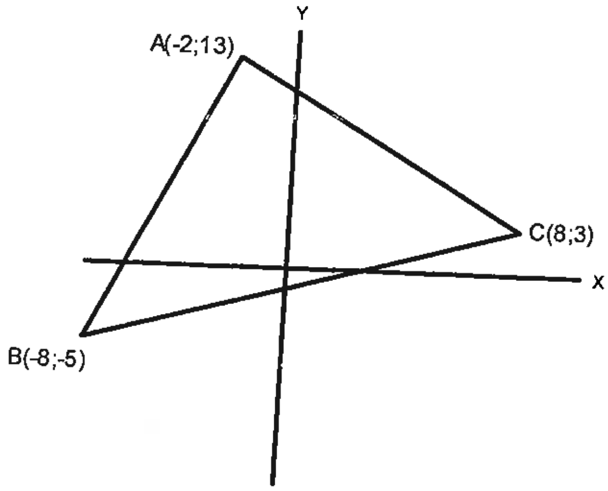
Prelim P2 September 2017

Section A

Question 1

Points A(−2;13), B(−8;−5) and C(8;3) form a triangle as illustrated below.



Determine

(a) the length of AB in simplest surd form. (3)
(b) the equation of the circle which has AB as a diameter. (5)
(c) the gradient of AB (2)
(d) the value of k if A, B and D(2;k), are collinear points. (3)
(e) the size of $\hat{A}$ correct to two decimal places (4)
(f) the equation of BE if E is the point on AC which is closest to B. (4)

[21]

(a) $AB^2 = (-2+8)^2 + (13+5)^2$ ✓

$= 36 + 324$ ✓

$AB = 6\sqrt{10}$ units ✓

3

(b) midpt AB $\left(\frac{-10}{2} ; \frac{8}{2}\right) = (-5;4)$ ✓✓

$r = 3\sqrt{10}$ ✓

$\therefore$ Eqn of ⊙ $(x+5)^2 + (y-4)^2 = 90$. ✓✓

5

(c) $m_{AB} = \frac{13-(-5)}{-2-(-5)} = 3$ ✓ ✓

(2)

(d) $m_{AB} = m_{AD}$ ✓

$3 = \frac{13-k}{-2-2}$ ✓

$-12 = 13 - k$

$k = 25$ ✓

(3)

(e) $m_{AB} = 3$

$\therefore$ incl(AB) $= 71{,}57°$ ✓

$m_{AC} = \frac{13-3}{-2-5} = -1$ ✓

$\therefore$ inclination (AC) $= 135°$ ✓

$\therefore \hat{A} = 135° - 71{,}57° = 63{,}43°$ ✓

(4)

(f) $BE \perp AC$ ✓

$\therefore m_{AC} = 1$ ✓

$\therefore y - y_1 = m(x - x_1)$

$y + 5 = 1(x + 5)$

$y = x + 3$. ✓ ✓

(4)

Question 2

A gardener plants a seedling and measures it over a 20 day period.

Here are the results

Days after planting	8	11	14	15	18	20
Height(cm)	3	4	6	8	10	11

(a) Determine the equation of the line of best fit. (3)

(b) Determine the correlation coefficient and comment on it. (3)

(c) If the plant has a height of 9cm can you determine the number of days it has been growing since being planted? Justify your answer. (2)

(a) $A = -3{,}308$ ✓ $B = 0{,}719$ ✓

$\therefore Y = -3{,}31 + 0{,}72X$ ✓ (3)

(b) $r = 0{,}98$ ✓

Very ✓ strong positive ✓ linear correlation (3)

(c) No. ✓

Height is not the independent ✓ variable &

$\therefore$ we cannot substitute into $Y = A + Bx$ (2)

[8]

Question 3

(a) If $\sin 61° = p$, determine the following in terms of p.

(i) $\sin 241°$

(ii) $\cos 122°$

(5)

(i) $\sin 241° = -\sin 61° = -p$

(ii) $\cos 122° = \cos(2 \times 61°)$

$= 1 - 2\sin^2 61°$

$= 1 - 2p^2$

5

(b) Prove that $\dfrac{\sin 2x - \tan x}{\cos 2x} = \tan x$

(5)

$$LHS = \frac{2\sin x\cos x - \frac{\sin x}{\cos x}}{\cos 2x}$$

$$= \frac{\frac{2\sin x\cos^2 x - \sin x}{\cos x}}{\cos 2x}$$

$$= \frac{\sin x(2\cos^2 x - 1)}{\cos x} \times \frac{1}{\cos 2x}$$

$$= \frac{\sin x(\cos 2x)}{\cos x} \times \frac{1}{\cos 2x}$$

$$= \frac{\sin x}{\cos x}$$

$= \tan x$

$= RHS$

5

[10]

Question 4

(a) Determine x if

$$\cos(x - 45^\circ) = \sin 2x\ ;\ x \in [-180^\circ; 180^\circ]$$

Gen soln:

$x - 45^\circ = 90^\circ - 2x + 360^\circ k$ or $x - 45^\circ = -(90^\circ - 2x) + 360^\circ k$

$3x = 135^\circ + 360^\circ k$; $x - 45^\circ = -90^\circ + 2x + 360^\circ k$

$x = 45^\circ + 120^\circ k$; $-x = -45^\circ + 360^\circ k$; $k \in \mathbb{Z}$

$x = 45^\circ + 360^\circ k$

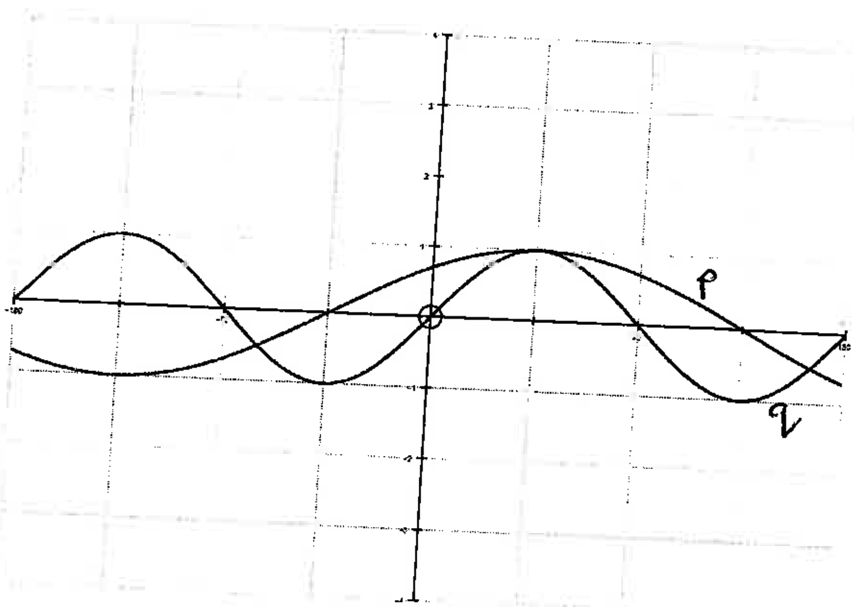
$x \in [-180^\circ; 180^\circ]$

$x = 45^\circ; 165^\circ; -75^\circ$

6

(6)

(b) On the set of axes below the graphs of
$p(x) = \cos(x - 45^\circ)$ and $q(x) = \sin 2x$ are shown.
$x \in [-180^\circ; 180^\circ]$



Using your result from (a) find x if p(x) > q(x)

(3)

$x \in (-75^\circ; 45^\circ) \cup (45^\circ; 165^\circ)$

3

[9]

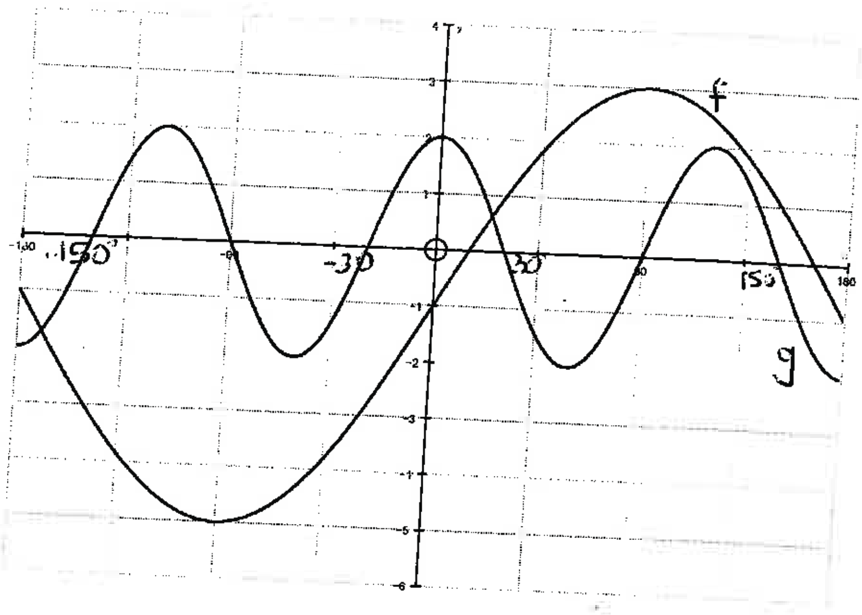
Question 5

The sketch below shows the graphs of the functions

$f(x) = a\sin x + b$

and

$g(x) = p\cos(qx)$ for $x \in [-180^\circ;180^\circ]$



(a) Determine the period of g. (1)

120° ✓

(b) Determine the values of a, b, p and q (4)

$a = 4$ ✓

$b = -1$ ✓

$p = 2$ ✓

$q = 3$ ✓

(c) Determine the values of x for which $f(x).g(x) \geq 0$ if $x < 0^\circ$ (4)

✓ ✓ ✓ ✓

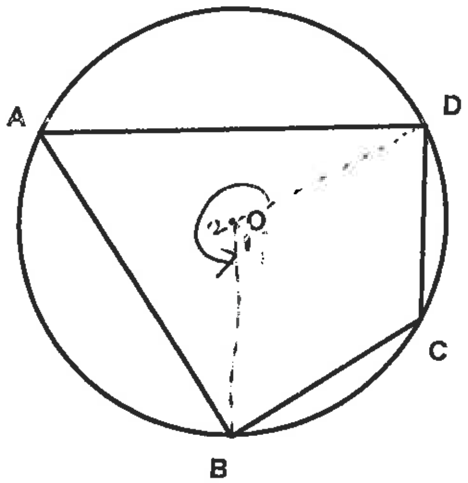
$x \in [-180^\circ;-150^\circ] \cup [-90^\circ;-30^\circ]$

[9]

Question 6

(a) Use the drawing below to prove the theorem that states the opposite angles of a cyclic quadrilateral are supplementary. O is the centre of the circle.
i.e prove that $\hat{A}+\hat{C}=180^0$

(5)



Join BO & DO

$\hat{O}_1 = 2\hat{A}$ } ($\angle$ at centre = 2 × $\angle$ at circumf)

$\hat{O}_2 = 2\hat{C}$

but $\hat{O}_1 + \hat{O}_2 = 360^\circ$ (revolution).

$\therefore 2\hat{A} + 2\hat{C} = 360^\circ$

$\therefore \hat{A} + \hat{C} = 180^\circ$

(b) ABCD is a cyclic quad. P and Q are points on AD and BC respectively so that PQ// AB.

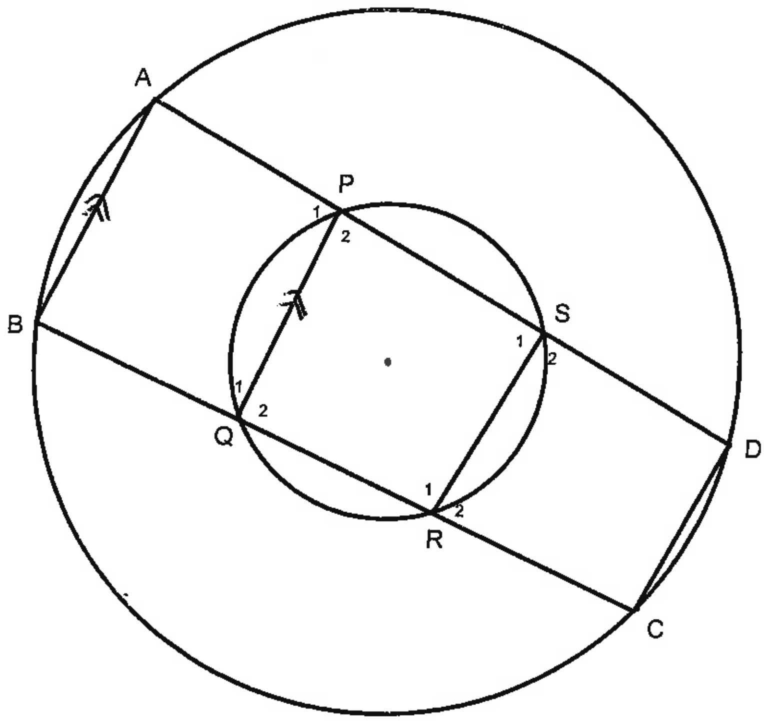
PQ is a chord of another circle which intersects AD at S and BC at R.

Prove:

(i) ABRS is a cyclic quadrilateral (4)

(ii) $\hat{R}_1 = \hat{C}$ (3)

[12]



(i) $\hat{Q}_2 + \hat{S} = 180°$ (opp ∠s of cyclic quad suppl)

$\hat{Q}_2 = \hat{B}$ (corr ∠s = ; AB‖PQ)

$\therefore \hat{B} + \hat{S}_1 = 180°$

$\therefore$ ABRS is cyclic quad (opp ∠s suppl).

(ii) $\hat{C} + \hat{A} = 180°$ (opp ∠s of cyclic quad ABCD suppl)

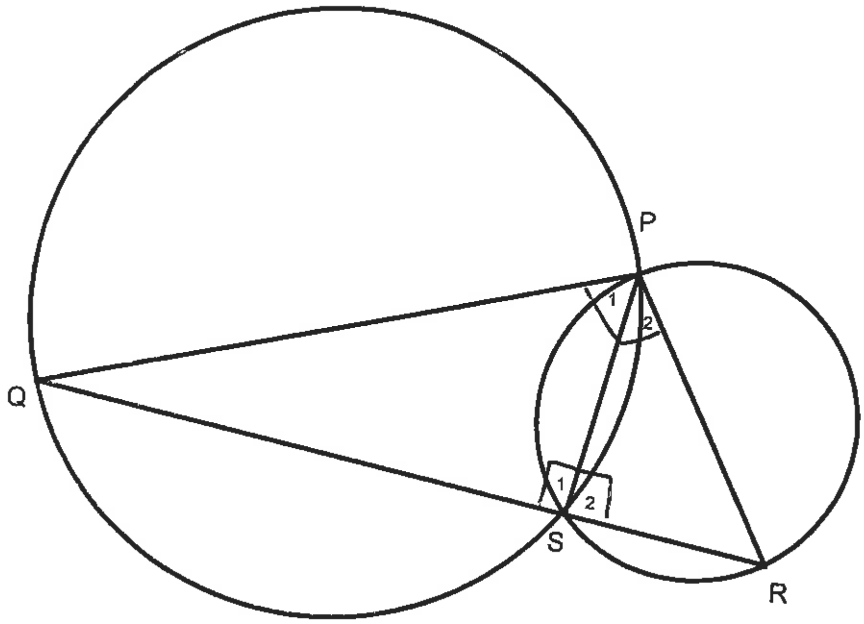
$\hat{R}_1 + \hat{A} = 180°$ (opp ∠s of cyclic quad ABRS suppl)

$\therefore \hat{R}_1 = \hat{C}$

Question 7

In the figure the two circles intersect at P and S.

$Q\hat{P}R = 90^0$ and $PS \perp QR$



a) Prove that $\Delta SPR /// \Delta PQR$ (2)
b) HENCE prove that PR is a tangent to circle PQS (2)
c) Prove that $PS^2 = QS.SR$ (3)+1
d) If QS = 45 units and SR = 31.25 units. Calculate the length of PS (2)

[9]+1

(a) $\triangle SPR ||| \triangle PQR$ ✓ (right-angled $\triangle$ thm) ✓ (2)

(b) $\therefore \hat{P}_2 = \hat{Q}$ ✓ ($\angle$s $|||$)

$\therefore$ PS is a tangent to circle PQS (converse tan-chrd thm) ✓ (2)

(c) In $\triangle SPR$ & $\triangle SQP$

1) $\hat{P}_2 = \hat{Q}$ (proved)

2) $\hat{S}_1 = \hat{S}_2 = 90°$

3) $\hat{R} = \hat{P}_1$ (3rd $\angle$s of $\triangle$s)

OR $\triangle SPR ||| \triangle SQP$ (rt - $\angle$d $\triangle$ thm)

$\therefore \triangle PSR ||| \triangle QSP$ (A.A.A) ✓✓

$\therefore \frac{PS}{QS} = \frac{SR}{SP}$ ($\triangle$s $|||$) ✓ (4)

$\therefore PS^2 = QS.SR$. ✓

(d) $PS^2 = 45 \times 31{,}25$ ✓

$PS = 37{,}5$ units ✓ (2)

TOTAL Section A = ~~78~~ +1

Section B

Question 8

(a) The following information is given about the test results of a class:

$$\sum_{k=1}^{20}(x_k - \bar{x})^2 = 156 \quad \text{and} \quad \sum_{k=1}^{20} x_k = 1220$$

Determine

(i) The number of pupils in the class (1)

(ii) The mean mark (1)

(iii) The standard deviation (2)

(i) $n = 20$ ✓ 1

(ii) $\frac{1220}{20} = 61$ ✓ 1

(iii) $\sigma^2 = \frac{156}{20}$ ✓

$\therefore \sigma = 2{,}79$ ✓ 2

(b) In a certain data set the interval that is one standard deviation from the mean is [350;380]

Calculate the mean and the standard deviation. (4)

$\bar{x} + \sigma = 380$ ✓

$\bar{x} - \sigma = 350$ ✓

$\therefore 2\bar{x} = 730$

$\bar{x} = 365$ ✓

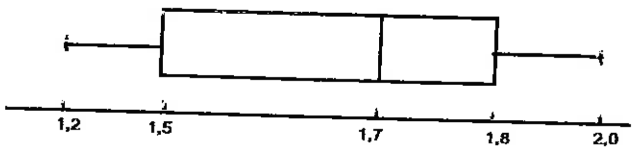
$\sigma = 15.$ ✓

[8]

Question 9

The mass of 60 small dogs is summarised in the box and whisker diagram below.

Mass (in kilograms)



(a) Fill in the missing values in the table below:

Mass interval	Frequency	Midpoint
$1,2 \leq x < 1,3$	2	1,25
$1,3 \leq x < 1,4$	6	1,35
$1,4 \leq x < 1,5$	7	1,45
$1,5 \leq x < 1,6$	7	1,55
$1,6 \leq x < 1,7$	8	1,65
$1,7 \leq x < 1,8$	15	1,75
$1,8 \leq x < 1,9$	13	1,85
$1.9 \leq x < 2,0$	2	1,95

(4)

(b) Determine the estimated mean mass of the dogs. (2)

$\bar{x} = 1,65$ kg

(c) Discuss the skewness of the graph. Give a reason for your answer. (2)

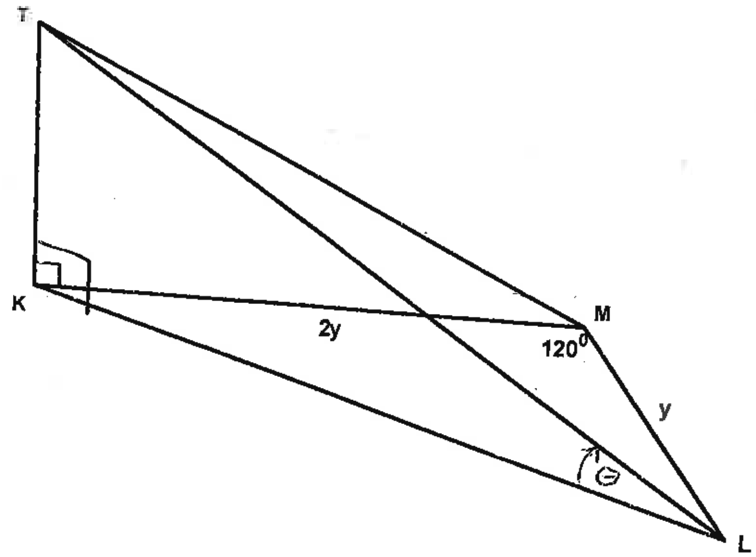
Skewed left.

mean $<$ median or $1,7 - 1,2 > 2 - 1,7$.

[8]

Question 10

(a) In the figure K,M and L are 3 points in the same horizontal plane.
$K\hat{M}L = 120°$ T represents the position of a point directly above K.
ML = y and MK = 2y. The angle of elevation of T from L is θ.



(i) Show that $KL = \sqrt{7}y$ (5)

(ii) Hence, or otherwise determine TL in terms of y and θ (2) [7]

(i) In ΔMKL:

$KL^2 = (2y)^2 + y^2 - 2(2y)(y)\cos 120°$ ✓

$= 4y^2 + y^2 - 4y^2(-\frac{1}{2})$ ✓ ✓

$= 5y^2 + 2y^2$ ✓

$= 7y^2$ ✓

$\therefore KL = \sqrt{7}y$

5

(ii) In ΔTKL: $\frac{TL}{KL} = \frac{1}{\cos\theta}$ ✓

$TL = \frac{KL}{\cos\theta}$

$= \frac{\sqrt{7}y}{\cos\theta}$ ✓

2

(b) Show that $\dfrac{\cos 330^\circ}{1+\sin(90^\circ-\theta)} - \dfrac{\cos 150^\circ}{1-\cos(360^\circ-\theta)} = \dfrac{\sqrt{3}}{\sin^2\theta}$

(6)

$$LHS = \frac{\frac{\sqrt{3}}{2}}{1+\cos\theta} - \frac{\left(-\frac{\sqrt{3}}{2}\right)}{1-\cos\theta}$$

$$= \frac{\frac{\sqrt{3}}{2}(1-\cos\theta) + \frac{\sqrt{3}}{2}(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\cos\theta + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\cos\theta}{1-\cos^2\theta}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\right)}{\sin^2\theta}$$

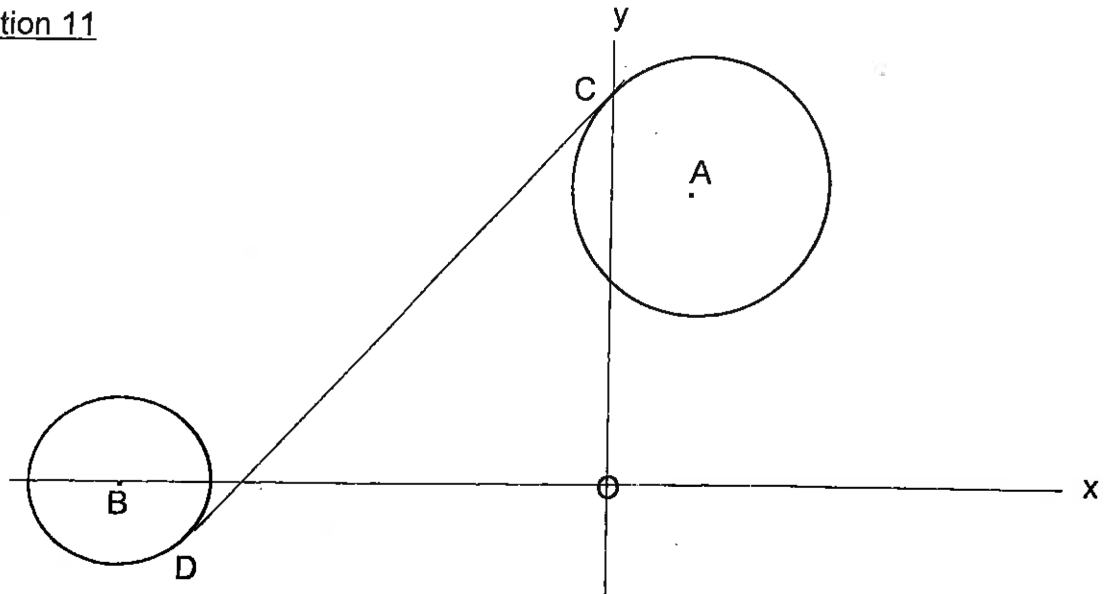
$$= \frac{\sqrt{3}}{\sin^2\theta}$$

6

$= RHS$.

[13]

Question 11



In the diagram, two circles with equations

$$(x-4)^2 + (y-6)^2 = 25 \quad \text{and}$$

$$(x+10)^2 + y^2 = 1 \text{ are drawn.}$$

CD is a common tangent to the circles with centres A and B at C and D respectively.
C is a point on the y axis.

(a) Determine the co-ordinates of C. (4)

(b) Determine the length of the common tangent. HINT: Join BD (5)

[9]

(a) At C, $x = 0$ $\quad \therefore (0-4)^2 + (y-6)^2 = 25$ ✓

$(y-6)^2 = 9$ ✓

$\therefore y - 6 = \pm 3$ ✓

$\therefore C(0;9)$ ✓ 4

(b) $B(-10;0)$ ✓ $\left.\begin{matrix} \\ C(0;9) \end{matrix}\right\}$ $CB^2 = (10)^2 + (9)^2 = 181$ ✓

In $\triangle BDC$: $BC^2 = BD^2 + CD^2$ (Pythag) ✓

$181 = 1 + CD^2$ ✓

$\sqrt{180} = CD$

$CD = 13{,}42$ units ✓ 5

Question 12

Prove that the radius of the circle having the equation

$$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$$

can never exceed $\sqrt{13}$ for any value of θ. [6]

Show all your calculations.

$x^2 + 4x\cos\theta + y^2 + 8y + \sin\theta + 3 = 0$

$(x + 2\cos\theta)^2 + (y + 4\sin\theta)^2 = -3 + 4\cos^2\theta + 16\sin^2\theta$

$r^2 = -3 + 4\cos^2\theta + 16\sin^2\theta$

$= -3 + 4\cos^2\theta + 4\sin^2\theta + 12\sin^2\theta$

$= -3 + 4(\cos^2\theta + \sin^2\theta) + 12\sin^2\theta$

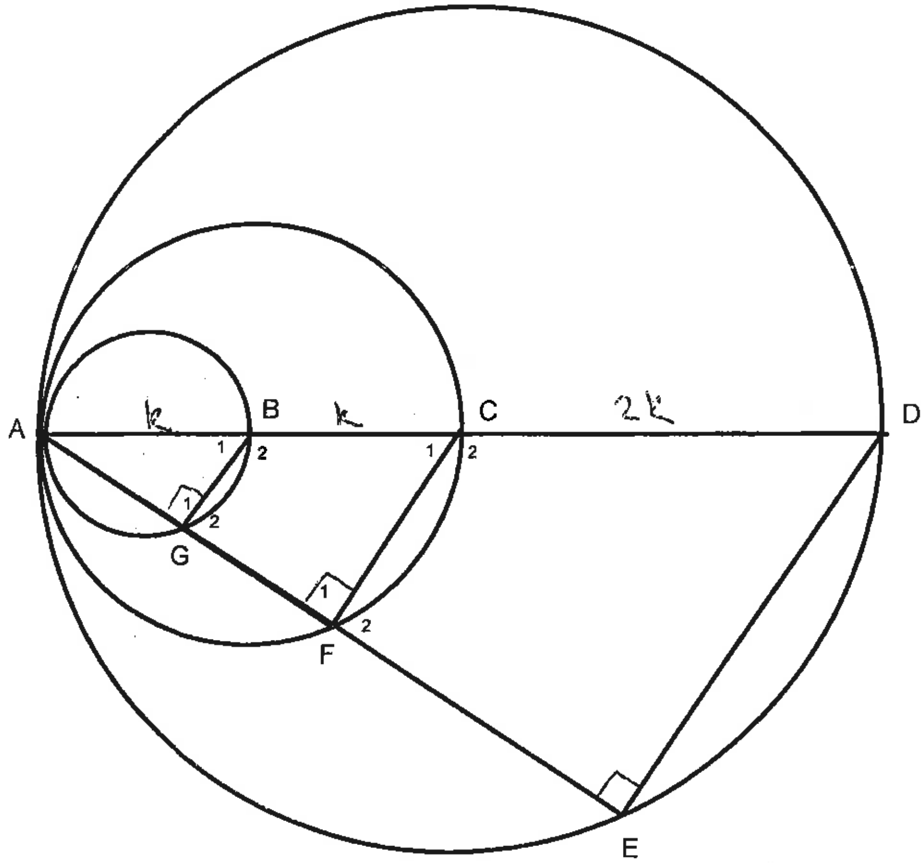
$= -3 + 4(1) + 12\sin^2\theta$

$r = \sqrt{1 + 12\sin^2\theta}$

max value of $\sin^2\theta = 1$

$\therefore$ max $r = \sqrt{1 + 12(1)} = \sqrt{13}$.

Question 13



Three circles touch internally at A and have diameters AB, AC and AD respectively.

AC = 2AB and AD= 4AB.

(a) Prove: GB//FC//ED (3)

(b) Calculate the value of the ratio FC:GB. Show all working (5)

(c) Calculate the value of the ratio $\frac{\text{Area}\Delta AFC}{\text{Area}\Delta AED}$ (5)

[13]

(a) $\hat{G}_1 = \hat{F}_1 = \hat{E} = 90°$ ✓ ($\angle$ subtended by diam $= 90°$) ✓

$\therefore GB \parallel FC \parallel ED$ (corres $\angle$s $=$). ✓ 3

(b) In ΔAGB and ΔAFC

① A is common. ✓

② $\hat{G}_1 = \hat{F}_1 = 90^\circ$ (proved)

③ $\hat{B}_1 = \hat{C}_1$ (3rd ∠s of ∆s) or (corres ∠s =; $GB \parallel FC$). ✓

$\therefore \Delta AGB \parallel\!| \Delta AFC$ (A.A.A) ✓

$\therefore \frac{AB}{AC} = \frac{GB}{FC}$ ✓

$\therefore \frac{GB}{FC} = \frac{1}{2}$ ✓ (5)

(c) $\Delta AFC \parallel\!| \Delta AED$ (A.A.A). ✓

$\frac{AC}{AD} = \frac{2}{4} = \frac{1}{2}$.

$\therefore \frac{AF}{AE} = \frac{2}{4}$

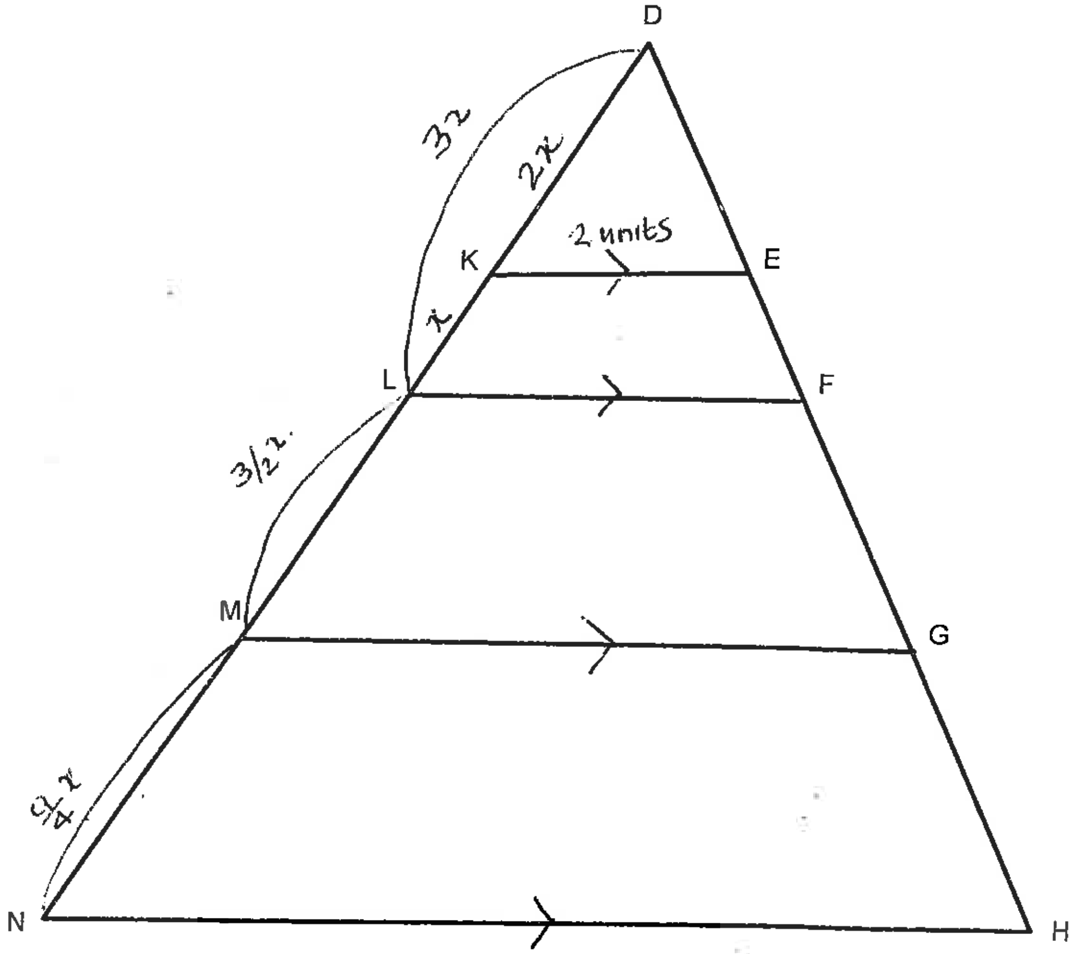
Let $AF = 2p$ $\therefore AE = 4p$. ✓

area $\Delta AFC = \frac{1}{2}(2p)(2k)\sin A$ ✓

area $\Delta AED = \frac{1}{2}(4p)(4k)\sin A$ ✓

$\therefore \frac{\text{area } \Delta AFC}{\text{area } \Delta AED} = \frac{1}{4}$ ✓ (5)

Question 14



ΔDNH is not drawn to scale.

KE//LF//MG//NH

DK:KL = 2:1

DL:LM=2:1

DM:MN=2:1 etc

If KE =2 units and NH **is** the 4[th] parallel line, calculate the length of the 10[th] parallel line, rounded off to one decimal digit. [7]

Ratios are shown on the diagram.

$\triangle DKE \,|||\, \triangle DLF \,|||\, \triangle DMG$, etc (A.A.A)

corres $\angle s =$ $\because$ $KE \parallel LF \parallel MG$
$\hat{D}$ is common to all $\triangle s$

$\therefore$ If $KE = 2$ units, $LF = 3$ units

$MG = 4,5$ units

$NH = 6,75$ units

$2; 3; 4,5; 6,75; \ldots$ is a G.S with $r = \frac{3}{2}$

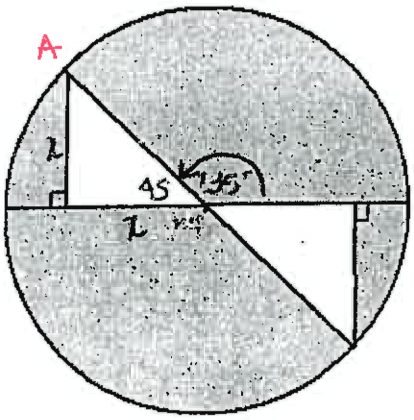
$T_n = a \cdot r^{n-1}$

$T_n = (2)\left(\frac{3}{2}\right)^9 = 76,9$ units

7

Question 15

The diameter of the circle is $8\sqrt{2}$ cm and the angle at the centre of the circle is 135^0 as indicated on the diagram. Determine the shaded area in terms of π. [7]



diam $= 8\sqrt{2}$ $\therefore r = 4\sqrt{2}$ ✓

$\angle$ in $\Delta = 45°$ $\therefore \Delta$ is isosceles ✓

$\therefore x^2 + x^2 = (4\sqrt{2})^2$ ✓

$2x^2 = 32$

$x^2 = 16$

$\therefore x = 4$ ✓

or $\frac{4\sqrt{2}}{\sin 90°} = \frac{AB}{\sin 45°}$

$\therefore AB = 4cm$

area $\Delta = \frac{1}{2} b.\perp h = \frac{1}{2}(4)(4) = 8 cm^2$ ✓

area $\odot = \pi r^2 = \pi(4\sqrt{2})^2 = 32\pi$ ✓

$\therefore$ shaded area $= 32\pi - 2(8)$

$= (32\pi - 16) cm^2$ ✓

7

Total section B = 7