MATHS PRELIM PAPER ONE 2017

1 (a) $(3x+2)(x+3) \geq 0$

-3 , $-\frac{2}{3}$

$x \in (-\infty; -3] \cup [-\frac{2}{3}; \infty)$ (3)

(b)(i) $\sqrt{10+k} + 2 = k$

$\sqrt{10+k} = k-2$

$10+k = (k-2)^2$

$10+k = k^2-4k+4$

$0 = k^2-5k-6$

$0 = (k-6)(k+1)$

$\therefore k = 6$ or $k=-1$ N/A (5)

(ii) Let $3^x = k$

$\therefore 3^x = 6$

$x = \log_3 6$

$x = 1{,}63$ (3)

[11]

2 (a) $\Delta = 13-2k$

For real roots $\Delta > 0$

$13-2k>0$

$-2k>-13$

$k<6{,}5$

For rational roots Δ is a perfect square

If $k=6$, $\Delta = 1$. (3)

(b) $54 = \frac{1}{2}n(n-3)$

$108 = n^2-3n$

$0 = n^2-3n-108$

$0 = (n-12)(n+9)$

$\therefore n = 12$ (4)

[7]

3 (a) $T_7 - T_4 = 9$

$a+6d-(a+3d) = 9$

$\therefore 3d = 9$

$d = 3$

$T_6 = 5$

$\therefore a+5d = 5$

$a+5(3) = 5$

$a = -10$ (3)

(ii) $S_n = \frac{n}{2}[2a+(n-1)d]$

$1005 = \frac{n}{2}[2(-10)+(n-1)(3)]$

$2010 = n[-23+3n]$

$0 = 3n^2-23n-2010$

$n = \dfrac{23 \pm \sqrt{23^2-4(3)(-2010)}}{2(3)}$

$n = 30$ (4)

(b) $2-3+4-6+8-9+\ldots$

Two series

GS: $2+4+8+\ldots$ and AS: $-3+(-6)+(-9)$

$S_{13} = \dfrac{2(2^{13}-1)}{2-1}$

$= 16382$

$S_{12} = \frac{12}{2}[2(-3)+11(-3)]$

$= -234$

$\therefore S_{25} = 16148$ (6)

(c) $5; \boxed{T_2}; \boxed{T_3}; \ldots$

First diff: $3; 7; 11; 15; \ldots$

(i) $\therefore T_2 = 8$, $T_3 = 15$ (2)

(ii) Second diff $= 4$ $\therefore a = 2$

$T_0 = 6$ $\therefore c = 6$

$\therefore T_n = 2n^2+bn+6$

$5 = 2(1)^2+b(1)+6$ $\therefore b=-3$

$\therefore T_n = 2n^2-3n+6$ (4)

[19]

4 (a) $r = x - 2$ (1)

(b) $-1 < x - 2 < 1$

$1 < x < 3$ (3)

(c) $S_\infty = 4 \quad \therefore \frac{a}{1-r} = 4$

$\therefore \frac{(x-2)^2}{1-(x-2)} = 4$

$\frac{(x-2)^2}{3-x} = 4$

$x^2 - 4x + 4 = 4(3-x)$

$x^2 - 8 = 0$

$\therefore x = \pm 2\sqrt{2}$

(4)

[8]

5 (a)(i) Loan $= 2 \times 10^6 - 400000$

$= 1600000$

$P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right]$

$1600000 = x\left[\frac{1-\left(1+\frac{0{,}09}{12}\right)^{-240}}{\frac{0{,}09}{12}}\right]$

$x = R\,14395{,}62$ (3)

(ii) Interest $= (14395{,}62 \times 240) - 1600000$

$= R\,1854947{,}67$ (2)

(iii) Bal $= 14395{,}62\left[\frac{1-\left(1+\frac{0{,}09}{12}\right)^{-90}}{\frac{0{,}09}{12}}\right]$

$= R\,939668{,}47$ (3)

(b) $A = P(1-i)^n$

$15000 = 100000(1-0{,}07)^n$

$\frac{3}{20} = (0{,}93)^n$

$\therefore \log_{0{,}93}\left(\frac{3}{20}\right) = n$

$n = 26{,}14$

$\therefore$ After 27 years (3)

6 (a) $y = p^x$

Subst $A(2;9)$:

$9 = p^2$

$\therefore p = 3 \quad (p > 0)$ (2)

(b) $\therefore B(0;1)$ (y-int of exponential funct)

Eqn of parabola is

$y = a(x-p)^2 + q$

$\therefore y = a(x-2)^2 + 9$ —— (*)

Subst $B(0;1)$:

$1 = a(0-2)^2 + 9$

$-8 = 4a$

$-2 = a$

Subst into (*):

$\therefore y = -2(x-2)^2 + 9$

$= -2x^2 + 8x + 1$

$\therefore a = -2;\ b = 8;\ c = 1$ (4)

(c)(i) $f^{-1}(x) = \log_3 x$ (2)

(ii) $x \in (0;1)$ (2)

(d)(i) $y = -3$ (2)

(ii) y-int $(0;-2)$

For x-int: $3^x - 3 = 0$

$\therefore 3^x = 3$

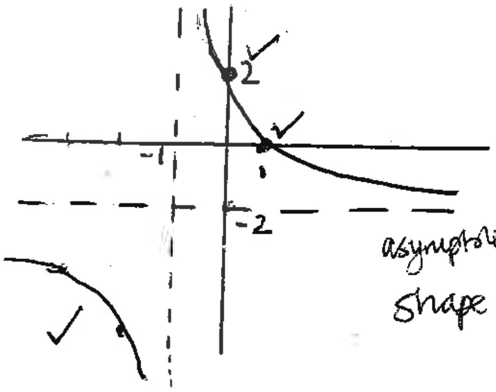
$x = 1$

x-int: $(1;0)$ (4)

[16]

4 (a) $y + 2 = \frac{4}{x+1}$

$\therefore y = \frac{4}{x+1} - 2$



x-int: $0 = \frac{4}{x+1} - 2$

$0 = 4 - 2x - 2$

$2x = 2$

$x = 1$

(6)

(b) $y = -x + c$

Subst $(-1, -2)$

$-2 = -(-1) + c$

$-3 = c$

$\therefore y = -x - 3$

(3)

(c) $y = \frac{4}{x+1} - 2$

for inverse:

$x = \frac{4}{y+1} - 2$

$\frac{x+2}{1} = \frac{4}{y+1}$

$\frac{1}{x+2} = \frac{y+1}{4}$

$\frac{4}{x+2} = y + 1$

$\therefore y = \frac{4}{x+2} - 1$

(3)

[12]

5 (a) $f(x) = 2x^2 + x$

$f(x+h) = 2(x+h)^2 + (x+h)$

$= 2x^2 + 4xh + 2h^2 + x + h$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h}$

$= \lim_{h \to 0} \frac{h(4x + 2h + 1)}{h}$

$= 4x + 1$

(4)

(b) $y = \frac{\sqrt{x} + 2}{3x}$

$y = \frac{x^{\frac{1}{2}}}{3x} + \frac{2}{3x}$

$= \frac{1}{3} x^{-\frac{1}{2}} + \frac{2}{3} x^{-1}$

$\therefore \frac{dy}{dx} = -\frac{1}{6} x^{-3/2} - \frac{2}{3} x^{-2}$

$= \frac{-1}{6x^{3/2}} - \frac{2}{3x^2}$

(4)

(c) $f(x) = x^3 + ax^2 + bx + c$

(i) $f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

At non-hor. pt of inflection $f''(x) = 0$

$\therefore 0 = 6(2) + 2a$

$\therefore a = -6$

(5)

(ii) $\therefore f'(x) = 3x^2 - 12x + b$

$f'(2) = -3$

$\therefore -3 = 3(2)^2 - 12(2) + b$

$9 = b$

$\therefore f(x) = x^3 - 6x^2 + 9x + c$

$f(2) = (2)^3 - 6(2)^2 + 9(2) + c$

$12 = 2 + c$

$10 = c$

(5)

9 a $y = 2x^3 - 5x^2 - 4x + 3$

a $y = (x+1)(2x^2 + kx + 3)$

$2 + k = -5$

$\therefore k = -7$

$\therefore y = (x+1)(2x^2 - 7x + 3)$

$0 = (x+1)(2x-1)(x-3)$ Can get from calc

$\therefore B(\frac{1}{2}; 0)$ ✓✓ ; $C(3;0)$ ✓✓ 4

b At D, $\frac{dy}{dx} = 0$ ✓

$\therefore 6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$ ✓

$(3x+1)(x-2) = 0$

$\therefore x_D = -1/3$ ✓ 3

c $\frac{d^2y}{dx^2} = 12x - 10$ ✓

at G, $\frac{d^2y}{dx^2} = 0$

$\therefore x = 5/6$ ✓

$\therefore$ Concave up: $x \in (\frac{5}{6}; \infty)$ ✓ 3

d $y_G = 2\left(\frac{5}{6}\right)^3 - 5\left(\frac{5}{6}\right)^2 - 4\left(\frac{5}{6}\right) + 3$

$= -\frac{143}{54}$ ✓

$G\left(\frac{5}{6}; -\frac{143}{54}\right)$ ✓

$A(-1; 0)$

$GA^2 = \left(\frac{5}{6} + 1\right)^2 + \left(-\frac{143}{54} - 0\right)^2$ ✓

$GA = 3{,}22$ ✓ 4

[14]

10 a $\lim_{x \to 3} \frac{x^3 - 9x}{x - 3}$

$= \lim_{x \to 3} \frac{x(x-3)(x+3)}{(x-3)}$ ✓

$= \lim_{x \to 3} x(x+3)$ ✓

$= 18$ ✓ 3

10 b As $k \to \infty$ the shape will be a circle 2

c $R + r = 200$

$\therefore R = 200 - r$ ✓

Area figure $= \pi R^2 + \pi r^2$ ✓

Let $A(r) = \pi(200 - r)^2 + \pi r^2$ ✓

$= \pi(40000 - 400r + r^2) + \pi r^2$

$= \pi[40000 - 400r + 2r^2]$

For min A, $A'(r) = 0$ ✓

$0 = \pi[40000 - 400r + 2r^2]$

$0 = r^2 - 200r + 20000$ ✓

$0 = (r - 100)(r + 200)$

$\therefore r = 100$ mm ✓ 6

$R = 100$ mm

[11]

11 a 1, 2, 3, 4, 5, 6, 7

4 digit even codes < 4000

2	5	4	"2"
3	5	4	"4" or "6"

$\therefore n\left(\text{even codes} < 4000\right) = (2 \times 5 \times 4 \times 1) + 2 \times (3 \times 5 \times 4 \times 1)$

$= 160$ ✓✓

total n(codes) $= 7 \times 6 \times 5 \times 4$

$= 840$ ✓

$\therefore P\left(\text{even code} < 4000\right) = \frac{160}{840} = \frac{4}{21} = 0{,}19$ ✓ 4

b (i) $n(\text{arr}) = \frac{16!}{5!\,4!\,4!\,3!}$ ✓✓ $= 50450400$ 2

(ii) $n\left(\text{arr - banana tog}\right) = \frac{14!}{5!\,4!\,4!}$ ✓✓ $= 1261260$

$\therefore P(\text{banana tog}) = \frac{1}{40} = 0{,}025$ ✓ 3

11c $P(A \cup B) = 0{,}75$

$P(A) = 0{,}3$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

mutually exclusive: $P(A \cap B) = 0$

$\therefore 0{,}75 = 0{,}3 + P(B)$ ✓

$P(B) = 0{,}45$ ✓ (2)

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

independent events: $P(A).P(B) = P(A \cap B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A).P(B)$. ✓

Let $P(B) = x$

$\therefore 0{,}75 = 0{,}3 + x - 0{,}3.x$ ✓

$0{,}45 = 0{,}7x$

$\frac{9}{14} = P(B)$

$0{,}64 = P(B)$ ✓ (3)

[14]

12 (a) $f(x) = 2x^2 + 3px - 3$ $\quad$ $g(x) = 2x^2 + (p-2)x - 1$

$f(r) = 2r^2 + 3pr - 3$ ✓ $\quad$ $g(r) = 2r^2 + (p-2).r - 1$ ✓

$0 = 2r^2 + 3pr - 3$ ✓ $\quad$ $0 = 2r^2 + (p-2).r - 1$

$\therefore \cancel{2r^2} + 3pr - 3 = \cancel{2r^2} + pr - 2r - 1$ ✓

$4pr + 2r = 2$

$pr + r = 1$ ✓

$r(1+p) = 1$

$r = \frac{1}{1+p}$ ✓ (5)

(b) $0 = 2r^2 + 3pr - 3$

$0 = 2\left(\frac{1}{1+p}\right)^2 + 3p\left(\frac{1}{1+p}\right) - 3$ ✓

$0 = \frac{2}{(1+p)^2} + \frac{3p}{(1+p)} - 3$

$0 = 2 + 3p(1+p) - 3(1+p)^2$ ✓✓

$0 = 2 + 3p + \cancel{3p^2} - 3 - 6p - \cancel{3p^2}$

$0 = -1 - 3p$

$3p = -1$

$p = -\frac{1}{3}$ ✓ (4)