DURBAN GIRLS' COLLEGE

TRIALS EXAMINATION 2017 **MATHEMATICS PAPER II**

TIME: 3 HOURS **150 MARKS**

EXAMINERS: Mrs M Dwyer, Mrs C Jacobsz, Mrs T Thorne

Name: MEMO

Maths teacher: ____________

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

- This question paper consists of 21 pages.
- Read the questions carefully.
- All questions must be answered on the question paper.
- Number your answers exactly as the questions are numbered.
- Diagrams are not necessarily drawn to scale.
- All answers must be given correct to **one decimal place** where necessary, unless stated otherwise.
- Approved calculators may be used, unless stated otherwise.
- All the necessary working details must be clearly shown.
- Reasons must be given for **ALL** geometry statements.

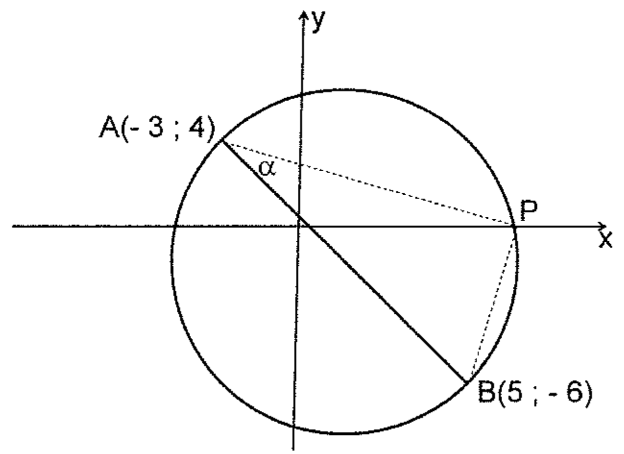
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
18	12	10	6	6	6	9	9

Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	TOT
8	9	7	8	7	7	14	6	8	150

Section A

Question 1

Points A(– 3 ; 4) and B(5 ; – 6) are points on the circle. (Line AB **does NOT** pass through the origin).



Determine:

a) the length of AB (**leaving your answer in surd form**). (2)

$AB^2 = 64 + 100$

$AB = 2\sqrt{41}$

b) the equation of AB. (3)

$m = \frac{10}{-8}$

$= -\frac{5}{4}$

$y - 4 = -\frac{5}{4}(x+3)$

$y = -\frac{5}{4}x + \frac{1}{4}$

c) the inclination of AB. (2)

$\tan\theta = -\frac{5}{4}$

$\therefore \theta = 128{,}7^\circ$

d) the coordinates of M, the midpoint of AB. (2)

M is $(1;-1)$

e) the equation of circle centre M and diameter AB. (2)

$(x-1)^2 + (y+1)^2 = 41$

f) the coordinates of P, the positive x-intercept of the circle. (3)

$(x-1)^2 + (0+1)^2 = 41$

$(x-1)^2 = 40$

$x-1 = \pm\sqrt{40}$

$x = 1 \pm \sqrt{40}$

at P, $x = 7{,}3$

g) i) the inclination of line AP. (3)

$m_{AP} = \frac{4}{-10{,}3}$

$\therefore A\hat{P}X = 158{,}8^\circ$

ii) hence determine α (BÂP) (1)

$\alpha + 128{,}7^\circ = 158{,}8^\circ$ (ext $\angle$ of Δ)

$\alpha = 30{,}1^\circ$

[18]

Question 2

Simplify

a) $\dfrac{\sin(360°-\theta)-\cos(90°+\theta)+\cos^2(180°-\theta)}{(1+\sin\theta)(1-\sin\theta)}$ (6)

$\dfrac{-\sin\theta + \sin\theta + \cos^2\theta}{1-\sin^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

$= 1$

b) $\dfrac{\cos^2 15°-\sin 15°\cos 75°}{\cos^2 15°+\sin 15°.\cos 15°.\tan 15°}$ (6)

$\dfrac{\cos^2 15° - \sin 15°.\sin 15°}{\cos^2 15° + \sin 15° \cancel{\cos 15°} \frac{\sin 15°}{\cancel{\cos 15°}}}$

$= \dfrac{\cos^2 15° - \sin^2 15°}{\cos^2 15° + \sin^2 15°}$

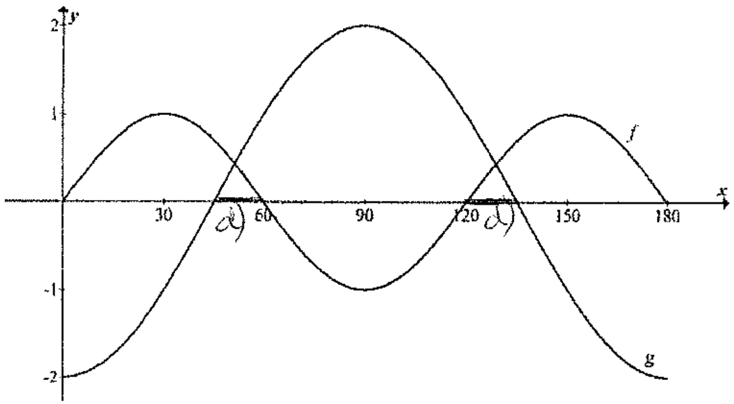
$= \cos 30°$

$= \dfrac{\sqrt{3}}{2}$

[12]

Question 3

The graph below shows the graphs of $f(x) = \sin bx$ and $g(x) = a \cos 2x$ for $x \in [0°; 180°]$



a) Write down the values of a and b. (2)

$a = -2$; $b = 3$

b) Write down the period of g. (1)

$180°$

c) For what value(s) of x is $f(x) - g(x) = 2$? (2)

$0°$; $30°$; $150°$; $180°$

d) **On your diagram**, mark the segment(s) on the x-axis that will satisfy the inequality $\frac{g(x)}{f(x)} \geq 0$ (2)

e) If $g(x)$ is shifted up by 2 units, write down amplitude and the range of the resulting graph. (3)

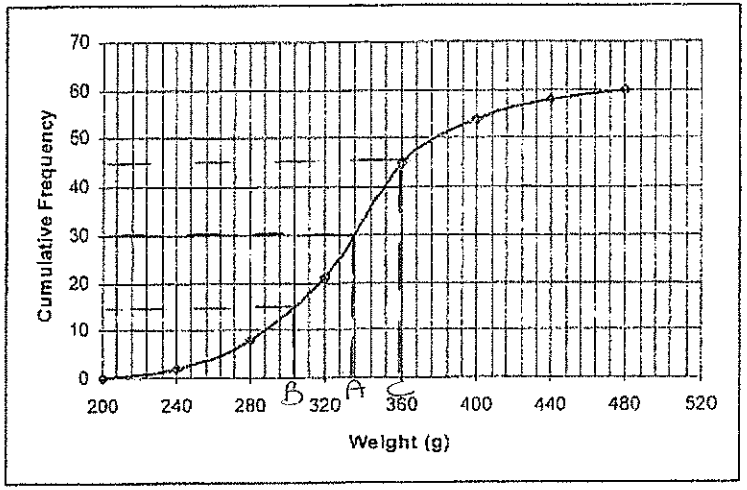
Amplitude: 2

Range: $y \in [0; 4]$

[10]

Question 4

A sample of 60 red onions was taken at Veggie City. Each one was weighed, and the results are shown on this cumulative frequency diagram.



Use this ogive to answer the following questions:

a) What was the median weight of the sample of red onions? Indicate with the letter A where you read this value from on the ogive. (2)

336g

b) What was the interquartile range (IQR) of the red onions? Indicate with the letters B and C where you read off the values (3)

360g − 304g = 56g

c) What percentage of the red onions have a weight of less than 360g? (1)

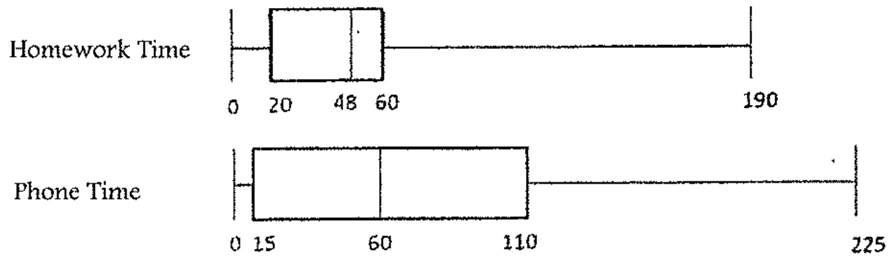
75%

[6]

Question 5

A group of grade 12s were asked how much time they spent on their phones per night compared to how much time they spent on their homework. Two box and whisker plots are drawn below to compare the results.

Phone and homework minutes per night



a) What percentage of grade 12's use their phones for at least 15 minutes per night? (1)

75%

b) What is the upper quartile for the phone time data? (1)

110

c) Is it more common for the grade 12 group of pupils at this school to spend more than 1 hour on their homework or more than 1 hour on their phones? Explain. (2)

25% on homework

50% on phones

∴ More common on phones

d) Is the homework box and whisker diagram positively or negatively skewed? (2)

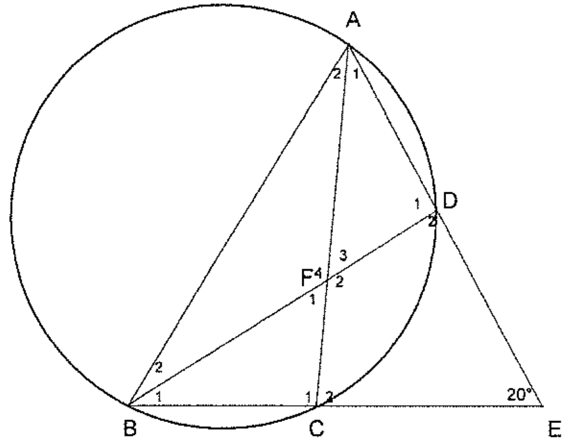
Positively.

Explain

Median > mean (or other)

[6]

Question 6



AB is the diameter and AE and BE are straight lines. $\hat{E} = 20°$.

a) Determine the size of $\hat{D}_1$ (1)

$\hat{D}_1 = 90°$ (∠ in semi circle)

b) Prove that DFCE is a cyclic quadrilateral. (3)

$\hat{D}_2 = 90°$ (∠s on str. line)

$\hat{C}_1 = 90°$ (∠ in semi circle)

∴ DFCE is cyclic (ext ∠ = int opp ∠)

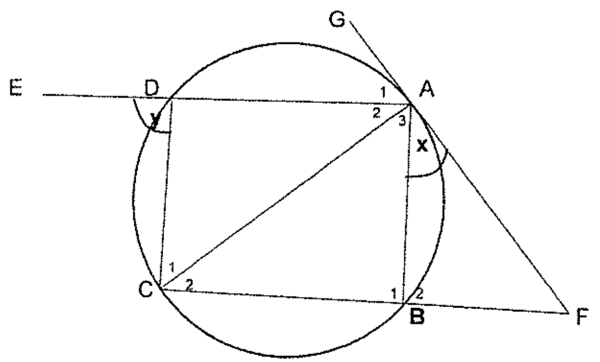
c) Calculate the sizes of $\hat{F}_1$ and $\hat{A}_1$. (2)

$\hat{F}_1 = 20°$ (ext ∠ cyclic quad)

$\hat{A}_1 = 70°$ (ext ∠ △ACE)

[6]

Question 7



ABCD is a cyclic quadrilateral with AD produced to E and CB produced to F. GA is a tangent to the circle at A.

a) Determine another angle equal to:

i) x (1)

$\hat{C}_2 = x$ (tan-chord)

ii) y (1)

$\hat{B}_1 = y$ (ext ∠ cyclic quad)

b) Determine the following angles in terms of x and y:

i) $\hat{F}$ (2)

$\hat{F} + x = y$ (ext ∠ △ABF)

$\therefore \hat{F} = y - x$

ii) $\hat{A}_3$ (2)

$\hat{A}_3 = 180° - x - y$ (∠s of △ABC)

c) If CA is a tangent to the circle through ABF at A, prove that y = 90°. (3)

$\hat{A}_3 = \hat{F}$ (tan-chord)

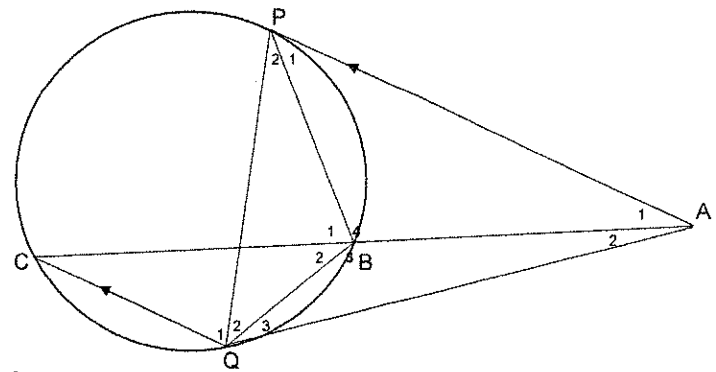
$\therefore 180° - x - y = y - x$

$-2y = -180°$

$y = 90°$

[9]

Question 8



AP and AQ are tangents to the circle. At P and Q respectively. AP //QC.
Prove:

a) ΔPBA /// ΔQBP (5)

In △PBA and △QBP

$\hat{P}_1 = \hat{Q}_2$ (tan-chord)

$\hat{A}_1 = \hat{C}$ (alt ∠s; AP∥QC)

$\hat{P}_2 = \hat{C}$ (∠s in same seg.)

$\therefore \hat{A}_1 = \hat{P}_2$

$\hat{B}_4 = P\hat{B}Q$ (3rd ∠)

$\therefore$ △PBA ||| △QBP (AAA)

b) PB = 4cm, QB = 6cm, and QP = 10cm. Calculate the length of QA. (4)

$\frac{PB}{QB} = \frac{PA}{QP}$ (sim △s)

$\frac{4}{6} = \frac{PA}{10}$

$6PA = 40$

$PA = \frac{20}{3}$

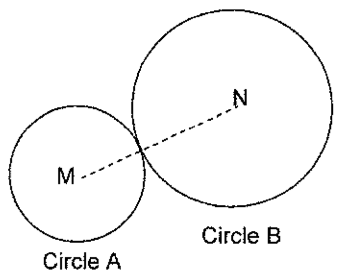
$\therefore QA = \frac{20}{3}$ (tans from same point)

[9]

76 marks for Section A

Section B

Question 9



Circles A and B touch each other externally. Circle A has centre M and the equation $x^2 + y^2 - 2x - 2y = 2$. Circle B has centre N and equation $x^2 + y^2 - 8x - 10y = k$, where k is a constant.
Determine the co-ordinates of M and N and hence calculate the value of k.

A: $x^2 - 2x + 1 + y^2 - 2y + 1 = 2 + 1 + 1$

$(x-1)^2 + (y-1)^2 = 4$

$M(1;1)$

B: $x^2 - 8x + 16 + y^2 - 10y + 25 = k + 16 + 25$

$(x-4)^2 + (y-5)^2 = k + 41$

$N(4;5)$

$MN^2 = 9 + 16$

$MN = 5$

Circles touch $\therefore r_A + R_B = MN$

$2 + \sqrt{k+41} = 5$

$\sqrt{k+41} = 3$

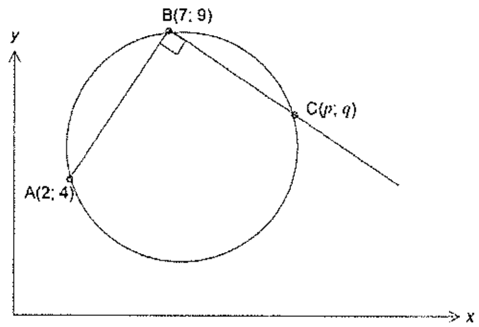
$k + 41 = 9$

$\underline{k = 32}$

[8]

Question 10

A(2; 4), B(7; 9) and C(*p*; *q*) are three points on a circle. AB ⊥ BC.



a) Show that $p + q = 16$ (3)

$AB \perp BC \therefore M_{AB} = -\frac{1}{M_{BC}}$

$\frac{5}{5} = -\frac{q-9}{p-7}$

$p - 7 = -q + 9$

$p + q = 16$

b) If the radius of the circle is $\sqrt{17}$, determine the values of *p* and *q* (6)

AC is diameter (subtends 90°)

$\therefore AC = 2\sqrt{17}$

$(p-2)^2 + (q-4)^2 = (2\sqrt{17})^2$

Sub $p = 16 - q$

$(16 - q - 2)^2 + (q-4)^2 = 4(17)$

$(14 - q)^2 + (q-4)^2 = 68$

$196 - 28q + q^2 + q^2 - 8q + 16 = 68$

$2q^2 - 36q + 144 = 0$

$(q - 12)(q - 6) = 0$

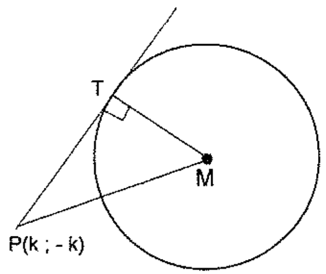
$q = 12$ or $q = 6$.

From diagram, $\underline{q = 6} \therefore \underline{p = 10}$

[9]

Question 11

M is the centre of the circle with equation $(x-1)^2 + (y-2)^2 = 2$.
P is any point outside the circle with coordinates $(k\,;-k)$. A tangent PT is drawn from P touching the circle at T.



a) Show that the length of the tangent PT is given by $PT^2 = 2k^2 + 2k + 3$ (4)

Centre M is $(1;2)$; $r = \sqrt{2}$

$PT^2 + 2 = (k-1)^2 + (-k-2)^2$ (Pythag)

$PT^2 + 2 = k^2 - 2k + 1 + k^2 + 4k + 4$

$PT^2 = 2k^2 + 2k + 3$

b) Determine the value of k in order for PT to be the **shortest possible** tangent that can be drawn from P to the circle. (3)

$PT^2 = 2k^2 + 2k + 3$

$\frac{dPT^2}{dk} = 4k + 2$

At max, $4k + 2 = 0$

$k = -\frac{1}{2}$

OR use $x = -\frac{b}{2a}$

OR complete the square

[7]

Question 12

a) Show that the equation $2sinx = \cos(x + 30°)$ can also be written in the form $tanx = \frac{\sqrt{3}}{5}$. (5)

$2\sin x = \cos x \cos 30° - \sin x \sin 30°$

$2\sin x = \cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right)$

$4\sin x = \sqrt{3}\cos x - \sin x$

$5\sin x = \sqrt{3}\cos x$

$\tan x = \frac{\sqrt{3}}{5}$

b) Hence, determine the solution to the equation, $2sinx = \cos(x + 30°)$, in the interval $x \in [-90°; 270°]$. (3)

$\tan x = \frac{\sqrt{3}}{5}$

$RA = 19{,}1°$

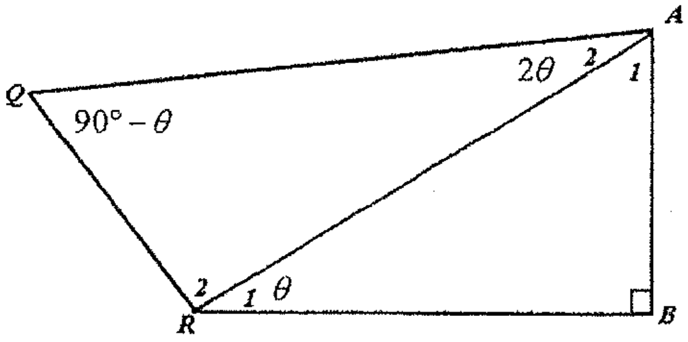
$x = 19{,}1° + k\,180°;\ k \in \mathbb{Z}$

$x = 19{,}1°$ or $199{,}1°$

[8]

Question 13

ABRQ is an aerial view of a plot of land where $\hat{R}_1 = \theta, \hat{A}_2 = 2\theta$ and $\hat{Q} = (90° - \theta)$.
QR = α units.



Determine AR in terms of θ and α, and hence show that $AB = \frac{a}{2}$.

In △ARQ

$\frac{AR}{\sin(90° - \theta)} = \frac{\alpha}{\sin 2\theta}$

$AR = \frac{\alpha \cos\theta}{2\sin\theta\cos\theta}$

$= \frac{\alpha}{2\sin\theta}$

In △ARB

$\frac{AB}{AR} = \sin\theta$

$AB = \frac{\alpha \sin\theta}{2\sin\theta}$

$= \frac{\alpha}{2}$

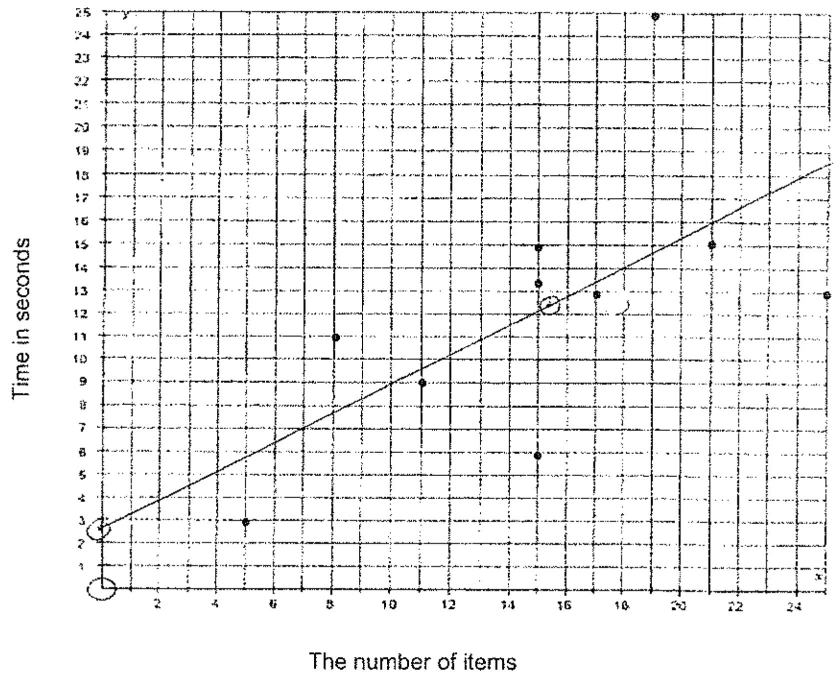
[7]

Question 14

Veggie City decided to do a survey to see how long, in seconds (y) it takes to scan (x) items at a till point. They decide to select the results from 9 shoppers. The results are indicated in the table below:

	A	B	C	D	E	F	G	H	I
x (the number of items)	5	8	12	15	15	17	20	21	25
y (time in seconds)	3	11	9	6	15	13	25	15	13

This information is represented on the scatter plot below:



$(\bar{x}; \bar{y}) = (15{,}3; 12{,}2)$

a) Use your calculator to determine the equation of the regression line. Give values correct to two decimal places. (2)

$y = 0,62x + 2,68$

b) Draw this line on the graph above, showing at least two points. (2)

c) Calculate the value of r, the correlation co-efficient for the data. (1)

$r = 0,628$

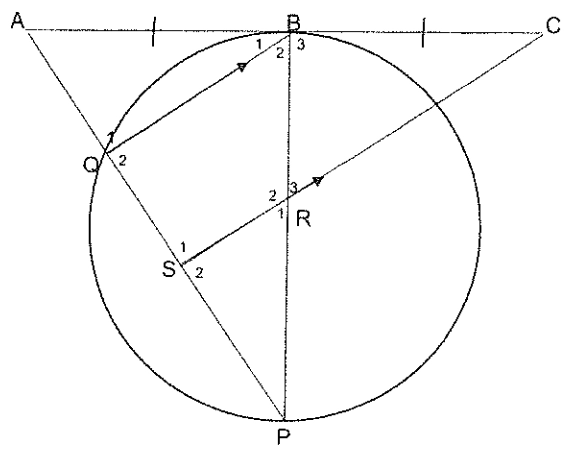
d) It was found that an error was made with the time value G(20;25). The point should have been (20;20). If the correlation co-efficient for the new regression line was now calculated, would it indicate a stronger or weaker correlation? Validate your answer without any further calculations. (2)

Stronger correlation

Less dispersion of points

[7]

Question 15



ABC is a tangent to the circle at B with AB = BC. BP is a diameter. AP meets the circle at Q. CRS is parallel to BQ.

a) Prove that:

i) $\Delta BCR \;|||\; \Delta SPR$ (5)

In ΔBCR and ΔSPR

$\hat{R}_3 = \hat{R}_1$ (vert. opp)

$\hat{B}_3 = \hat{Q}_2$ (tan-chord)

$\hat{S}_2 = \hat{Q}_2$ (corr. ∠s; CS || BQ)

$\therefore \hat{B}_3 = \hat{S}_2$

$\hat{C} = \hat{P}$ (3rd ∠)

$\therefore \Delta BCR \;|||\; \Delta SPR$ (AAA)

ii) 2RB.SP = SR.CA (3)

$\frac{BC}{SP} = \frac{BR}{SR}$ (sim ∆s)

But $BC = \frac{1}{2}AC$

$\therefore \frac{AC}{2SP} = \frac{BR}{SR}$

$\therefore 2BR.SP = SR.AC$

b) If BR : RP = 2 : 3, calculate AQ : QP (6)

$\frac{BR}{RP} = \frac{2}{3}$ (given)

$\therefore \frac{QS}{SP} = \frac{2a}{3a}$ (prop int)

BUT $\frac{AQ}{QS} = \frac{AB}{BC}$ (prop int

$\therefore \frac{AQ}{QS} = \frac{1}{1}$

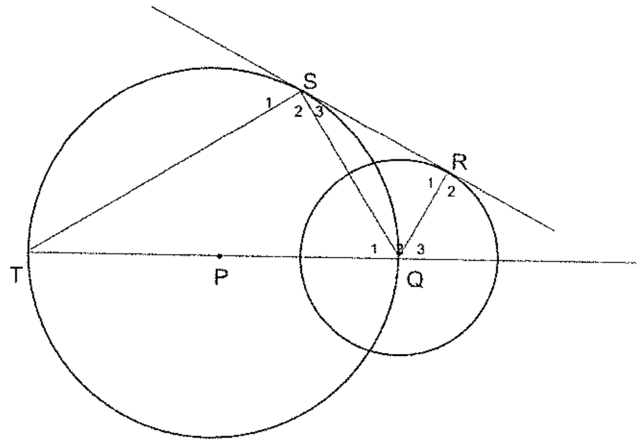
$\frac{AQ}{QS} = \frac{2a}{2a}$

$\therefore \frac{AQ}{QP} = \frac{2a}{5a}$

$= \frac{2}{5}$.

[14]

Question 16



The radius of the larger circle with centre P is twice that of the smaller circle with centre Q. SR is a tangent to both circles. ΔTSQ /// ΔSRQ

a) If the radius of the smaller circle is r, determine the length of SQ in terms of r. (3)

$TQ = 4r$

$\frac{SQ}{RQ} = \frac{TQ}{SQ}$ (sim Δs)

$\frac{SQ}{r} = \frac{4r}{SQ}$

$SQ^2 = 4r^2$

$SQ = 2r$

b) Calculate the size of $\hat{T}$ without using a calculator. (3)

$\hat{S}_2 = 90^\circ$ (∠ in semi circle)

$\therefore \sin \hat{T} = \frac{SQ}{TQ}$

$= \frac{2r}{4r}$

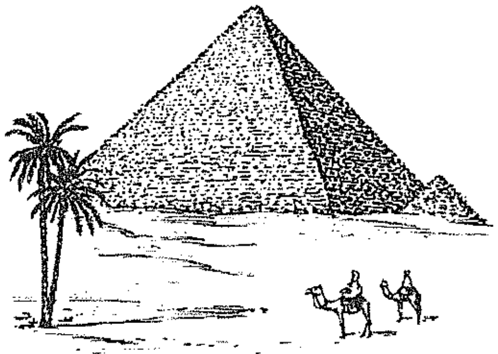
$= \frac{1}{2}$

$\therefore \hat{T} = 30^\circ$

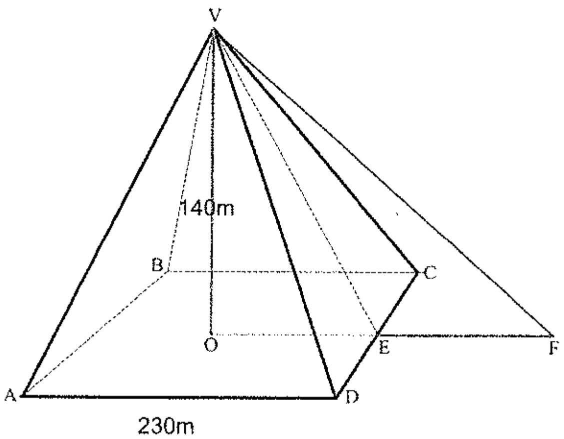
[6]

Question 17

The pyramid of Giza is situated near Cairo. It has a height of 140 metres and a square base with sides of length 230 metres.



In the sketch below, F is on the same horizontal plane as ABCDE. VO is the perpendicular height of the pyramid. The shadow that the pyramid is casting is represented by EF and has a length of 52 metres.



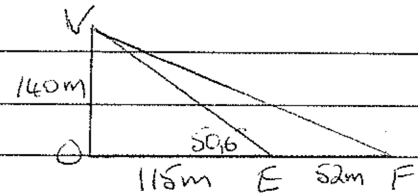
a) Calculate the magnitude of $V\hat{E}O$. (2)

$VO = 140m$; $OE = 115m$ ($\frac{1}{2}$ side)

$\therefore \tan V\hat{E}O = \frac{140}{115}$

$V\hat{E}O = 50{,}6^\circ$

b) Susie used Pythagoras to calculate the distance between the top of the pyramid and the end of the shadow, i.e. VF, to be 218m..
Using a **different** method to calculate the length of *VF*, determine whether you agree with her or not. (6)



$\tan F = \frac{140}{167}$

$\hat{F} = 40{,}0^\circ$

$\therefore E\hat{V}F = 10{,}6^\circ$ (ext ∠ of △VEF)

$V\hat{E}F = 129{,}4^\circ$ (∠s on str. line)

$\therefore \frac{VF}{\sin 129{,}4^\circ} = \frac{52}{\sin 10{,}6^\circ}$

$VF = \frac{52 \sin 129{,}4^\circ}{\sin 10{,}6^\circ}$

$VF = 218{,}4$ m

∴ Same answer if rounded to the nearest metre

[Note: if all angles were NOT rounded off, VF would $= 217{,}9197098$ m]
≈ 218 m

[8]

74 marks for Section B