Domino Servite School

Accreditation Number 13SCH0100008 Registration Number 122581

Mathematics Paper II

Grade 12

2017 Trial Examination

Name: Memo

Time: 3 hours
Examiner: H Pretorius

Total: 150
Moderators: B Hlongwane
J Bebb

Question 1 **[8 marks]**

Eight students entered a juggling competition. Their performances were scored by two judges. The scores (out of 20) are given in the table below.

Student	1	2	3	4	5	6	7	8
Judge 1 (x)	18	4	6	8	5	12	10	14
Judge 2 (y)	15	6	3	5	5	14	8	15

a) Determine the equation of the line of best fit of the scores given by the two judges. Round off to the third decimal place. (2) K

$y = 0,925x - 0,031$ ✓ ✓

b) A ninth student entered late for the competition and received a score of 15 from Judge 2. Estimate the score that might have been assigned by Judge 1. (3) RP

$15 = 0,925(x) - 0,031$ ✓

$0,925x = 15,031$ ✓

$x = 16$ ✓

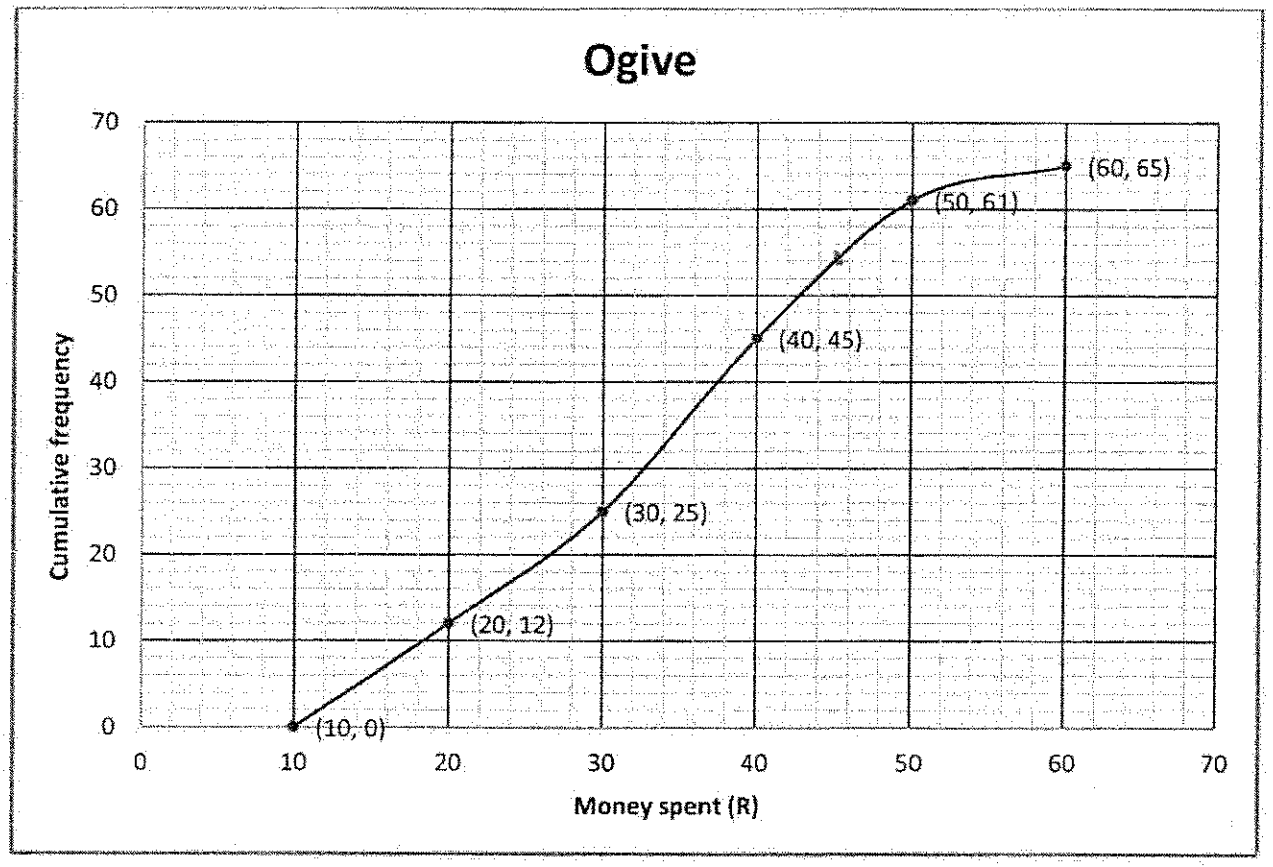
c) Comment on whether the judges are consistent in assigning scores to the performance of the students. Provide your answer with some evidence from the statistics. (3) CP

$r = 0,896$ ✓ Strong correlation ✓

∴ generally consistent

Question 2 **[6 marks]**

The amount of money, in rands, that learners spent while visiting a tuck shop at school on a specific day was recorded. The data is represented in the ogive below.



An incomplete frequency table is also given for the data.

Amount of money (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	13	20	b	4

a) How many learners visited the tuck shop on that day? (1) K

65 ✓

b) Determine the values of a and b in the frequency table. (2) K

$a = 12$ ✓

$b = 16$ ✓

c) Write down the modal class of this data. (1) K

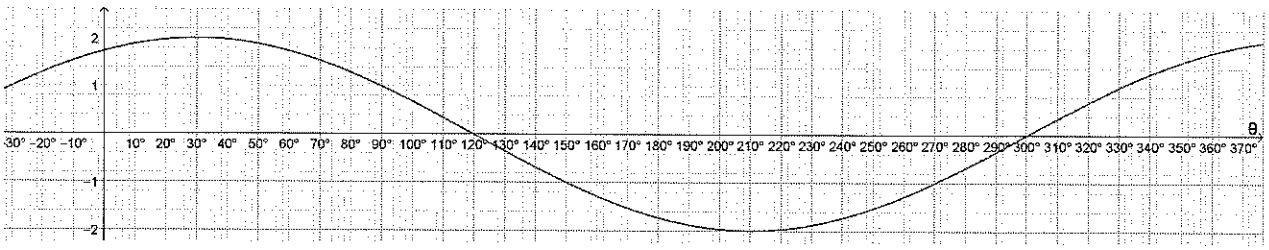
$30 \leq x < 40$ ✓

d) Use the ogive to estimate the number of learners who spent at least R45 on the day the data was recorded at the tuck shop. (2) RP

$65 - 54 = 11$ ✓

Question 3 **[23 marks]**

a) Find the equation for the graph shown below:



(3) RP

$y = 2\cos(\theta - 30^\circ)$ or $y = 2\sin(\theta - 120^\circ)$ etc...

b) Express $\cos 110°$ in terms of p if $\sin 20° = p$. (3) RP

$\cos 110^\circ = -\sin 20^\circ$

$= -p$

c) Find $\cos 2\theta$ in terms of m if $\sin\theta = \frac{1}{m}$. (3) LP

$\cos 2\theta = 1 - 2\sin^2\theta$ ✓

$= 1 - 2\left(\frac{1}{m}\right)^2$ ✓

$= 1 - \frac{2}{m^2}$ ✓

$= \frac{m^2 - 2}{m^2}$

d) If $\cos 36° = m$, find $\tan 216°$ in terms of m. (5) RP

$\tan 216° = \tan 36°$ ✓

$= \frac{\sin 36°}{\cos 36°}$ ✓

$= \frac{\sqrt{1-m^2}}{m}$ ✓

$\sin 36° = \sqrt{1 - \cos^2 36°}$ ✓

$= \sqrt{1-m^2}$ ✓

e) Simplify the expression $\tan(180° + x) - \frac{\cos x}{\sin(180°+x)}$. (5) RP

$\tan x - \frac{\cos x}{-\sin x}$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \frac{1}{\cos x \sin x}$

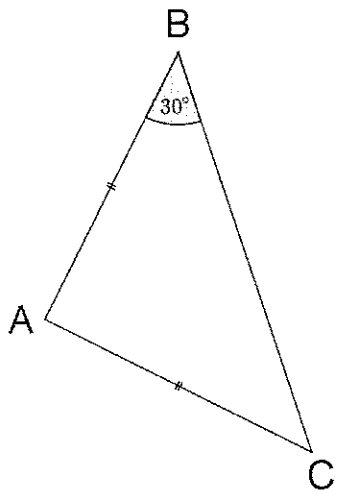
(OR)

$\tan x - \frac{\cos x}{-\sin x}$

$= \tan x + \frac{1}{\tan x}$

$= \frac{\tan^2 x + 1}{\tan x}$

f) In triangle ABC, AB = AC = $\sqrt{3}$ cm and $A\hat{B}C = 30°$. Find the area of triangle ABC. (4) RP



$\hat{C} = 30°$ base $\angle$'s isosc. $\triangle$ ✓

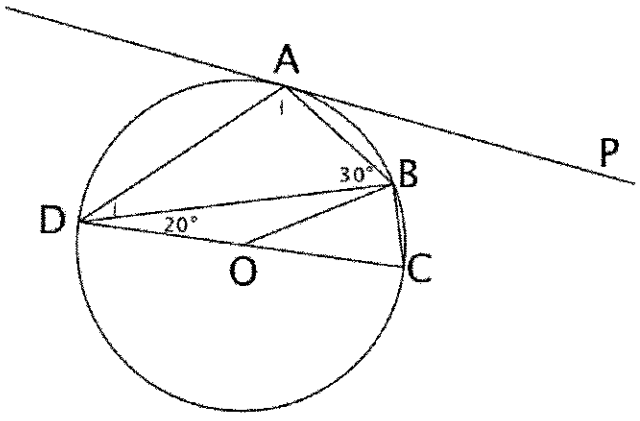
$\hat{A} = 120°$ int. $\angle$'s of $\triangle$ ✓

$\therefore$ Area $= \frac{1}{2} \times \sqrt{3} \times \sqrt{3} \times \sin 120°$ ✓

$= 1,3 \text{ cm}^2$ ✓ $\left(\frac{3\sqrt{3}}{4} \text{ cm}^2\right)$

Question 4 **[10 marks]**

a) In the given diagram, DOC is a diameter of the circle, and a tangent line AP is drawn at A. $O\hat{D}B = 20°$ and $A\hat{B}D = 30°$. DABC is a cyclic quadrilateral.



(1) Determine the magnitude of $D\hat{B}C$. (1) K

$D\hat{B}C = 90°$ ∠ on diameter

(2) Determine the magnitude of $A\hat{D}B$. (2) RP

$\hat{D}_1 = 180° - (20° + 90° + 30°)$ opp. ∠s of c.q.

$= 40°$ ✓

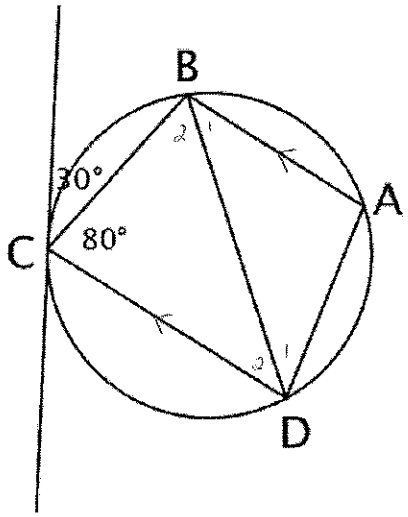
(2) Determine the magnitude of $D\hat{A}B$. (2) K

$\hat{A}_1 = 180° - (40° + 30°)$ int. ∠s of △

$= 110°$ ✓

b) The diagram shows a tangent to the circle at C, making an angle of 30° with chord BC. BADC is a cyclic quadrilateral.

If chord BA is **parallel** to chord CD and $B\hat{C}D = 80°$, find the size of $A\hat{D}B$.



(5) RP

$\hat{A} = 100°$ opp. ∠ of c.q. ✓

$\hat{D}_2 = 30°$ tan-ch th ✓

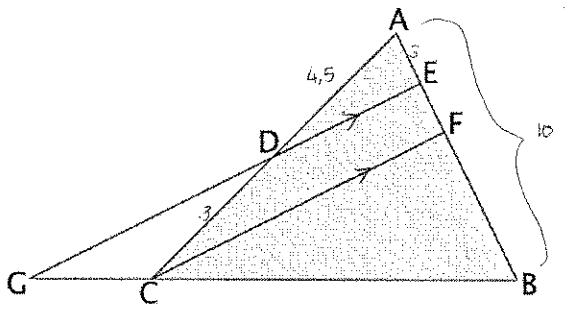
$\therefore \hat{B}_1 = 30°$ alt. ∠s; BA || CD ✓

so, $\hat{D}_1 = 180° - 130°$ int. ∠s of △ ✓

$= 50°$ ✓

Question 5 **[10 marks]**

a) In the given figure, AE = DC = 3, AB = 10 and AD = 4,5. Line GDE is **parallel** to CF,



(1) Name a triangle which is similar to triangle BFC. (1) K

$\triangle BEG$ ✓

(2) Calculate the length of EF. (2) RP

$\frac{EF}{3} = \frac{3}{4,5}$ line // 3rd side of $\triangle$ ✓

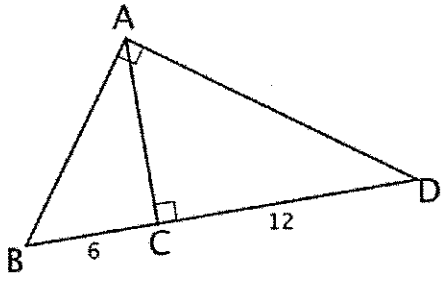
$\therefore EF = 2$ units ✓

(3) Find FC : EG. (2) RP

$\frac{FC}{EG} = \frac{BF}{BE}$ similar $\triangle$s ✓

$= \frac{5}{7}$ ✓

b) In the given diagram, angles BAD and ACD are right angles, BC = 6 and CD = 12. Find the length of AC.



(5) RP

$\triangle ACB \parallel\!\parallel \triangle DCA$ ⊥ from right angle

$\frac{AC}{BC} = \frac{DC}{AC}$ similar $\triangle$'s

$\frac{AC}{6} = \frac{12}{AC}$

$AC^2 = 72$

$\therefore AC = 6\sqrt{2} = 8,5u$

Question 6 **[10 marks]**

a) A circle of radius 13 has its centre at the point O (4; −2). If the point P (−1; b) lies on the circumference of the circle, find the possible values of b. (5) RP

$(x-4)^2 + (y+2)^2 = 13^2$ ✓

$(-1;b) \Rightarrow (-1-4)^2 + (b+2)^2 = 169$ ✓

$25 + (b+2)^2 = 169$

$(b+2)^2 = 144$

$b+2 = \pm 12$ ✓

$\therefore b = -2+12$ OR $b = -2-12$

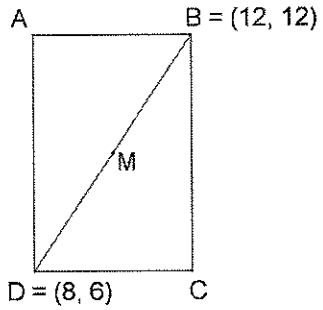
$= 10$ ✓ $= -14$ ✓

OR $b^2 + 4b - 140 = 0$ ✓

$(b+14)(b-10) = 0$ ✓

$b = -14$ OR $b = 10$ ✓

b) Rectangle ABCD is drawn in the Cartesian plane with sides parallel to the x- and y-axes and one diagonal through the points B (12; 12) and D (8; 6). Find the equation of the circle with diameter AC. (5)



$M = \left(\frac{12+8}{2}; \frac{12+6}{2}\right) = (10; 9)$ ✓

$AC = BD$ ✓ diagonals of rectangle

$\therefore r^2 = (10-8)^2 + (9-6)^2$ ✓

$= 13$ ✓

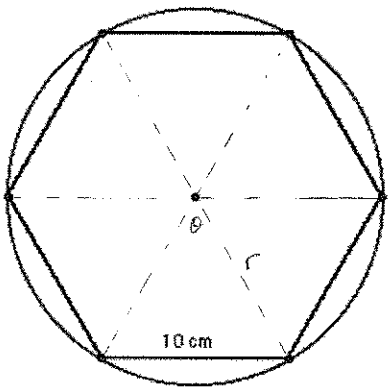
$(x-10)^2 + (y-9)^2 = 13$ ✓

Question 7 **[5 marks]** RP

A regular hexagon is drawn inside a circle, with all vertices on the circumference. Find an expression for the radius of the circle if the hexagon has a perimeter of 50 cm.

60 cm error

Mark with error



$\theta = \frac{360^\circ}{6} = 60^\circ$ ✓ $\angle$s round a point

$10^2 = r^2 + r^2 - 2r^2\cos 60^\circ$ ✓

$100 = 2r^2 - 2r^2\cos 60^\circ$

$100 = 2r^2(1 - \cos 60^\circ)$ ✓

$\therefore r^2 = \frac{100}{2(1-\cos 60^\circ)}$ ✓

$\therefore r = 10$ cm ✓

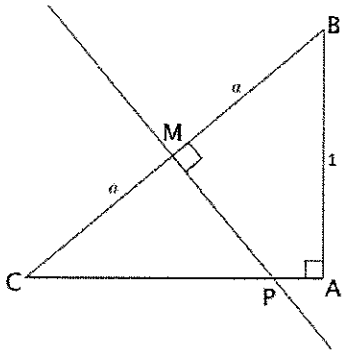
OR

∴ each $\triangle$ is equilateral ✓ int. $\angle$s of $\triangle$, isosc. $\triangle$ ✓

$\therefore r = 10$ cm ✓

Question 8 **[13 marks]**

a) In the diagram, MP is the perpendicular bisector of CB so that CM = MB = a. AB = 1 unit.



(1) Express AC in terms of a. (2) RP

$AC^2 = (2a)^2 - 1$ Pyth ✓

$\therefore AC = \sqrt{4a^2 - 1}$ ✓

(2) Prove that ΔMCP ||| ΔACB. (3) RP

$\hat{M}_1 = \hat{A}$ given ✓

$\hat{C}$ is common ✓

$\therefore \triangle MCP \,|||\, \triangle ACB$ equiangular ✓

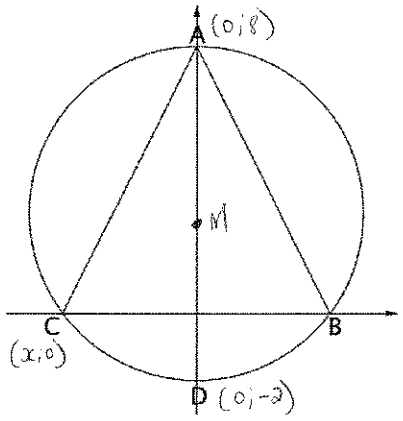
(3) Find the length of MP. (2) RP

$\frac{MP}{AB} = \frac{MC}{AC}$ similar Δ^s

$\therefore MP = \frac{a \times 1}{\sqrt{4a^2 - 1}}$

$MP = \frac{a}{\sqrt{4a^2 - 1}}$

b) A circle passes through the points A (0; 8) and D (0; −2), and has x-intercepts at B and C. If AD is a diameter of the circle, find the length of AC.



(6) CP

$M = (0;3)$ ✓ and $r = 5$ ✓

$(x-0)^2 + (y-3)^2 = 25$

$(x;0) \Rightarrow \quad x^2 + 9 = 25$ ✓

$x^2 = 16$

$\therefore x = \pm 4$ ✓

$C(-4;0) \Rightarrow \quad AC^2 = 8^2 + (-4)^2$ Pyth. ✓

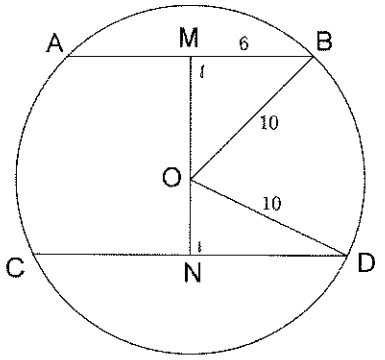
$= 80$

$\therefore AC = \sqrt{80}$

$= 4\sqrt{5} = 8{,}9\ u$ ✓

Question 9 **[12 marks]**

a) Two **parallel** chords AB and CD lie 14 cm apart on opposite sides of the centre of a circle of radius 10 cm (i.e. MN = 14 cm). If AB is 12 cm long, find the length of CD.



(6) CP

$\hat{M}_1 = 90°$ line midpt chord ✓

$\therefore MO^2 = 10^2 - 6^2$ pyth. ✓

$MO = 8$ cm ✓

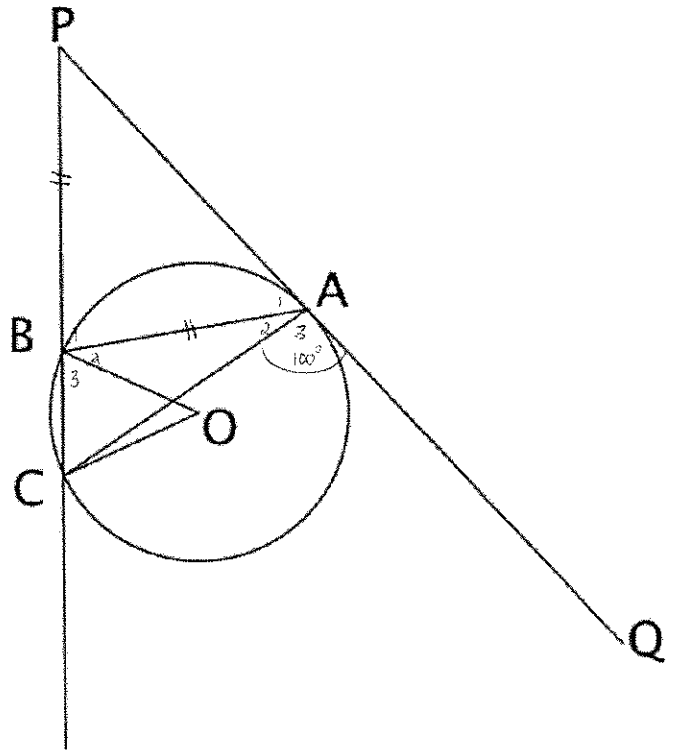
$ON = 6$ cm

$\hat{N}_1 = 90°$ co-int. $\angle$s; AB ∥ CD ✓

$\therefore ND = 8$ cm pyth. ✓

$\therefore CD = 16$ cm line ⊥ from midpt. ✓

b) In the diagram (not drawn to scale), PAQ is a tangent to the circle centre O at A, PBC is a secant to the circle, and PB = BA. If $C\widehat{A}Q = 100°$, find the size of $B\widehat{O}C$



(6)

$\hat{B}_2 + \hat{B}_3 = 100^\circ$ tan-ch th ✓

$\hat{A}_1 = \hat{P}$ base $\angle^s$ isosc. $\triangle$ ✓

$\therefore \hat{B}_2 + \hat{B}_3 = 2\hat{A}_1$ ext. $\angle$ of $\triangle$ ✓

$2\hat{A}_1 = 100^\circ$

$\hat{A}_1 = 50^\circ$ ✓

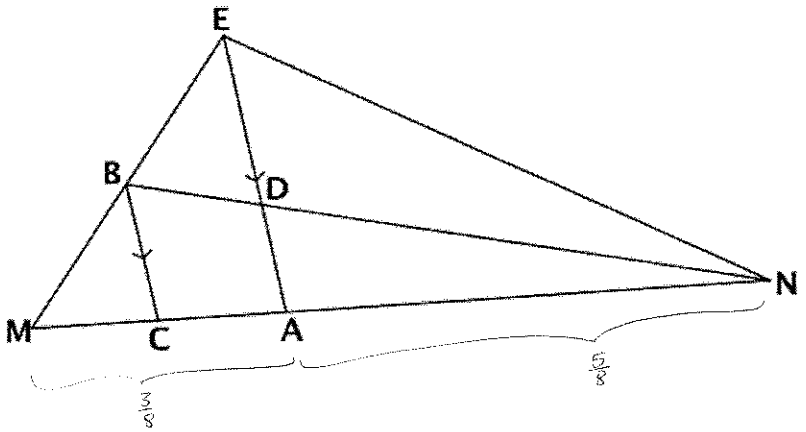
$\therefore \hat{A}_2 = 180^\circ - (100^\circ + 50^\circ)$ $\angle^s$ on str. line ✓

$= 30^\circ$

$\therefore \hat{O} = 60^\circ$ $\angle$ at centre = 2x $\angle$ at circ. ✓

Question 10 **[7 marks]**

a) In the diagram (not drawn to scale), BC is parallel to EA, $MA = \frac{3}{8}MN$ and $2MB = BE$. Find the value of BD:DN. (3)



$\frac{MB}{BE} = \frac{1}{2}$ given

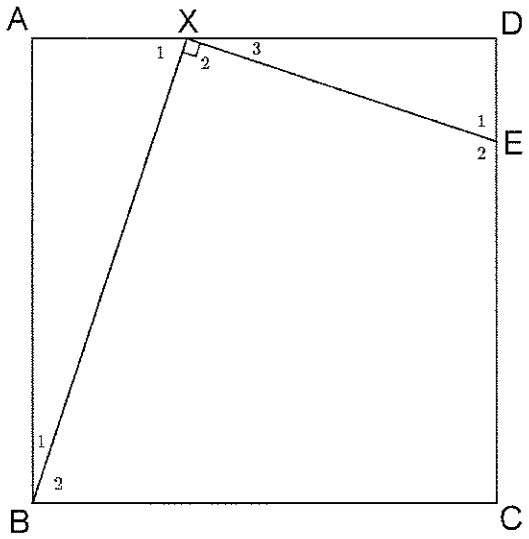
$\frac{MC}{CA} = \frac{MB}{BE} = \frac{1}{2}$ line // 3rd side of Δ

$\therefore CA = \frac{2}{5}$

$\frac{BD}{DN} = \frac{CA}{AN}$ line // 3rd side of Δ

$= \frac{2}{5}$

a) ABCD is a square. Given BX ⊥ XE, prove that Δ BAX ||| Δ XDE. (4)



$\hat{A} = \hat{D}$ properties of square ✓

$\hat{X}_2 + \hat{X}_3 = \hat{B}_1 + \hat{A}$ ext. ∠s of Δ ✓

but $\hat{X}_2 = 90^\circ$

$\therefore \hat{X}_3 = \hat{B}_1$ ✓

✓

$\therefore$ Δ BAX ||| Δ XDE equiangular