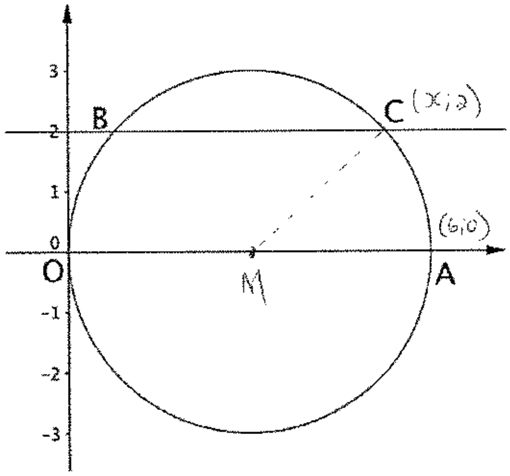
Question 11 **[6 marks]** 6

The diagram shows a circle in the Cartesian plane with diameter OA, where O is the origin and A is the point (6; 0). The horizontal line through the point (0; 2) intersects the circle at B and C. Find the x-coordinate of point C.



M (3;0) ✓ $\therefore r = 3$ ✓

$(x-3)^2 + (y-0)^2 = 9$ ✓

$(x;2) \Rightarrow$ $(x-3)^2 + (2)^2 = 9$ ✓

$(x-3)^2 = 5$

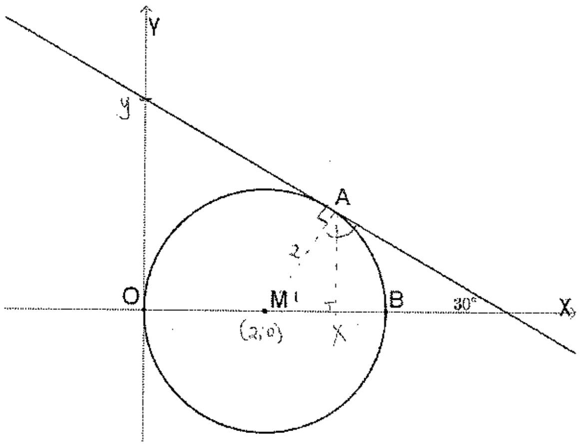
$\therefore x - 3 = \pm\sqrt{5}$ ✓

$C \Rightarrow x = 3 + \sqrt{5}$

$= 5{,}24$ ✓

Question 12 **[9 marks]** CP

The line $y = mx + c$, with $m < 0$, makes an angle of 30° with the x-axis. The line is a tangent to the circle defined by $x^2 - 4x + y^2 = 0$, with the point of contact in the first quadrant. Find the y-intercept of the tangent line.



$x^2 - 4x + \left(-\frac{4}{2}\right)^2 + y^2 = \left(-\frac{4}{2}\right)^2$ ✓

$(x-2)^2 + y^2 = 4$ ✓

$\therefore M(2;0)$ and $r = 2$

$\hat{A}_1 = 90°$ rad ⊥ tan ✓

$\therefore \hat{M}_1 = 60°$ int. ∠s of Δ ✓

$\sin 60° = \frac{y_A}{2}$

$\therefore y_A = \sqrt{3}$ ✓

$\cos 60° = \frac{MX}{2}$

$\therefore MX = 1$ ✓

$x_A = 3$

$A(3;\sqrt{3})$

$y - \sqrt{3} = \tan 150°(x-3)$ ✓

$y = -\frac{\sqrt{3}}{3}x + 2\sqrt{3}$ ✓

$\therefore y_{tan} = 2\sqrt{3}$

$(0;2\sqrt{3})$ ✓

Question 13 **[8 marks]** CP

Find the general solution to the equation $\cos 2x \, . \cos x = \sin 2x \, . \sin x$.

Only 4 marks if $\cos x = 0$ is discarded.

$$(\cos^2 x - \sin^2 x)\cos x = 2\sin x \cos x \sin x$$

$$\cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x = 0$$

$$\cos^3 x - 3\sin^2 x \cos x = 0$$

$$\cos x(\cos^2 x - 3\sin^2 x) = 0$$

$$\cos x = 0 \quad \text{or} \quad \cos^2 x = 3\sin^2 x$$

$$\therefore \frac{1}{3} = \tan^2 x$$

$$\tan x = \pm\sqrt{\frac{1}{3}}$$

$$x = 90^\circ + 180^\circ.k \quad \text{or} \quad x = \pm 30^\circ + 180^\circ.k \quad \text{where } k \in \mathbb{Z}$$

OR

$$\cos 2x \cos x - \sin 2x \sin x = 0$$

$$\cos(2x + x) = 0$$

$$\cos 3x = 0$$

$$\therefore 3x = \pm 90^\circ + 360^\circ.k$$

$$x = \pm 30^\circ + 120^\circ.k \quad \text{where } k \in \mathbb{Z}$$

OR

$$(1 - 2\sin^2 x)\cos x = 2\sin x \cos x \sin x$$

$$\cos x - 2\sin^2 x \cos x = 2\sin^2 x \cos x$$

$$\cos x - 4\sin^2 x \cos x = 0$$

$$\cos x(1 - 4\sin^2 x) = 0$$

$$\cos x = 0 \quad \text{OR} \quad \sin^2 x = \frac{1}{4}$$

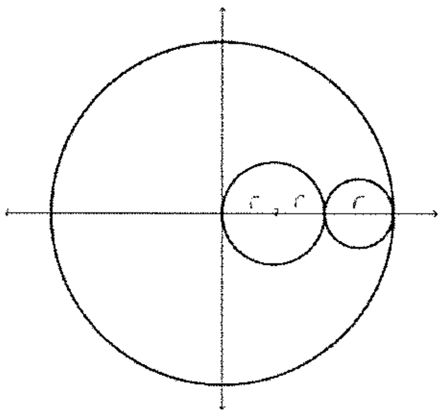
$$\sin x = \pm\frac{1}{2}$$

$$\therefore x = 90^\circ + 180^\circ.k$$

$$\text{or } x = \pm 30^\circ + 180^\circ.k \quad \text{where } k \in \mathbb{Z}$$

Question 14 **[5 marks]** PS

The diagram shows two tangential circle with their centres on the x-axis. These circles fit within the larger circle defined by $x^2 + y^2 = 144$, as shown. If the radius of the smallest circle is half the radius of the middle-sized circle, find the equation of the smallest circle.



$3r = 12$ ✓

$r = 4$ ✓ $\therefore r_{small} = 2$ ✓

centre of smallest ⊙ $\Rightarrow$ $(10;0)$ ✓

$\therefore (x - 10)^2 + y^2 = 4$ ✓

Question 15 **[5 marks]** PS

In a group of boys and girls, the average height of all the children in the group is 165 cm, the average height of the boys is 172 cm and the average height of the girls is 160 cm. Find the ratio of the number of boys to girls in the group.

$$\frac{172b + 160g}{b+g} = 165$$

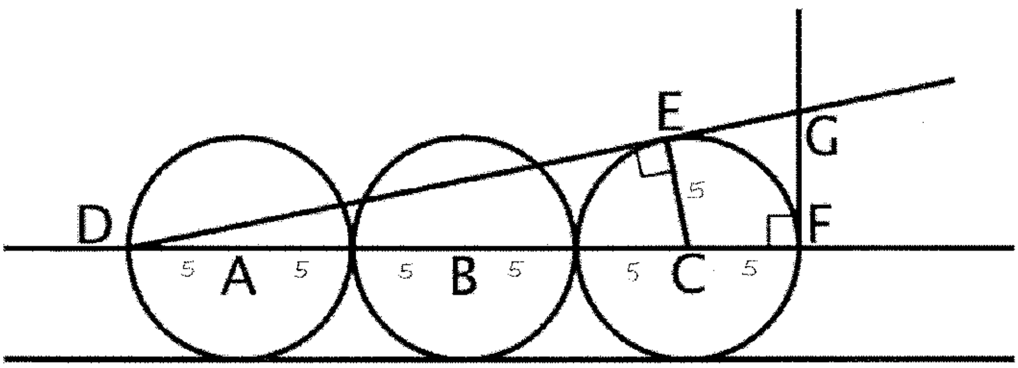
$$172b + 160g = 165b + 165g$$

$$7b = 5g$$

$$\frac{b}{g} = \frac{5}{7}$$

Question 16 **[7 marks]** PS

Three circles with centres A, B and C are tangent to each other. Each circle has a radius of 5 cm. Lines DE and FG are tangent to circle C and intersect at G. Find the length of FG.

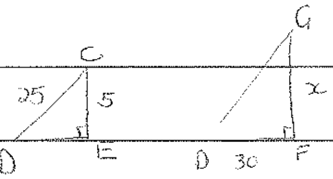


In $\triangle DCE$ and $\triangle DGF$

$\hat{D}$ is common

$\hat{E}_1 = \hat{F}_1 = 90°$ given

$\therefore \triangle DCE \,|||\, \triangle DGF$ equiangular



$DE^2 = (25)^2 - (5)^2$ Pyth.

$= 600$

$DE = 10\sqrt{6} = 24,5 \text{ cm}$

$\frac{FG}{5} = \frac{30}{10\sqrt{6}}$ similar $\triangle^s$

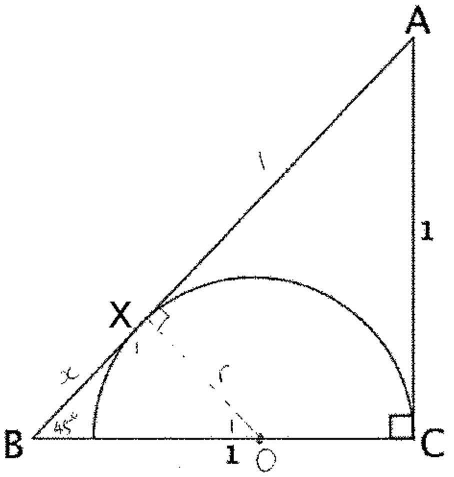
$\therefore FG = \frac{5\sqrt{6}}{2} = 6,1 \text{ cm}$

OR $\sin \hat{D} = \frac{5}{25}$ $\therefore \tan \hat{D} = \frac{FG}{30}$

$\therefore \hat{D} = 11,54\ldots°$ $\therefore FG = 6,1 \text{ cm}$

Question 17 **[6 marks]** PS

Triangle ABC is a right-angled isosceles triangle with AC = BC = 1. A semi-circle is drawn with its diameter on BC, so that AB and AC are tangents to the semi-circle. X is the point of contact of AB to the semi-circle. Determine the radius of the semi-circle.



$\hat{B} = 45°$ int. ∠s of isosc. Δ ✓

$(x+1)^2 = 1^2 + 1^2$ pyth. ✓

$(x+1)^2 = 2$

$x+1 = \sqrt{2}$

$x = -1 + \sqrt{2}$ ✓

$\hat{X}_1 = 90°$ tan ⊥ rad. ✓

$\hat{O}_1 = 90° - 45°$ int. ∠s of Δ ✓

$= 45°$

$\therefore x = r = -1 + \sqrt{2}$ base ∠s equal ✓

(0, 4 u)