



ST MARY'S DSG, KLOOF

GRADE: 12

SEPTEMBER 2016

MATHEMATICS: PAPER II

Examiner: S Drew

Moderators: J van Rooyen  
J Kinsey

TIME: 3 HOURS

TOTAL: 150 MARKS

INSTRUCTIONS:

1. This question paper consists of 27 typed pages. There are also 2 blank pages.
2. All answers must be written on the question paper.
3. There are 3 sections.
4. **Write your number and Maths teacher's name** on the top of each section.
5. A formula sheet has been provided.
6. Diagrams are not drawn to scale.
7. Please give all answers correct to **2 decimal places** unless otherwise indicated.
8. Read all the questions carefully.
9. An approved non-programmable and non-graphical calculator may be used, unless otherwise specified.
10. Make sure that your calculator is in degree mode.

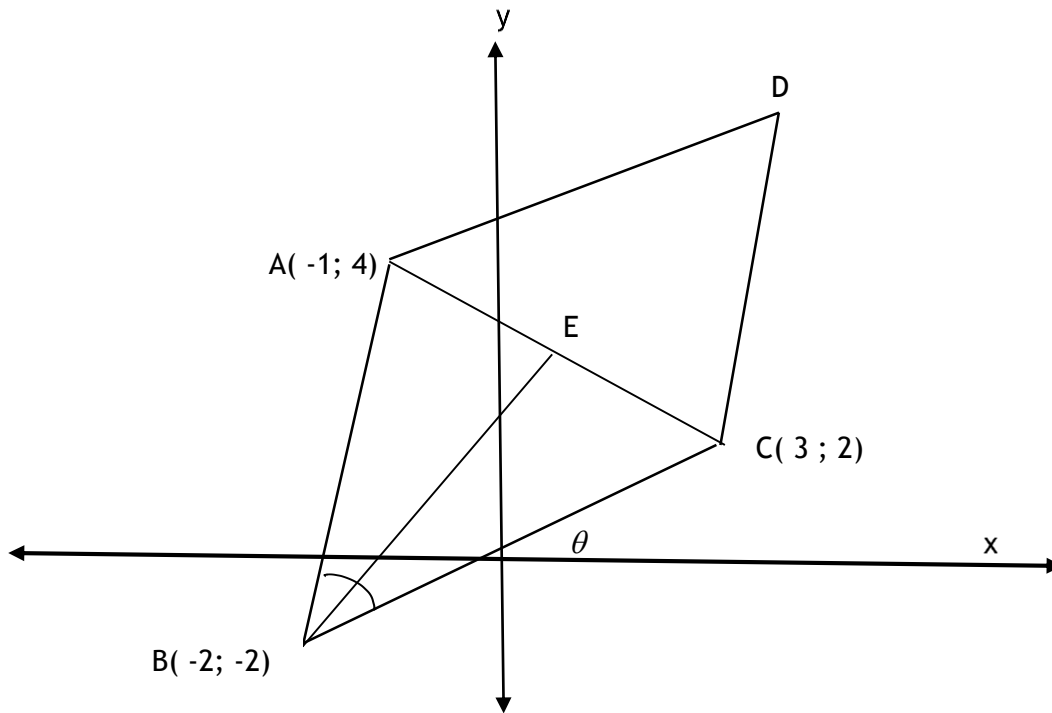
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TEACHER'S NAME: \_\_\_\_\_

**SECTION A (50 MARKS)**

**Question 1**

In the diagram below, A (-1 ; 4), B(-2 ; -2 ) , C (3 ; 2) and D are the vertices of parallelogram ABCD.



- a. Show that the co-ordinates of E, the midpoint of diagonal AC, are (1 ; 3).

\_\_\_\_\_ (1)

- b. Determine the equation of BE. (3)

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\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

c. Determine the co-ordinates of vertex D. (2)

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d. Calculate the size of  $\theta$ . (2)

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e. Hence, calculate the size of  $\widehat{ABC}$ . (2)

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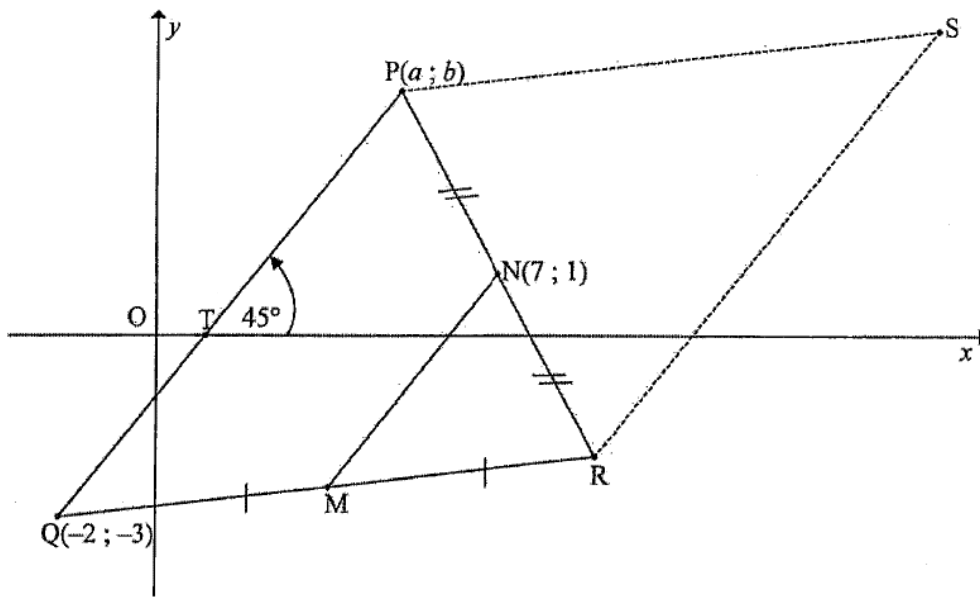
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[10]

### Question 2

In the diagram below, the line joining  $Q(-2 ; -3)$  and  $P(a ; b)$  makes an angle of  $45^\circ$  with the positive x-axis.  $QP = 7\sqrt{2}$  units.

$N(7 ; 1)$  is the midpoint of  $PR$  and  $M$  is the midpoint of  $QR$ .



- a. Show that the gradient of  $PQ$  is 1. (1)

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- b. Determine the equation of  $MN$ , with reasons. (4)

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c. Find the length of MN. Give a reason. (2)

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d. Calculate the numerical co-ordinates of P. (6)

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### Question 3

The masses (in kilograms) of the 20 girls in the St Marys' hockey squad are given below:

69    59    57    66    64    58    63    58    62    60  
59    53    60    51    60    48    47    60    40    61

a. Determine:

1. the mean for the masses of the girls. (1)

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2. the variance for the masses of the girls (2)

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b. The following information was obtained from the Kingsknee boys' hockey coach regarding the masses of the boys in his squad.

$$\sum_{n=1}^{22} x_n = 1320 \quad \text{and} \quad \sum_{n=1}^{22} (x_n - 60)^2 = 1012$$

1. How many boys are in the squad? (1)

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2. Determine the mean mass for the boys' squad. (1)

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3. Determine the standard deviation for the boys' squad. (2)

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c. If 5 girls of equal mass are added to St Marys' squad so that the mean masses of both schools are the same, what must the mass of each extra girl be? (4)

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[11]

#### Question 4

The charities committee recorded the amount of money raised each month over an extended period of time.

The data collected is summarised in the table below:

	Lowest value	Second lowest value	Lower quartile	Median	Upper quartile	Second highest value	Highest value
Amount raised	R3 000	R4 000	R7 000	R9 000	R14 000	R20 000	R25 000

A data value is an **outlier** if it is less than  $Q_1 - 1,5 \times IQR$  or it is greater than  $Q_3 + 1,5 \times IQR$ .

- a. Determine if there are any outliers in the data of the summary above. (5)

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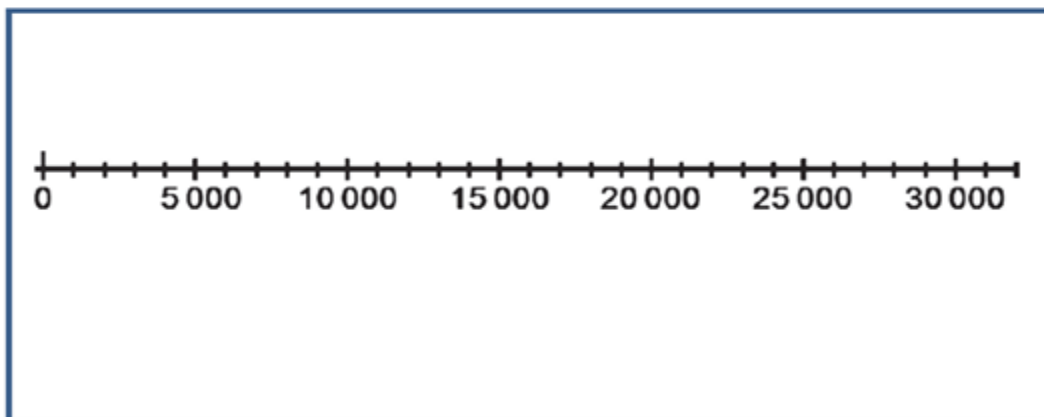
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- b. On the number line below, draw the box and whisker plot for the given data. Indicate clearly any outliers. (4)



- c. Comment on the skewness of the amount of money raised. (1)

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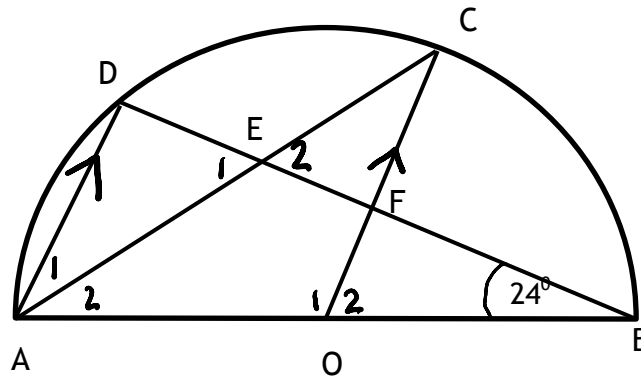
[10]

**Question 5**

a. In the diagram below, O is the centre of the semi-circle ABCD.

AD // OC. AC and BD intersect at E.

(3)



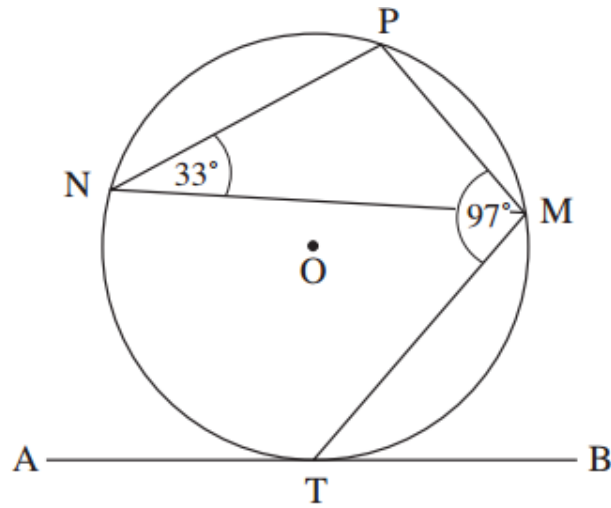
Fill in the missing statements/reasons (1-6) in the table below to determine  $\hat{E}_1$

STATEMENT	REASON
$\hat{D} = 90^\circ$	1.
2. $\hat{DAB} = \underline{\hspace{2cm}}$	Sum of angles of triangle
$\hat{DAB} = \hat{O}_2$	Corresponding angles; DA // OC
3. $\hat{A}_2 = \frac{1}{2} \times \underline{\hspace{2cm}}$	4.
5. $\therefore \hat{A}_1 = \hat{A}_2 = \underline{\hspace{2cm}}$	
6. $\hat{E}_1 = \underline{\hspace{2cm}}$	Sum of angles of triangle.



- b. In the diagram,  $ATB$  is the tangent to the circle at point  $T$ . Given that  $\widehat{PNM} = 33^\circ$  and  $\widehat{TMP} = 97^\circ$ , find  $\widehat{MTB}$

(3)




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**SECTION B (50 MARKS)**

**Question 6**

a. Given :  $\tan \theta = \frac{\sqrt{3}}{2}$  and  $\cos \theta < 0$ .

Using a sketch and without the use of a calculator, express each of the following in its simplest surd form:

1.  $\sin 2\theta$  (4)

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2.  $\cos (\theta - 90^\circ)$  (2)

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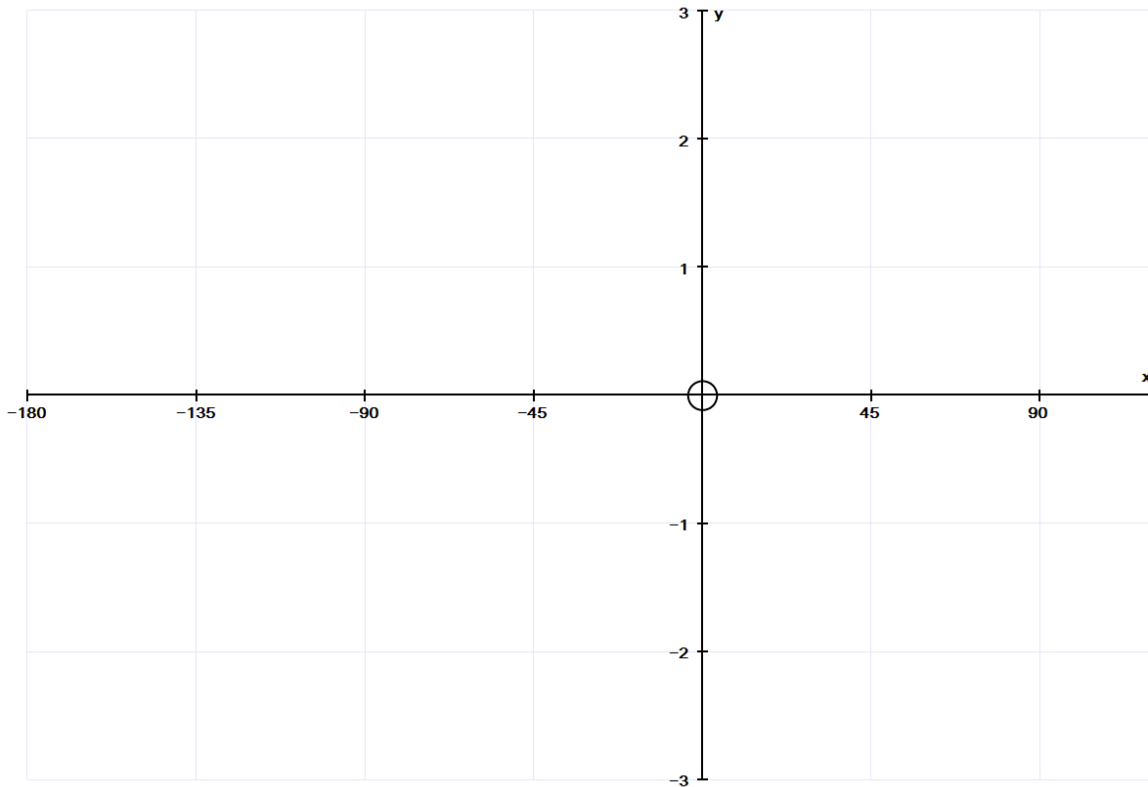


### Question 7

a. On the graph paper below, sketch the graphs of:

$$f(x) = \sin 2x \quad \text{and} \quad g(x) = \cos(x - 45^\circ)$$

$$\text{for } x \in [-180^\circ; 90^\circ] \quad (4)$$



b. Mark, using A; B; C ..... etc., where the solutions to the equation  $f(x) = g(x)$  can be read off on the graphs. (1)

c. Use the graphs to determine the value(s) of  $x$  for which  $f(x) - g(x)$  will be a maximum. (2)

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d. Give the values of  $x$  for which  $g'(x) > 0$  (2)

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### Question 8

Calculate the general solution to the equation:

$$2 \cos \theta - 3 \sin \theta = 0$$

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### Question 9

a. In each case, fill in the missing statement which would lead to the given deduction.

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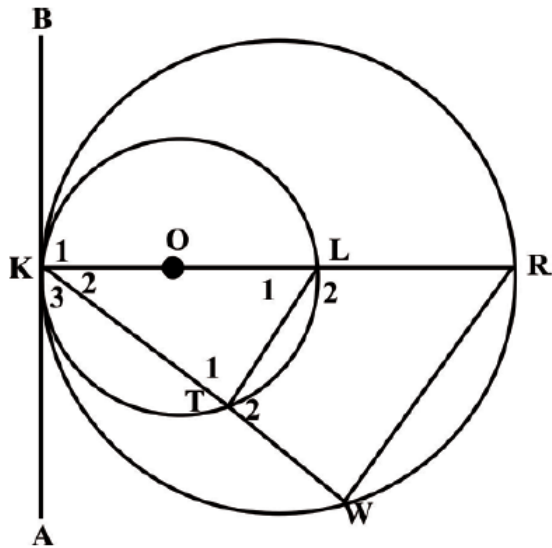
STATEMENT	$\hat{C} = \hat{A} + \hat{B}$
STATEMENT	
DEDUCTION	$\therefore \hat{C} = 2\hat{B}$

2.

STATEMENT	$\hat{C} = \hat{A} + \hat{B}$
STATEMENT	
DEDUCTION	$\therefore \hat{A} = \hat{D}$

(2)

- b. In the diagram below, two circles touch at K.  
 AKB is the common tangent and O is the centre of the smaller circle.  
 KO is produced to meet the circles at L and R.  
 KTW is a straight line. LT and RW are drawn.



1. Prove that KR is the diameter of the larger circle. (3)

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2. Show that:  $\frac{KL}{LR} = \frac{KT}{TW}$  (2)

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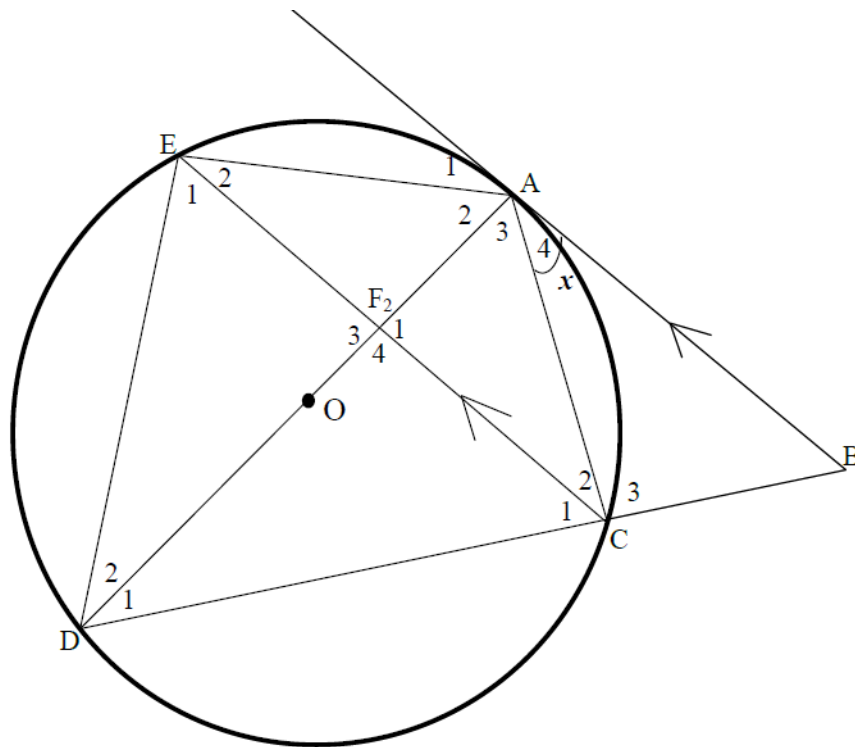
[7]

**Question 10**

a. Complete the statement of the theorem:

If two triangles are equiangular then their corresponding sides are \_\_\_\_\_ and the two triangles are similar. (1)

b. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA // CE. DC produced cuts tangent BA at B.



1. If  $\hat{A}_4 = x$ , determine with reasons, FIVE other angles equal to  $x$ .

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(5)

2. Prove that  $\triangle ACF \parallel \triangle ADC$  (2)

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3. Prove that  $AF = \frac{1}{2} AC$  (3)

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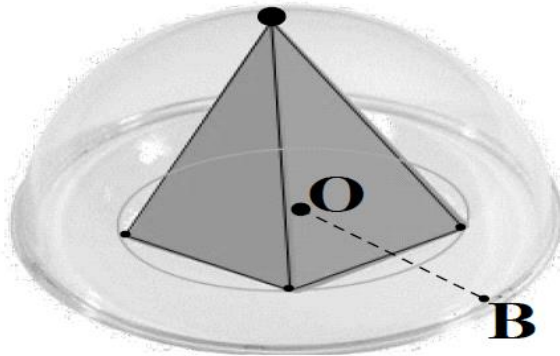
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[11]



### Question 11

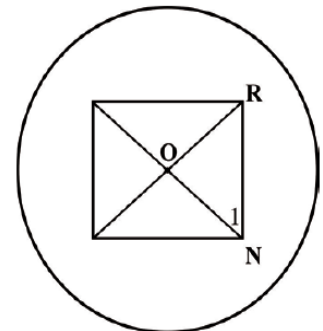
The diagram represents an aquarium in the shape of a hemispherical dome. Inside the dome is a solid right pyramid with a square base. The centre of the square is also the centre of the dome.



The top view of the hemispherical dome is represented in the diagram below.

ON = 10 cm and the radius of the dome is 25 cm.

Determine, to the nearest whole number, the volume of water required to completely fill the aquarium.



$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$$

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[4]

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**SECTION C (50 MARKS)**

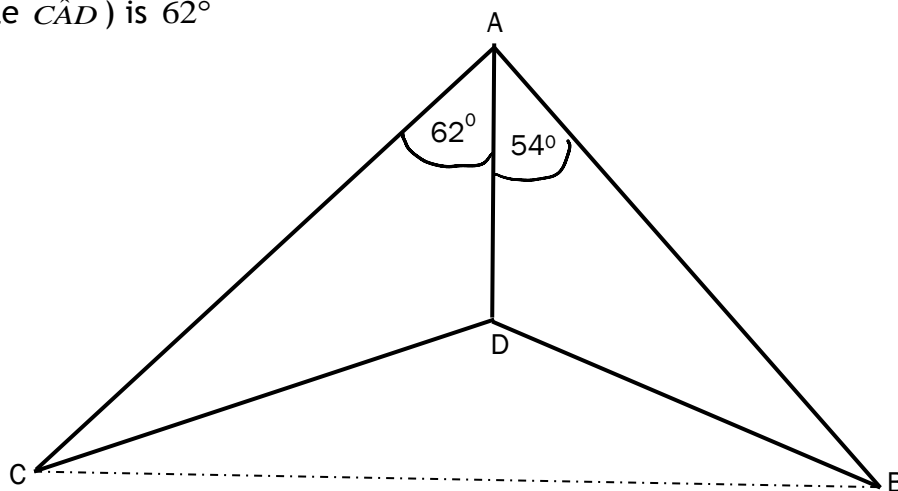
**Question 12**

The CN Tower in Toronto, Canada is the tallest free-standing structure in North America.

A woman at A on the observation deck, **11250 metres** above the ground, wants to determine the distance (CB) between two buildings at B and C on the ground below.

She observes the angle ( $\hat{CAB}$ ) formed by the lines of sight to these buildings is  $43^\circ$ .

She also observes that the angle ( $\hat{BAD}$ ) between the vertical AD and the line of sight to building B is  $54^\circ$  and to building C (angle  $\hat{CAD}$ ) is  $62^\circ$



Find the distance CB between the two buildings.

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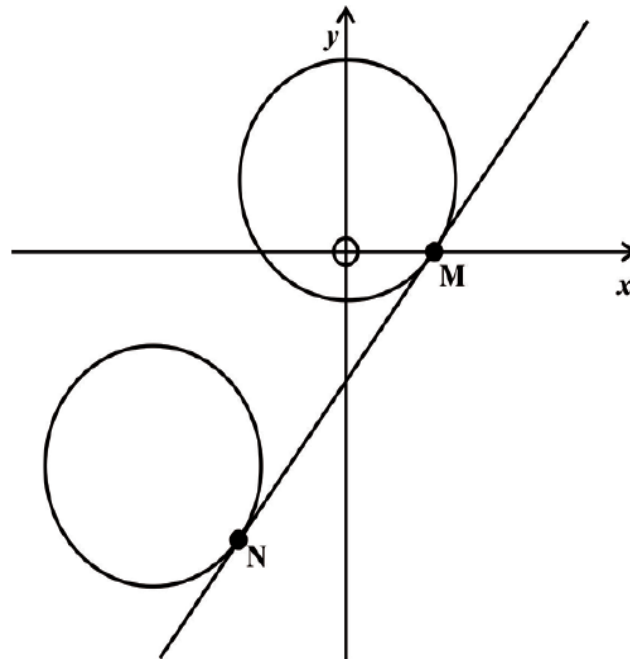
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[6]

**Question 13**

In the diagram below, the common tangent to the circles  $x^2 + y^2 - 6y = 16$  and  $(x + 9)^2 + (y + 9)^2 = 25$ , at M and N respectively, is drawn.



- a. Determine the co-ordinates of M, which lies on the x-axis. (2)

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- b. Determine the co-ordinates of the centre of the circle which passes through point M. (3)

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c. Give the equation of the common tangent MN. (3)

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d. Determine the co-ordinates of N . (5)

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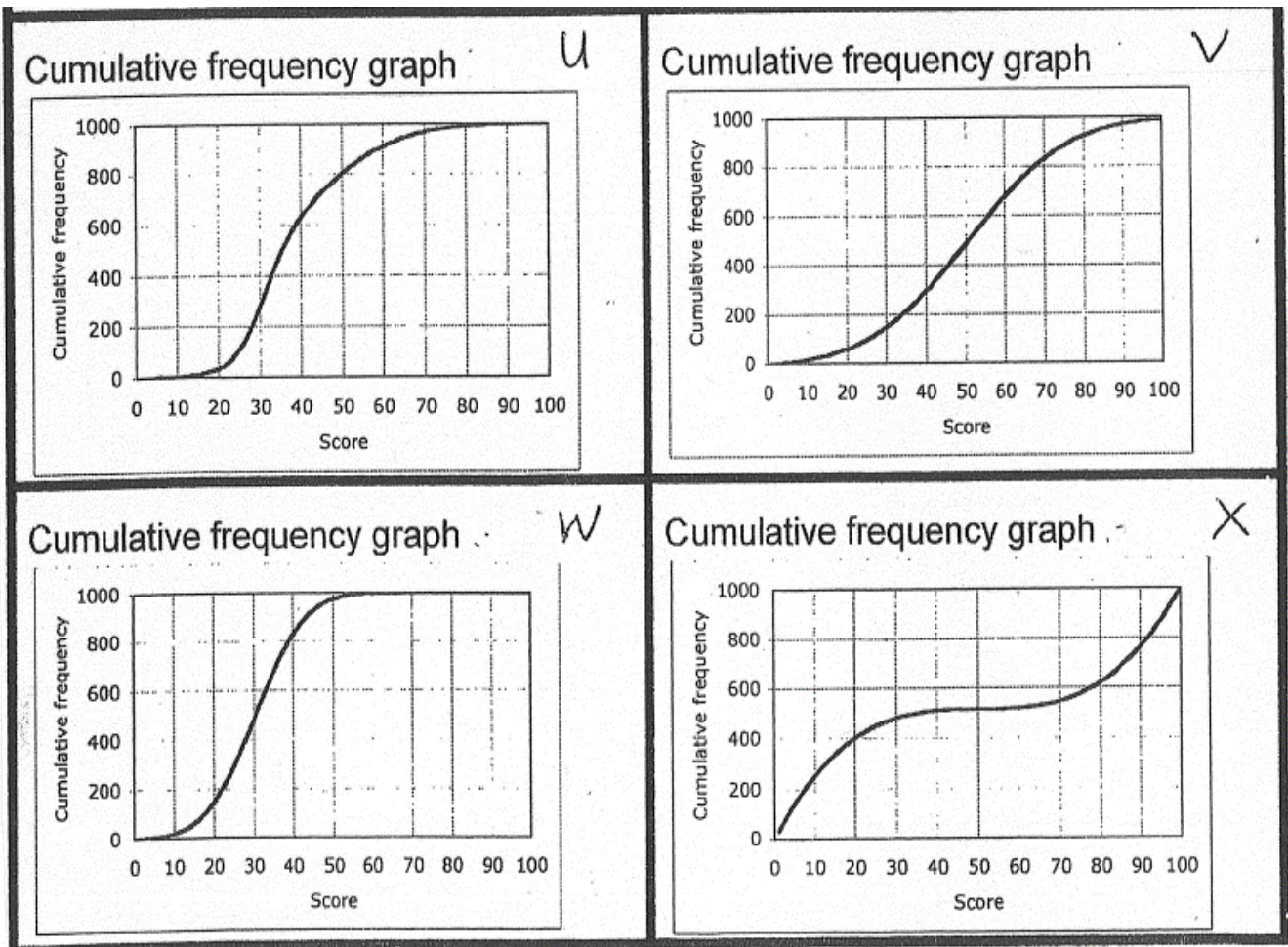
[13]

### Question 14

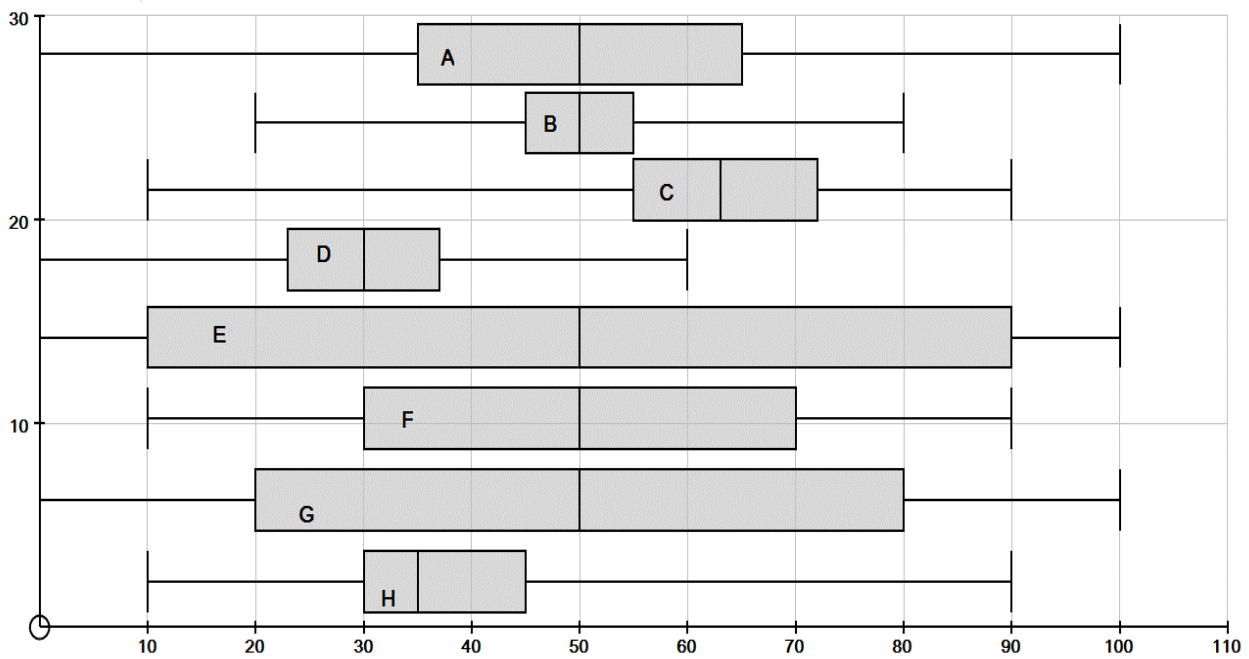
Match each of the following **four** Cumulative Frequency graphs (U - X) with the most fitting Box and Whisker plot (A-H).

Write your answers in the **table** on the next page:

### Cumulative Frequency Graphs:



### Box and Whisker Plots:



### Answers:

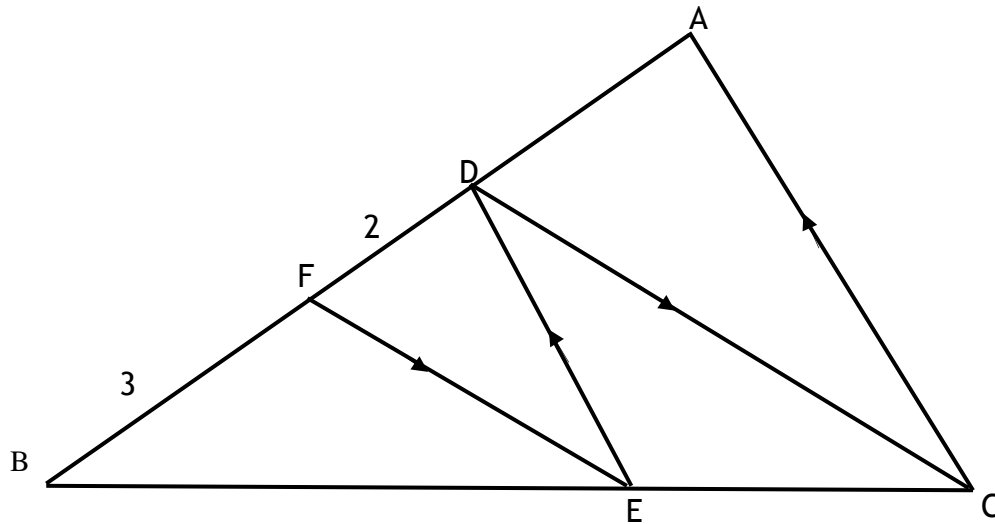
CUMULATIVE FREQUENCY GRAPH	BOX AND WHISKER PLOT
U	
V	
W	
X	

[8]

**Question 15**

In the diagram below, triangle ABC is drawn.  $DE \parallel AC$  and  $EF \parallel CD$

$BF = 3$  and  $FD = 2$



- a. Determine  $\frac{\text{Area } \triangle BED}{\text{Area } \triangle BCD}$  (3)

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- b. Determine the length of DA. (3)

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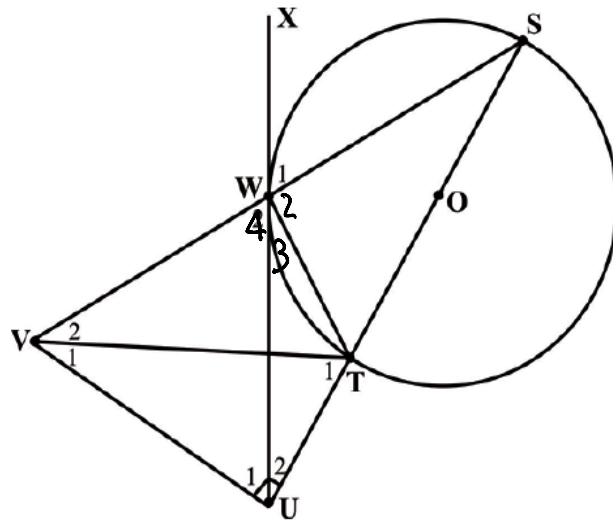


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[6]

**Question 16**

In the diagram,  $O$  is the centre of the circle with diameter  $ST$  produced to  $U$ .  $XU$  is a tangent to the circle at point  $W$  and chord  $SW$  is produced to  $V$ .  $VU \perp US$



- a. Prove that  $WVUT$  is a cyclic quadrilateral. (2)

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- b. Hence, show that  $\hat{V}_1 = \hat{S}$  (2)

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- c. Would it be possible to draw a circle through points  $S$ ,  $V$  and  $T$ ?  
Give a reason for your answer. (1)

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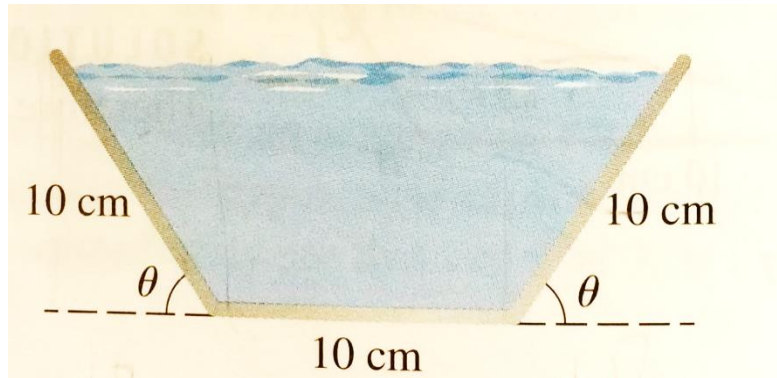
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[5]



### Question 17

A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle  $\theta$ .



Show that the cross-sectional area of the gutter is modelled by the function:

$$A(\theta) = 100 \sin \theta + 100 \sin \theta \cos \theta.$$

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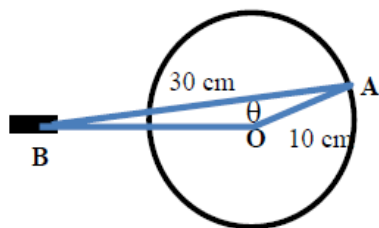
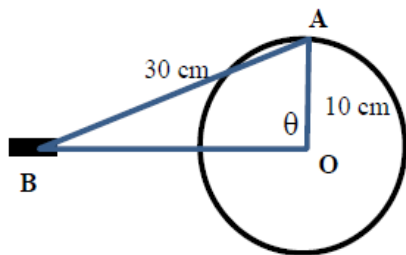
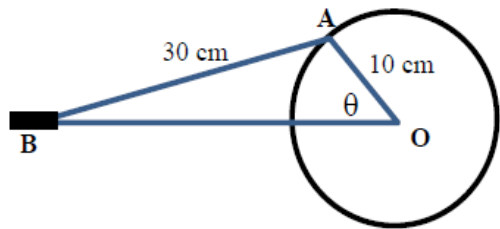
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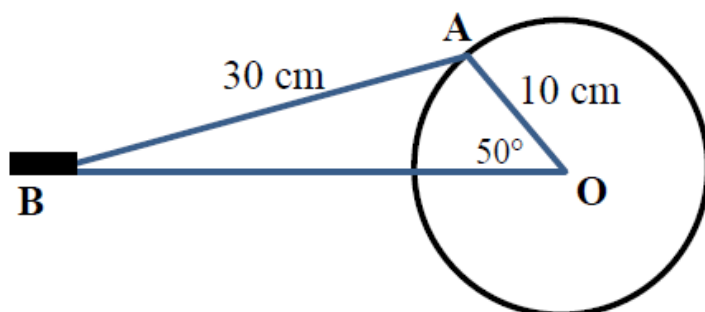
[3]

**Question 18**

In the diagrams below, arm OA is 10 cm long and rotates clockwise about centre O. The connecting rod AB is 30 cm long and point B moves on a fixed line through O.



In the diagram below,  $\hat{AOB} = 50^\circ$ ,  $\hat{BAO}$  is obtuse and the length of OB = 35,4 cm.



a. How far does B move when angle  $\hat{A}OB$  changes from  $50^\circ$  to  $120^\circ$ ? (6)

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b. How far does B move when angle  $\hat{A}OB$  changes from  $0^\circ$  to  $180^\circ$ ? (3)

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[9]