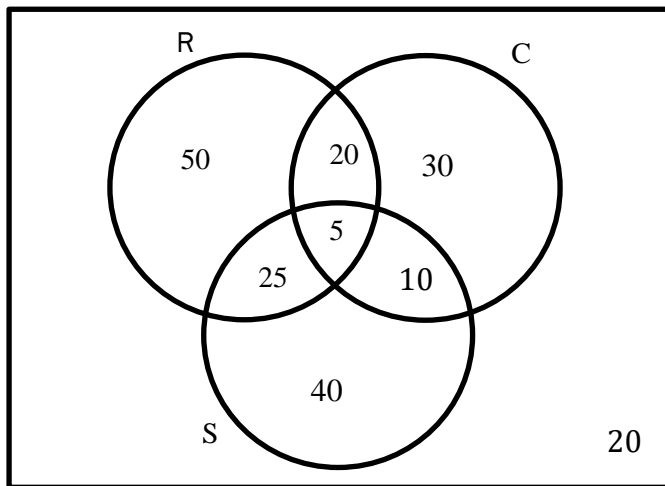
The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find

- (a) The number of houses built in 1986, the first year of the building programme (5)
- (b) The total number of houses built in the 25 years of the programme (2)

[7]

QUESTION 9

- (a) A group of 200 people was asked about the kind of sports they watch on television. 180 people watch rugby, cricket or soccer. The information collected is represented in the Venn diagram where R represents Rugby, S represents Soccer and C represents Cricket.



- (1) How many people watch Rugby? (1)
- (2) Give $P(R \text{ or } C)$ (1)
- (3) Give $P(\text{not}(R \text{ or } S \text{ or } C))$ (1)
- (4) Are watching Cricket and watching Rugby independent events? Confirm your answer using a calculation. (4)
- (b) Tamara invests R 8 500 in a special banking product which will pay 1% *p.a.* for 1 month, then 2% *p.a.* for the next 2 months, then 3% *p.a.* for the next 3 months, 4% *p.a.* for the next 4 months, and 0% for the rest of the year. If the bank charges her R 75 to open the account, how much can she expect to get back at the end of the year? (4)

[11]

QUESTION 10

A dairy farmer needs to buy new equipment, which costs R 200 450, for her dairy farm. She bought her old equipment 12 years ago for R 167 000. The value of the old equipment depreciates at a rate of 12,2% per year on reducing balance. She will need to arrange a loan for the remaining cost of the new equipment.

A bank supporting agricultural endeavours offers loans at a special interest rate of 10,01% *p. a.* compounded monthly for any loan up to R 175 000 and 9,61% *p. a.* compounded monthly for a loan above that amount. She arranges a loan so that she will not need to make any payments on the loan in the first six months (grace period) and she must pay back the loan over 20 years.

- (a) Determine the monthly payment (5)
- (b) What is the total amount of interest that she pays? (3)
- (c) By what factor is the interest she pays greater than the value of the loan? (3)

[11]

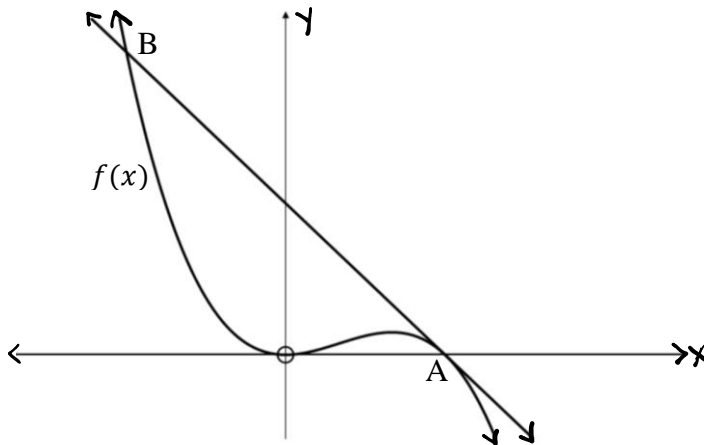
QUESTION 11

- (a) In how many ways can the letters of the word KARDASHIANS be arranged, if the repeated letters are treated as identical? (5)
- (b) What is the probability that this word will start with two A's? (3)

[8]

QUESTION 12

In the sketch, $f(x) = x^2 - \frac{1}{3}x^3$. $f(x)$ has x -intercepts at the origin and at A.



- (a) Show that the co-ordinates of A are $(3; 0)$ (1)
- (b) Show that the equation of the tangent at A is $y = -3x + 9$ (5)
- (c) The tangent at A meets the curve again at B. Find the x -coordinate of B. (4)
- (d) Give the x -coordinate of the point of inflection of $f(x)$. (2)
- (e) Find $f''(0)$ (2)
- (f) Sketch the graph of $f''(x)$ showing any intercept values. (3)
- (g) Give the value(s) of k for which $f(x) + k$ will have 3 distinct roots (5)

[22]

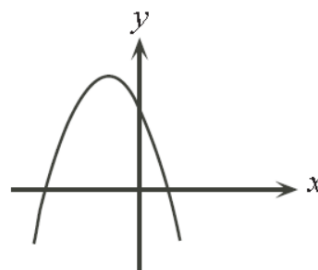
QUESTION 13

Write down the letter corresponding to the correct answer for the following:

The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following must be positive?

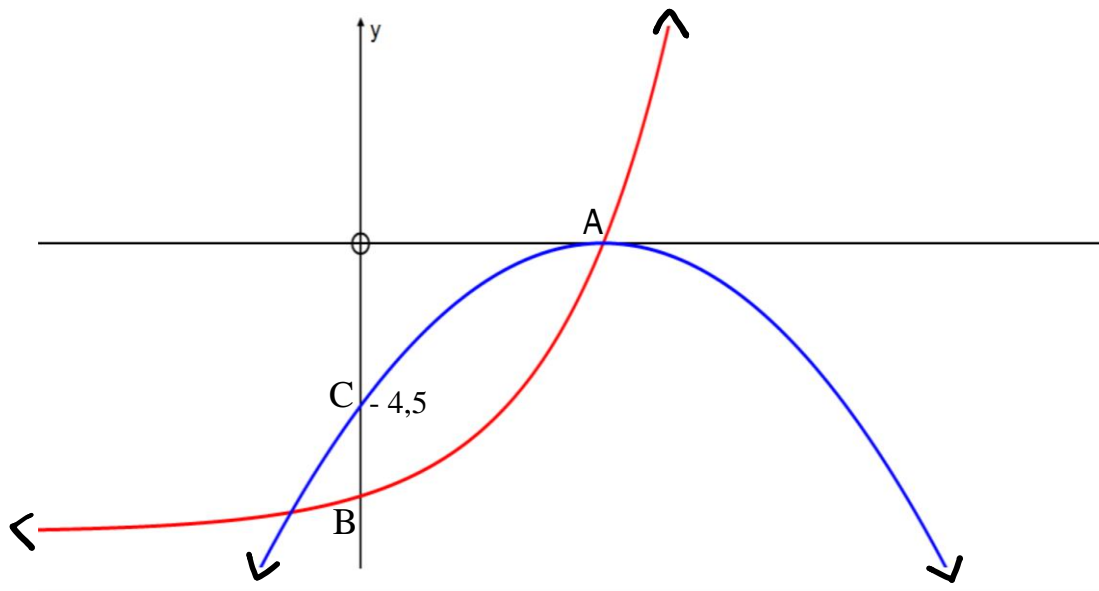
[2]

- (A) a
- (B) bc
- (C) ab^2
- (D) $b - c$
- (E) $c - a$



QUESTION 14

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below. B and C are the y -intercepts of the graphs of f and g respectively. The two graphs intersect at A, which is the turning point of graph $g(x)$ and the x -intercept of the graphs of $f(x)$ and $g(x)$. C has coordinates $(0; -4\frac{1}{2})$.



- Determine the coordinates of A and B (4)
- Write down the equation of the asymptote of $f(x)$ (1)
- Give the equation of $h(x)$ if $h(x) = f(2x) + 8$ (2)
- Determine the equation of $f^{-1}(x)$ in the form $y = \dots$ (2)
- Write down the equation of $p(x)$, if p is the reflection of $f(x)$ about the x -axis (1)
- Give the values of x where $f'(x) \cdot g(x) < 0$ (2)
- Determine

$$\sum_{n=0}^3 g(n) - \sum_{n=4}^5 g(n) \quad (4)$$

[16]

QUESTION 15

(a) A function f satisfies $f(x) + f(x + 3) = 2x + 5$ for all x . If $f(8) + f(2) = 12$, find the value of $f(5)$. (4)

(b) The total number of acres of genetically modified crops grown worldwide from 2007 through 2013 was changing at the rate of $2,718t^2 - 19,86t + 50,18$ million acres/year ($0 \leq t \leq 6$). (5)

The total number of acres of such crops grown in 2007 ($t = 0$) was 27,2 million acres.

How many acres of genetically modified crops were grown worldwide in 2013?

[9]

SUMMARY

| QUESTION NO. | L.O. | LEVEL (K / RP/ CP/ PS) | | | | MARKS |
|-----------------|---------|-------------------------|----|----|----|-------|
| | | K | RP | CP | PS | |
| 1 | Alg | 2 | 7 | | | 9 |
| 2 | S&S | | 8 | | | 8 |
| 3 | S&S | 3 | 3 | 6 | | 12 |
| 4 | Alg | | 6 | 1 | | 7 |
| 5 | Calc | | 10 | | | 10 |
| 6 | Alg | | | 4 | | |
| | Prob | | | 5? | | 9 |
| | | | | | | 55 |
| 7 | Calc | | 6 | | 3 | 9 |
| 8 | S&S | | 2 | 5 | | 7 |
| 9 | Prob | 1 | 2 | 4 | | |
| | Finance | | | 4 | | 11 |
| 10 | finance | | 3 | 8 | | 11 |
| 11 | Prob | | 5 | 3 | | 8 |
| | | | | | | 46 |
| 12 | Calc | 1 | 13 | 8 | | 22 |
| 13 | Func | | | | 2 | 2 |
| 14 | Func | 5 | 3 | 4 | 4 | 16 |
| 15 | Calc | | | | 5 | |
| | ALg | | | | 4 | 9 |
| | | | | | | 49 |
| | | 12 | 68 | 52 | 18 | |
| TOTAL: | | | 80 | | | 150 |
| TOTAL %: | | | 53 | 35 | 12 | |

COMMENTS FROM TEACHERS

Alg 24 S & S 27 Finance 22 Probability 20 Functions 18 Calculus 46
 WANT 25 25 15 15 35 35
