

Beaulieu College



Mathematics Department

GRADE 12

MATHEMATICS

PAPER 1

Time: 3 Hours 150 marks
Date: 13 July 2017
Examiner: Ms Smith Moderator: Mr Scholefield

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 12 pages and a separate Information Sheet of 2 pages (i – ii).
Please check that your paper is complete.
 2. Answer all the questions on the folio pages and read your questions carefully.
 3. Write your name on the question paper in the space provided below and hand in your question paper with the folio pages.
 4. Please note that diagrams are not necessarily drawn to scale.
 5. All necessary working details must be shown.
 6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 7. Number your answers exactly as the questions are numbered.
 8. Round off your answers to ONE decimal digit where necessary, unless stated otherwise.
 9. It is in your own interest to write legibly and to present your work neatly.
-

NAME: _____ Grade 12

MARKING GRID

Question	Algebra & Equations	Patterns & Sequences	Finance, Growth & Decay	Functions & Graphs	Probability	Differential Calculus
1	/22					
2				/10		
3			/15			
4		/19				
5						/9
6					/16	
7		/7				
8						/7
9				/24		
10						/9
11						/12
	/22	/26	/15	/34	/16	/37
TOTAL						/150

SECTION A

QUESTION 1

(a) Solve for x :

$$(1) \quad \frac{3}{x-2} = x-4 \quad (4)$$

$$(2) \quad (2x-1)(x-7) > 0 \quad (3)$$

$$(3) \quad \sqrt{5x+6} = -2x \quad (4)$$

(b) Solve for x in terms of k , leaving the answer in its simplest form:

$$5^{x-k} + 2(5^{x-k}) = 15(5^{k^2}) \quad (4)$$

(c) Given that $x = 3$ is a solution to the equation $mx^3 + x^2 - 25x + 6m = 0$, determine the value of m . (3)

(d) Given: $2x^2 - 3x + p = 0$

(1) Express the roots of the equation in terms of p . (2)

(2) For which values of p will the roots be equal? (2)

[22]

(Please turn over for Question 2.)

QUESTION 2

(a) Given: $f(x) = \frac{4}{x+5} - 7$

(1) Write down the equations of the asymptotes of $f(x)$. (2)

(2) Determine the equation of a tangent to the graph of $f(x)$ at $x = -1$ if it is given that $f'(-1) = -\frac{1}{4}$. (3)

(b) Given: $g(x) = -(x-4)^2 + 9$

Sketch the graph of g , clearly showing the turning point and the intercepts with the axes. (5)

[10]

QUESTION 3

ROUND OFF ALL ANSWERS IN THIS QUESTION TO TWO DECIMAL PLACES

(a) Jeffrey decides to open his own *Tea Merchant* franchise and takes out a loan of R940 000 from a bank, repayable over 7 years. The bank calculates the monthly payments of R17 357 at an interest rate of 13,5% per annum compounded monthly. Payments start 1 month after the loan is received.

(1) Four years after Jeffrey took out the loan, he decides to settle it. Determine the balance outstanding at the end of 4 years. (4)

(2) Determine the amount of interest Jeffrey paid on the loan in the first 4 years. (3)

- (b) Beaulieu College bought a new bus for R785 000. The school was advised to set up a sinking fund to ensure that there would be enough money to replace the bus at the end of the 5th year.

The following applies:

- The bus depreciates at 18% per annum on a reducing balance basis.
- The bus company will buy back the bus at the end of 5 years at the depreciated value.
- Inflation is estimated at 6% per annum over the five-year period.
- The sinking fund is set up so that all payments will receive 11% interest per annum compounded quarterly.

- (1) Determine the amount of money required at the end of 5 years to replace the bus. (4)

- (2) Determine the quarterly payments that should be made into the sinking fund to ensure that the bus can be replaced at the end of 5 years. (4)

[15]

QUESTION 4

- (a) The first three terms of an infinite geometric series are 27 ; 18 and 12 .

- (1) Write the series in Σ -notation. (3)

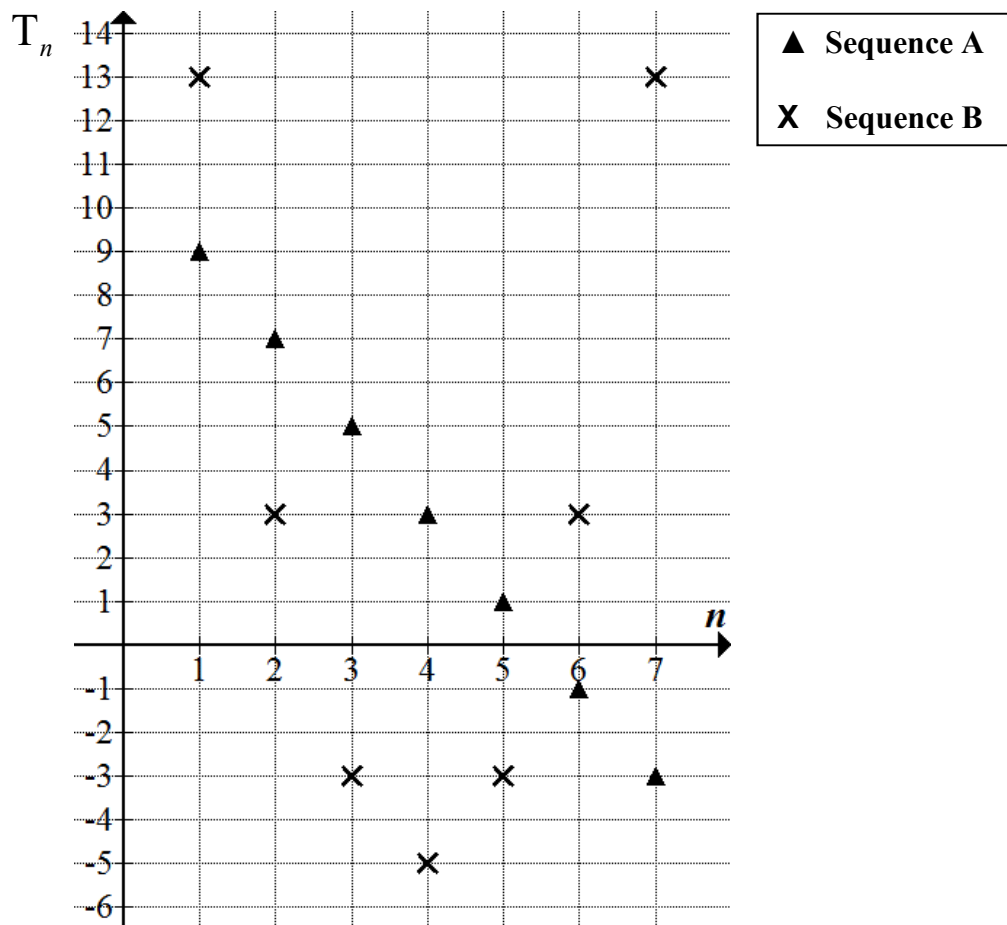
- (2) Why is the sum of the series a finite number? (1)

- (3) Determine the sum of the series. (2)

- (b) In an arithmetic series $S_4 - S_3 = 5$ and the eleventh term has a value of -37 .

- Calculate T_1 . (5)

- (c) The following graph represents two different sequences or number patterns, Sequence A and Sequence B:



- (1) What type of sequence is Sequence A? Give a reason for your answer. (2)
- (2) What type of sequence is Sequence B? Give a reason for your answer. (2)
- (3) Determine the n^{th} term of Sequence B. (4)

[19]

(Please turn over for Question 5.)

QUESTION 5

(a) Given $f(x) = 2x^2 - 5x$, determine $f'(x)$ from first principles. (5)

(b) Differentiate with respect to x : $y = \frac{4x^5 - \sqrt{x}}{x^2}$ (4)

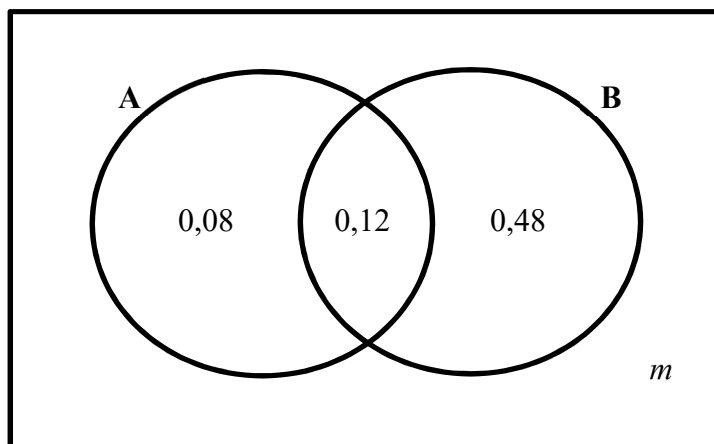
[9]

SECTION A: [75]

SECTION B

QUESTION 6

(a) The following Venn diagram is given, illustrating Events A and B:



(1) Determine the value of m , correct to two decimal places. (1)

(2) Are Events A and B mutually exclusive? Give a reason for your answer. (2)

(3) Are Events A and B independent? Give a reason for your answer. (2)

(4) Determine $P(A' \text{ or } B)$, correct to two decimal places. (1)

- (b) Antonio decides to go gambling on his birthday. He plays a computerised card game at the casino. The game is set up so that everybody who plays wins the first round. However, if you win a round, the probability of winning the next round is 85% that of winning the previous round. Antonio will only win the grand prize if he can win 24 rounds in a row, where the first counts as one of the 24 rounds.

Determine the probability of Antonio winning the grand prize. (3)

- (c) A survey was conducted among 80 boys and 100 girls to determine how many of them go to shopping malls while away on holiday at the coast. Their responses are shown in the partially completed table below.

	WENT TO A SHOPPING MALL WHILE ON HOLIDAY	DID NOT GO TO A SHOPPING MALL WHILE ON HOLIDAY	TOTAL
MALE	54	26	
FEMALE	<i>A</i>	48	
TOTAL	106	<i>B</i>	180

- (1) Determine the missing values, *A* and *B*. (2)

- (2) If it is given that the first event is choosing a male or a female, draw a tree diagram to represent the information in the table. Clearly indicate ALL the probabilities on the diagram. (5)

[16]

(Please turn over for Question 7.)

QUESTION 7

A sequence is given as: $p ; pq ; pq^2 ; pq^3 ; \dots$

The first differences of the sequence is given as: $12 ; 18 ; 27 ; \dots$

Determine the first three terms of the sequence.

(7)

[7]

QUESTION 8

Given: $y = f(x) = (x-a)(x-b)^2$ where $a, b \in \mathbb{R}$ and $0 < a < b$

It is further given that $f'(c) = 0$, $c \neq b$ and the graph has a point of inflection at $(d ; f(d))$.

Sketch a possible graph of $y = f(x)$, clearly indicating a, b, c and d on the x -axis and use the graph to solve the inequality $f'(x) \cdot f''(x) < 0$ in terms of a, b, c and/or d .

(7)

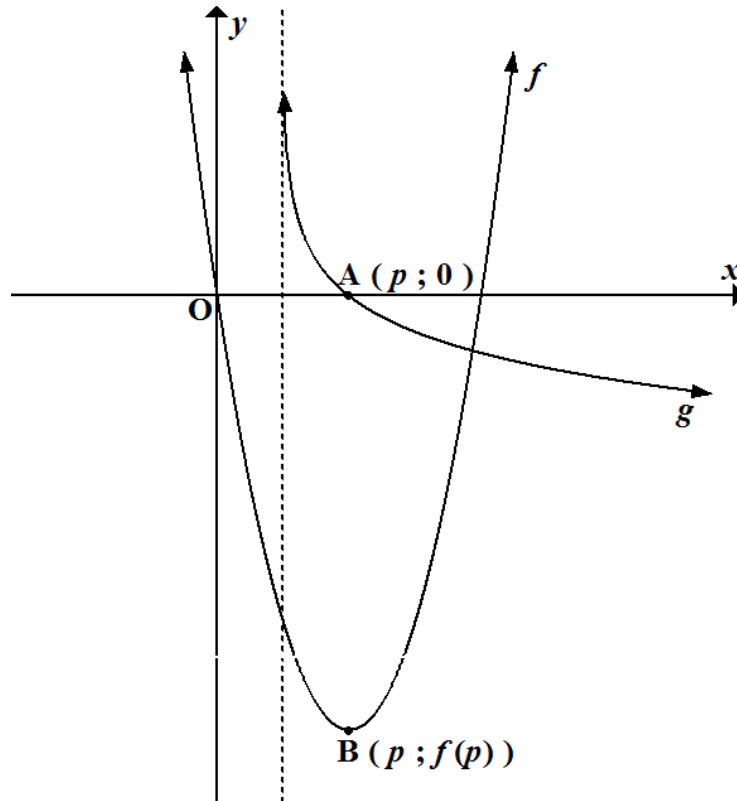
[7]

(Please turn over for Question 9.)

QUESTION 9

The sketch represents the graphs of the functions $f(x) = ax^2 + bx + c$ and $g(x) = \log_{\frac{1}{2}}(x-1)$.

$A(p; 0)$ is the x -intercept of the graph of g and $B(p; f(p))$ is the turning point of the graph of f .



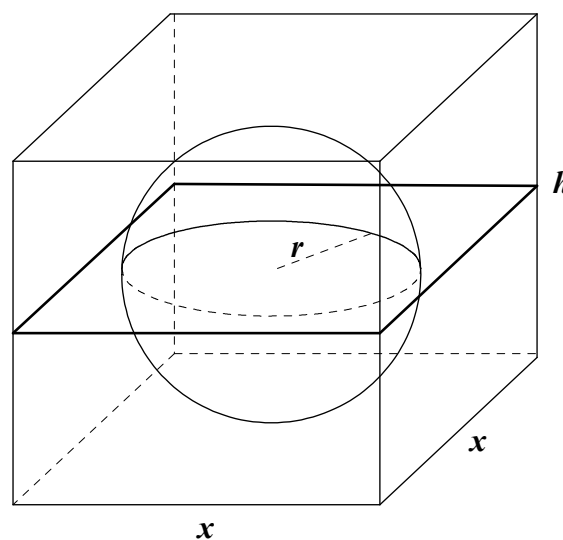
- Determine the value of p . (3)
- Use the graph to determine the values of x where $\frac{f(x)}{g(x)} < 0$. (4)
- Determine g^{-1} , the inverse of g in the form $y = \dots$ (3)
- State the range of g^{-1} . (2)
- Determine a, b and c if the average gradient between points O and B is -6 . (4)

- (f) Sketch the graph of the inverse of f clearly indicating all intercepts with the axes as well as the turning point. (5)
- (g) How will you restrict the domain of f so that its inverse is also a function? (1)
- (h) Determine the values of k for which $f(x) + k = 5$ has two roots that both have positive signs. (2)

[24]

QUESTION 10

In the diagram alongside, a sphere is cut out of a rectangular prism with a square base to make a ring box. The sphere has a radius of r cm and the prism has side lengths of x and h cm.



The radius of the sphere is given as 2,2 cm and the volume of the remaining wood, after the sphere has been cut out of the prism, is equal to $130,4 \text{ cm}^3$.

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

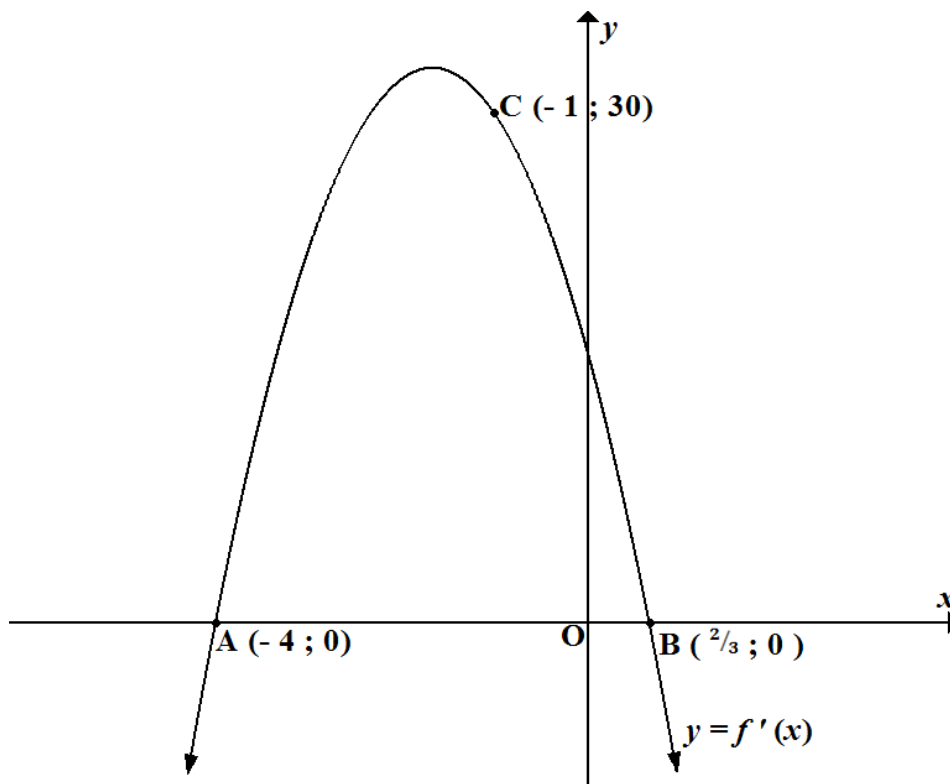
- (a) Determine the volume of the prism, correct to the closest whole number, before the sphere was cut out. (2)
- (b) Use your rounded off answer from (a) to show that the surface area of the prism can be written as $A = 2x^2 + \frac{700}{x}$. (4)
- (c) Using Calculus, determine the value of x for which the surface area of the prism will be a minimum. (3)

[9]

QUESTION 11

The graph of $y = f'(x)$ is sketched below where $f(x) = -2x^3 + ax^2 + bx + c$.

$A(-4; 0)$ and $B\left(\frac{2}{3}; 0\right)$ are the x -intercepts of the graph of $y = f'(x)$. $C(-1; 30)$ is a point on the graph of $y = f'(x)$.



- (a) Determine the values of a and b . Show all working. (6)
- (b) The line $y = 30x + 30$ is a tangent to the graph of $f(x)$ at the point $(p; 0)$.
Determine the values of p and c . (3)
- (c) Determine the interval(s) where $f(x)$ is concave up. (3)

[12]

SECTION B: [75]

TOTAL: [150]