



Mathematics Paper 1

Grade 12

Preliminary Examination 2017

DURATION: 3 hours

EXAMINER: R. Obermeyer

MARKS: 150

MODERATOR: A. Janisch

Date: 2017

External Moderator: J.L. Ruiz-Mesa

I. L. Atteridge

INSTRUCTIONS:

- See overleaf for Instructions.
- This paper consists of 10 pages (including cover) and an information sheet.

NAME: _____

ASSESSMENT					
Question	Level Tested	Topic	Suggested Time Allocation	Possible mark	Mark Obtained
SECTION A					
1	1 – 4	Equations & Inequalities	25 mins	21	
2	1 – 4	Sequence & Series	16 mins	13	
3	1 – 4	Finance Modelling	8 mins	7	
4	1 – 4	Calculus	20 mins	17	
5	1 – 4	Functions	13 mins	11	
6	1 – 4	Probability	7 mins	6	
7	1 – 4	Calculus	18 mins	15	
SECTION B					
8	1 – 4	Calculus	7 mins	6	
9	1 – 4	Sequence & Series	12 mins	10	
10	1 – 4	Probability	18 mins	15	
11	1 – 4	Functions	7 mins	6	
12	1 – 4	Financial Mathematics	11 mins	9	
13	1 – 4	Calculus	6 mins	5	
14	1 – 4	Equations & Inequalities	11 mins	9	
TOTAL:				150	
PERCENTAGE:					

Teacher's Signature: _____

Controller's Signature: _____

Moderator's Signature: _____

Instructions

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages and an Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **one** decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. Please hand in this question paper.
- 10. Answer all questions underneath each other.**
- 11. Start each new question on a new page.**

SECTION A

Question 1

a. Solve for x , correct to two (2) decimal places, where necessary:

1. $(2x - 3)(x + 1) = 0$ (2)

2. $2x^2 - 3x + 5 = 0$ (3)

b. If $4^x = 8^y$, find the ratio of $x : y$. (3)

c. Solve simultaneously: $x^2 + 2y = 3y^2 - 7$ and $3x - y = 1$. (7)

d. Given the expression: $2x^2 - 7x - 15$

1. Solve for x if $2x^2 - 7x - 15 < 0$ (3)

2. Hence, or otherwise, determine the value(s) of x for which the expression $\frac{\sqrt{2x^2 - 7x - 15}}{x - 8}$ will be real. (3)

[21]

Question 2

a. The first two terms of a sequence are 3 and 4. Each of the following terms in the sequence is the sum of the preceding terms.
Determine the sum of the first 5 terms of the sequence. (2)

b. The sum of the first n terms of an arithmetic series is given by the formula: $S_n = \frac{3n^2 - n}{2}$.

1. Determine S_{10} . (2)

2. Determine the 11th term of the series. (3)

3. How many terms have a sum of 590? Show all calculations. (4)

c. A chessboard has 64 squares. You place 1 grain of rice on the first square, 2 grains of rice on the second square, 4 grains of rice on the third square and 8 grains of rice on the fourth square. Following this pattern, how many grains of rice will you put on the 32nd square? Use an appropriate formula to work out your answer. (2)

[13]

Question 3

A swimming pool has been neglected and a certain type of bacteria is developing. At the start of the process, a 2 litre sample is tested and 2 000 bacteria are found. After 20 minutes another 2 litre sample is tested, and the bacteria count has increased to 6 000. The bacterium has a compound growth rate.

- a. Calculate an estimated growth rate of the bacteria per minute. (4)
- b. Using your answer in (a), how many bacteria will be present at the end of 1 hour. Round off to the nearest whole number. (3)

[7]

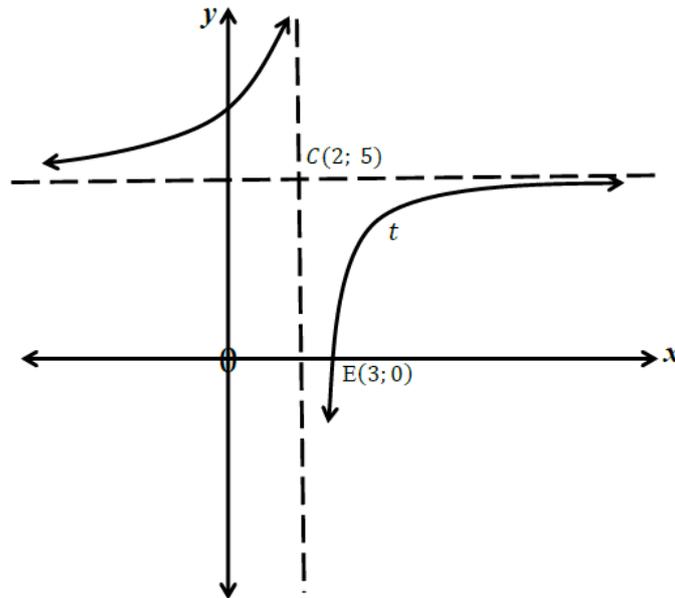
Question 4

- a. Given: $f(x) = -2x^2 + x$
1. Determine $f'(x)$ using first principles. (5)
2. A tangent to the graph of f has a gradient of -7 , find the equation of this tangent. (5)
- b. Determine:
1. $\frac{dy}{dx}$ if $y = (x^3 + 1)(x^2 - 2)$ (3)
2. $D_x \left[\frac{\sqrt{x^5 - 3}}{\sqrt{x}} \right]$ (4)

[17]

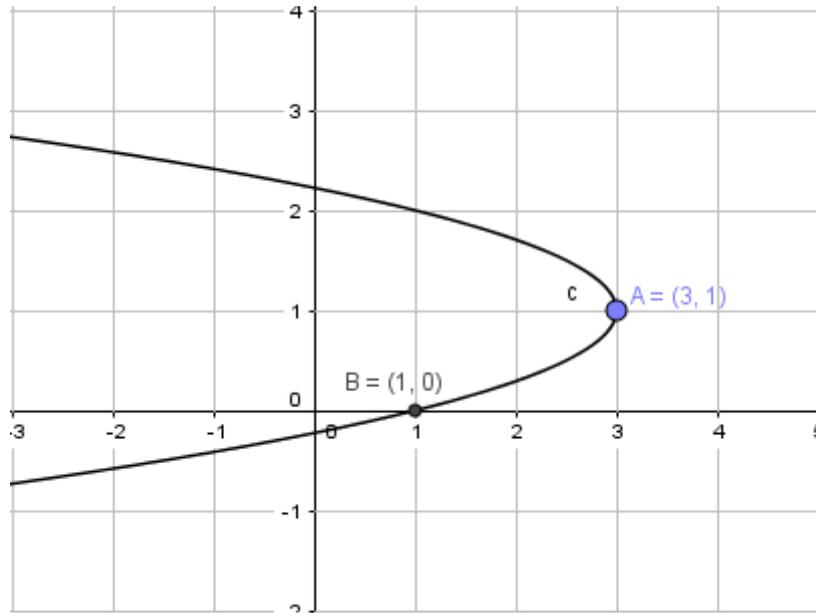
Question 5

- a. Sketched below is the graph of $t(x) = \frac{a}{x-p} + q$. $C(2; 5)$ is the point of intersection of the asymptotes of t . $E(3; 0)$ is the x -intercept of t .



1. Determine the equation of t in the form $t(x) = \frac{a}{x-p} + q$. (4)
2. Write down the equation of the axis of symmetry of t with a negative gradient in the form $y = \dots$. (2)

b. The graph of z^{-1} is shown below. Study the diagram and answer the questions that follow.



1. Find the equation of z given that the graph of z is a parabola. Write your answer in the form $= a(x - p)^2 + q$. (Do NOT simplify your answer) (4)
2. How would you restrict the domain of the graph of z such that the graph of z^{-1} is a function? (1)

[11]

Question 6

In a game of tennis, a player has two serves. If the first serve is successful, the game continues. If the first serve is not successful, the player serves again. If this second serve is successful, the game continues. If both serves are unsuccessful, the player has a “double fault” and loses the point.

Boris plays tennis. He is successful with 60% of his first serves and 95% of his second serves.

- a. Calculate the probability that Boris serves a double fault. (2)

If Boris is successful with his first serve, he has a probability of 0,75 of winning the point. If he is successful with his second serve, he has a probability of 0,5 of winning the point.

- b. Calculate the probability that Boris wins the point. (4)

[6]

Question 7

Given $g(x) = -x^3 - x^2 + x + 10$

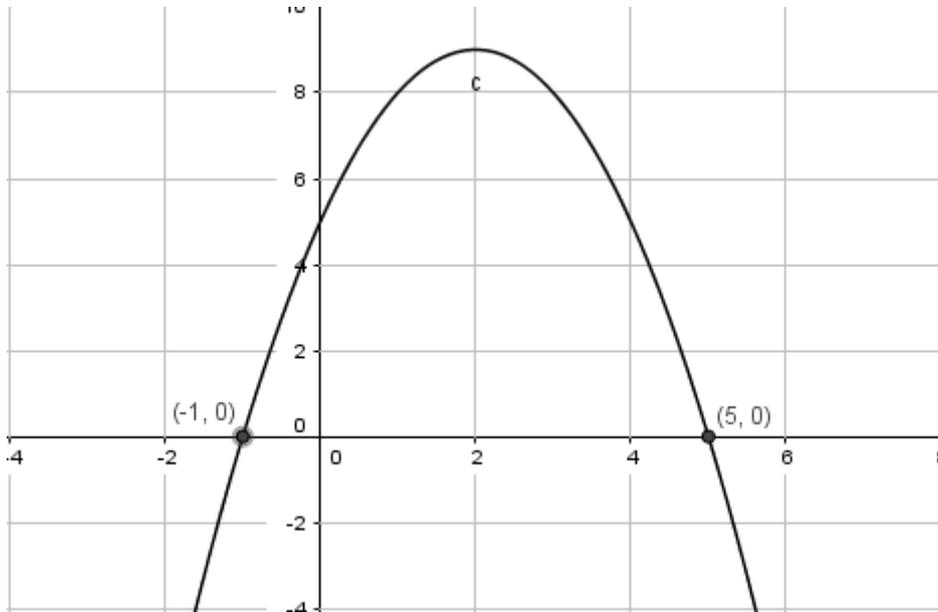
- a. Determine the x –intercept(s) of the graph of g . (5)
- b. Calculate the coordinates of the turning points of g . (5)
- c. Sketch the graph of g in your answer book. Show clearly all intercepts with the axes and all turning points. (3)
- d. For which value(s) of p will $g(x) = p$ have 3 unequal roots? (2)

[15]

SECTION B

Question 8

The graph of $y = p'(x)$, where $p(x)$ is a cubic function has been sketched below. Study the diagram and answer the questions that follow.



- a. For which value(s) of x is the graphs of $y = p'(x)$ decreasing? (2)
- b. At which value of x does the graph of p have a local maximum? Give a reason for your answer. (2)
- c. Sketch the graph of $p''(x)$, show the coordinates of the x –intercept. (2)

[6]

Question 9

- a. The first two terms of a convergent geometric series are m and 6 (where $m \neq 0$), in that order. The sum of the infinite series is 25. Calculate the value(s) of m . (5)
- b. The sum to infinity of the terms of an infinite geometric progression is 6. The sum of the first two terms is $4\frac{1}{2}$. Find the value(s) of the first term. (5)

[10]

Question 10

- a. A coin is biased so that the probability that it falls showing tails is 0,75.
1. Draw a tree diagram to show the outcomes if the coin is tossed two times. (3)
 2. Find the probability of obtaining at least one head when the coin is tossed **five** times. (2)
 3. What is the minimum number of times the coin must be tossed so that the probability of obtaining at least one head is greater than 0,98? (3)
- b. What is the probability that a leap year will contain exactly 53 Wednesdays and 52 Thursdays? (7)

[15]

Question 11

- a. For which value(s) of x would the points $(2x; x + 6)$ and $B(x + 5; x)$ NOT lie on a function? (2)
- b. If $h(x) = 5x + 7$ and $m(x) = \frac{x-7}{5}$, prove that $m(x)$ is the inverse of $h(x)$ by proving that $h(m(x)) = m(h(x)) = x$. (4)

[6]

Question 12

Nicky has been working for 5 years and gets an increase in her salary. She opens a savings account at USAVE bank and begins with deposits of R 350 every month. The account earns 5,53% interest per annum compounded monthly. Her plan is to continue saving on a monthly schedule until she retires. After 8 years, however, she stops making monthly payments and leaves the account to continue growing.

- a. How much money will Nicky have in her account 29 years after she has opened it? (6)
- b. Calculate the difference between total deposits made into the account and the amount of interest paid by the bank. (3)

[9]

Question 13

Veronica is designing a rocket which she shoots up into the air. The trajectory of the rocket is given by the formula $h(t) = 30t - 3t^2$, where h is the height measured in metres and t is time measured in seconds.

- a. How many seconds does it take for Veronica's rocket to reach its maximum height? (3)
- b. Hence, or otherwise, determine the maximum height. (1)
- c. Calculate the acceleration for this rocket until it reaches its maximum height. (1)

[5]

Question 14

- a. Solve for x if $3x^{\frac{2}{3}} - 13x^{\frac{1}{3}} - 10x^{\frac{0}{3}} = 0$ (4)
- b. The roots of the equation $ax^2 + bx + c = 0$ are $\sin Q$ and $\cos Q$
Prove that $a^2 - b^2 + 2ac = 0$ (5)

[9]