theking'sschool

Mathematics Paper 2
Grade 12
Preliminary Examination 2017

DURATION: 180 min
MARKS: 150
Date: 15 September 2017

EXAMINER: R. Obermeyer
MODERATOR: A. Janisch
External Moderator: I. Atteridge

INSTRUCTIONS:

- **See overleaf for instructions.**
- **This paper consists of 25 pages (including cover) and an information sheet.**

NAME: Memo

ASSESSMENT					
Question	**Level Tested**	**Topic**	**Time Allocation**	**Possible mark**	**Actual mark**
SECTION A					
1	1 – 4	Analytical Geometry	22 mins	18	
2	1 – 4	Trigonometry Graphs	10 mins	8	
3	1 – 4	Trigonometry	28 mins	23	
4	1 – 4	Euclidean Geometry	16 mins	13	
5	1 – 4	Euclidean Geometry	11 mins	9	
6	1 – 4	Statistics	16 mins	13	
SECTION B					
7	1 – 4	Analytical Geometry	26 mins	22	
8	1 – 4	Statistics	12 mins	10	
9	1 – 4	Trigonometry	10 mins	8	
10	1 – 4	Measurement	6 mins	5	
11	1 – 4	Euclidean Geometry	19 mins	16	
12	1 – 4	Euclidean Geometry	6 mins	5	
			TOTAL:	**150**	
			PERCENTAGE:		

Teacher's Signature: ____________________

Controller's Signature: ____________________

Moderator's Signature: ____________________

Instructions

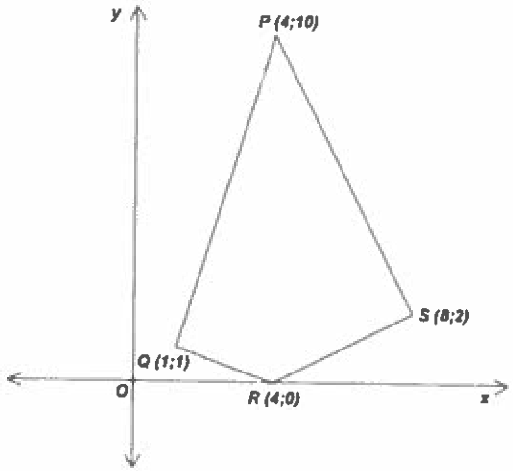
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 25 pages (including the cover page) and an Information Sheet of 2 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to ***one*** decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in DEGREE mode.
9. It is in your own interest to write legibly and to present your work neatly.

SECTION A

Question 1

In the figure below, $PQRS$ is a quadrilateral with $P(4;10)$; $Q(1;1)$; $R(4;0)$ and $S(8;2)$.



a. Determine the gradient of QR and RS. (4) (R)

$m_{QR} = \frac{1-0}{1-4} = -\frac{1}{3}$ • subst • Answer

$m_{RS} = \frac{2-0}{8-4} = \frac{1}{2}$ • subst • Answer

b. Determine the length of PS in simplest surd form. (2) R

$PS = \sqrt{(4-8)^2+(10-2)^2}$ • subst
$= \sqrt{80}$ • answer
$= 4\sqrt{5}$

c. Show that $PQ \perp QR$. (2) R

$m_{PQ} = 3$ • m_{PQ}
$m_{PQ} \times m_{QR} = -1$ • $m_1 \times m_2 = -1$
$\therefore PQ \perp QR$

d. Determine the coordinates of A to make $PARQ$ a rectangle. (2) R

$A(7,9)$ • each component

e. Prove that $PQRS$ is a cyclic quadrilateral. (3) C

$m_{PS} = -2$
• m_{PS}
$m_{RS} \times m_{PS} = -1$ • $m_1 \times m_2 = -1$
$\therefore PS \perp RS$ • Reason
$\therefore PQRS$ is cyclic opp ∠s supp.

f. Determine the equation of the circle passing through $P; Q; R$ and S. (5) C

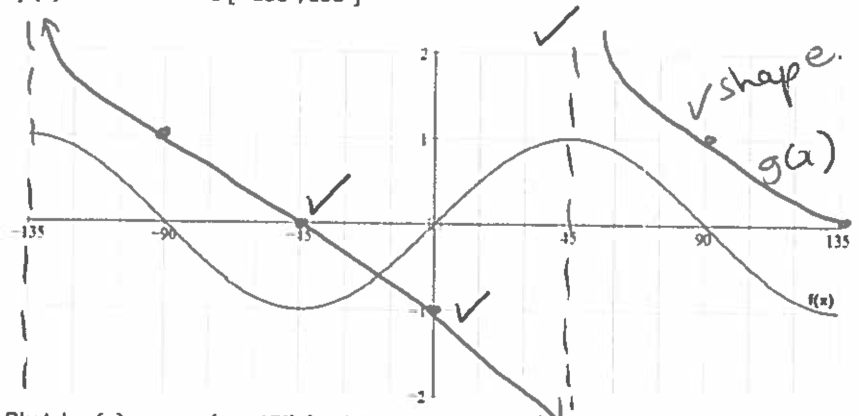
PR is the diameter • converse ∠ in semi ⊙
Midpoint $(4,5)$ • Centre
$(x-4)^2+(y-5)^2 = 25$ • 3 marks for equation

[18]

Question 2

Given: $f(x) = \sin 2x$ for $x \in [-135°; 135°]$

shape
x-int
y-int
asymp.



a. Sketch $g(x) = -\tan(x + 45°)$ for the given domain on the same axis as $f(x)$. (4) R

b. Find the value of $f(90°) - g(0°)$. (2) R

$\sin 2(90) - (-\tan(0+45))$
$\sin 180 + \tan 45$ subst
$0 + 1$ answer.
1

c. If the equation $y = a\sin(x + p)$ represents the graph of $f(x)$ reflected over the x –axis and shifted 60° to the right, find the values of a and p. (2) C

$a = -1$
$p = -30$

[8]

Question 3

a. Simplify the following without the use of a calculator:
$\cos 25° . \sin 55° - \cos 65° . \sin 35°$ (5) R

$\sin 65 \cos 35 - \cos 65 \sin 35$
$\sin(65 - 35)$
$\sin 30$
$\frac{1}{2}$

$\cos 25 \sin 55 - \sin 25 \cos 55$
alt.

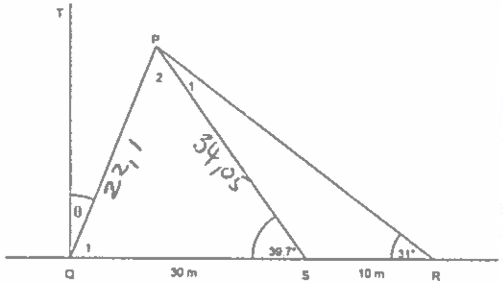
b. Prove that: $\frac{1-\cos 2x}{\sin 2x} = \tan x$ (4) R

LHS $\frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$
$= \frac{2\sin^2 x}{2\sin x \cos x}$
$= \frac{\sin x}{\cos x}$
$= \tan x$
$= RHS$

c. Determine the general solution of: $\sin x . \cos x = \frac{1}{3}$ (5) C

$2\sin x \cos x = \frac{2}{3}$
$\sin 2x = \frac{2}{3}$
r.a $= 41,81$
$2x = 41,81 + k.360$ $2x = 138,19 + k.360$
$k \in Z$
$x = 20,91 + k.180$ $x = 69,09 + k.180$

d. In the photograph below, a bridge is supported by a tower which is not perpendicular to the ground.



In the diagram, the tower PQ is shown and stays (steel ropes), PR and PS, help keep the tower stable. (There are also other cables). From base Q of the tower a vertical line QT is shown. $QS = 30m$, $SR = 10m$, $\hat{R} = 31°$ and $\hat{S} = 39,7°$.

1. Why is $\hat{P}_1 = 8,7°$? (1) K

Ext ∠ of △ = sum int opp ∠'s ✓

2. Show that $PS = 34,05\ m$. (2) R

$\frac{PS}{\sin 31} = \frac{10}{\sin 8,7}$ ✓

PS = 34,05 m ✓

3. Determine the length of the tower PQ correct to one decimal place. (2) R

$PQ^2 = 30^2 + (34,05)^2 - 2(30)(34,05)\cos 39,7$
$= 487,519209$ ✓

$= 22,1$ m ✓

C1

4. Now find the angle marked θ, the inclination of the tower from the vertical. (4) R3

$\frac{\sin Q_1}{34,05} = \frac{\sin 39,7}{22,1}$ ✓

$\sin Q_1 = 0,98416489$ ✓

$Q_1 = 79,79°$ ✓

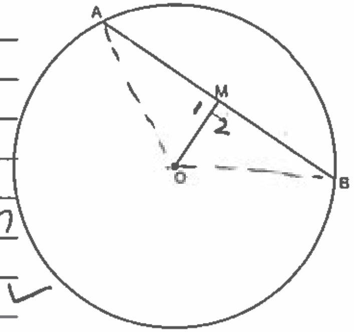
$\theta = 10,2°$ ✓

[23]

All statements must have reasons in Question 4 – 5

Question 4

a. Use the diagram below to prove the theorem that states:
The line from the centre of a circle perpendicular to a chord bisects the chord. (6) K



Given circle O with OM ⊥ AB ✓

RTP AM = MB ✓

Construct join AO & OB ✓

Proof In △AOM & △BOM

AO = OB radii

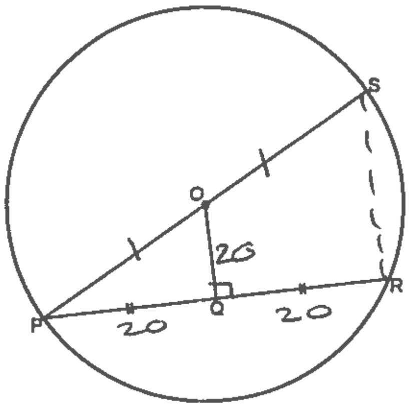
OM = OM common ✓

$M_1 = M_2$ given ⊥ ✓

∴ △AOM ≡ △BOM (RHS)

∴ AM = MB ✓ (≡)

b. In the diagram below, O is the centre of the circle, Q is the mid-point of PR. $OQ = 20\text{mm}$ and $PR = 40\text{mm}$



1. Determine, stating reasons, the length of SR. (3) K

PQ = QR given
PO = OS radii
OQ = ½ SR } midpnt th
OQ // SR }

SR = 40mm

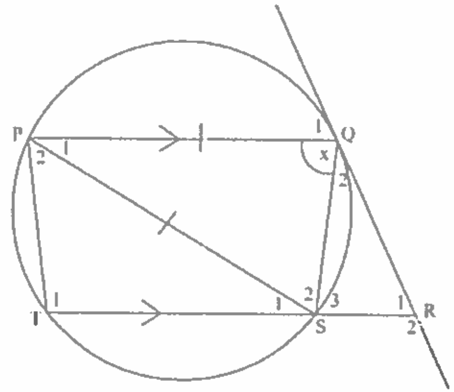
2. Hence, or otherwise, determine the length of the diameter PS. (4) K R

~~[illegible]~~
PQ = QR given
OQ ⊥ PR line bisecting chord
$PO^2 = 20^2 + 20^2$ Pyth
PO = 800
= $20\sqrt{2}$
× 2
PS = $40\sqrt{2}$

[13]

Question 5

$PQST$ is a cyclic quadrilateral. QR is a tangent to the circle at Q. TR is a straight line. $PQ//TR$, $PQ = PS$ and $P\hat{Q}S = x$.



a. Complete the given table: (5) R

STATEMENT	REASON
$\hat{S}_2 = x$	∠'s opp = sides
$\hat{S}_3 = x$	Alternate <' s; $PQ//TR$
$\hat{P}_{1+2} = x$	ext ∠ of cyclic quad
$\hat{Q}_1 = x$	tan chord
$\hat{R}_1 = x$	Corresponding <' s; $PQ//TR$

b. Prove that $PQRT$ is a parallelogram. (4) R

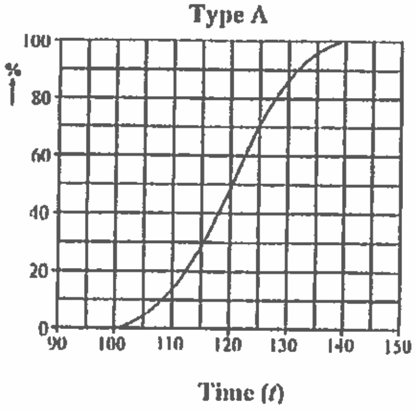
$T_1 = 180 - x$ co-int $PQ \parallel TS$ ✓
$PQ \parallel TR$ given
$PT \parallel RQ$ ✓ co-int suppl. ✓
$\therefore$ $PQRT$ is a parm opp sides $\parallel$ ✓

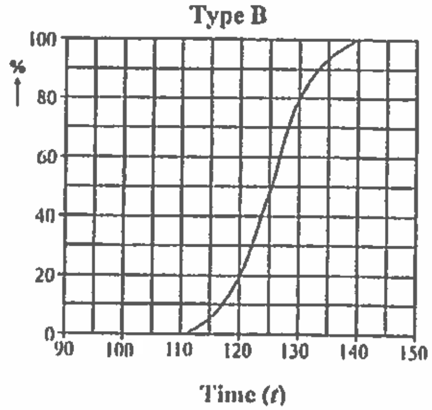
OR $P = R_1 = x$ proven
$PQR = 180 - x$ str line
$T_1 = 180 - x$ co-int $PQ \parallel TS$
$PQR = 180 - x$ proven
$\therefore$ $PQRT$ is a parm opp $\angle$s $=$

[9]

Question 6

a. The battery life of two different makes of laptops are compared. The following cumulative frequency curves about the battery lifespan of two types of laptops are given below. The graphs indicate the percentage of batteries that die after t minutes of usage.





1. Give the median battery lifespan of each type of laptop. (2) K

Type A 120 min ✓
Type B 125 min ✓

2. Which of the two types has a range which is more than 30? Show some working out. (1) K

Type A ✓
$140 - 100 = 40$ min

3. Which of the two types has an interquartile range which is less than 10? Show your working out. (1) K

Type B $Q_3 - Q_1$
$127 - 122 = 5$ min ✓

4. Using the information above, give TWO reasons why Type B laptop should be chosen over Type A laptop. (2) C

Type B laptop seems to be/constantly lasting longer as the range ✓ is less than Type A (spread is less) 50% of Type B have a battery life which is within 5 min of each other ✓

b. A large company employs 7 salespersons. The commission that each salesperson earned (in rands) in a certain month is shown below.

3900	5700	10600	13600	15100	15800	17100

1. Calculate the mean of the data. (1) K

R 11685,71 ✓✓

2. Calculate the standard deviation of the data. (2) K

R 4768,78 ✓✓

3. The company rates the staff according to the amount of commission earned. A salesperson whose commission is more than one standards deviation above the mean receives a rating of "good". How many salespersons will receive a rating of "good" for that month? Substantiate your answer. (4) R

$(\bar{x} - \sigma ; \bar{x} + \sigma)$

above

11685,71 + 4768,78 ✓

= R16454,49

∴ only one person would ✓ receive a rating of good

[13]

SECTION B

Question 7

The circle with centre A and equation $(x-2)^2 + (y+2)^2 = 4$ is given.

a. Write down the co-ordinates of the centre of the circle, A, and the radius of the circle. (3) R

$A(2; -2)$ • $A(x ; y)$ ✓✓

rad = 2 • rad

b. Determine the equation of the tangent to circle A at the point $T\left(\frac{2}{5}; -\frac{4}{5}\right)$. (5) R

$A(2; -2)$ $T(2/5 ; -4/5)$ • subst AT

$m_{AT} = \dfrac{-4/5 + 2}{2/5 - 2} = \dfrac{-3}{4}$ • m_{AT}

• m tan

$m_{tan} = \dfrac{4}{3}$ • subst EQ

• answer

$y + \dfrac{4}{5} = \dfrac{4}{3}\left(x - \dfrac{2}{5}\right)$

$3y = 4x - 4$

c. A second circle with centre B has the equation $x^2 + y^2 + 4x - 2y + k = 0$
Determine the value of k for which the two circles with centres A and B will touch each other externally. (8) C

$x^2 + y^2 + 4x - 2y + k = 0$

$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 1 - k$

$(x+2)^2 + (y-1)^2 = 5 - k$

Centre $B(-2; 1)$ rad: $\sqrt{5-k}$

centre $A(2; -2)$

$AB^2 = (-2-2)^2 + (1+2)^2 = 25$

$AB = 5$

Rad A + Rad B = dist AB

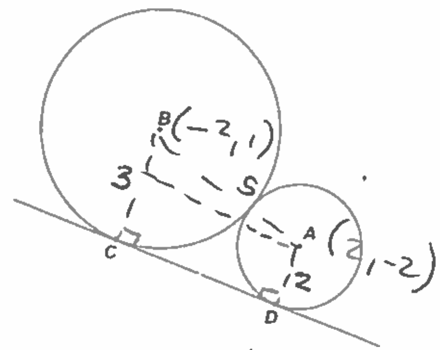
$\sqrt{5-k} + 2 = 5$

$5 - k = 9$

$k = -4$

- $(x+2)^2$
- $(y-1)^2$
- $5-k$
- Subst dist
- $AB = 5$
- Rad + Rad = dist
- Simplify
- answer

d. Assume that the radius of circle B is 3 units and circle B touch circle A externally.
Circle A and B have a common tangent that touches the circles at D and C respectively.
Determine the area of the trapezium $ABCD$ made by this common tangent. (6) P



$AD \perp DC$ $BC \perp DC$ tan ⊥ rad.

$AB = 5$

$BC = 3$ $CD^2 = 5^2 - 1^2$ Pyth

$AD = 2$ $CD = \sqrt{24}$

area $= \frac{1}{2}(BC + AD) \times CD$

$= \frac{1}{2}(2+3) \times \sqrt{24}$

$= 5\sqrt{6}$

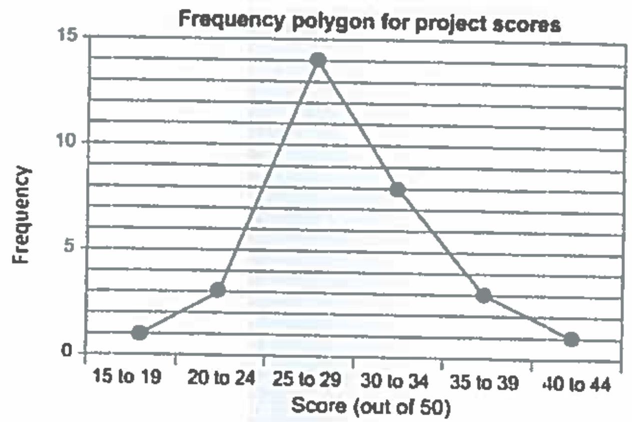
$= 12{,}25$

- Construction
- rad ⊥ tan
- CD
- Method 2 marks
- Answer

[22]

Question 8

The scores (out of 50) obtained for a project by the learners of a Grade 12 Mathematics class are shown in the frequency polygon below.



a. How many learners are there in the class? (1)

30 learners

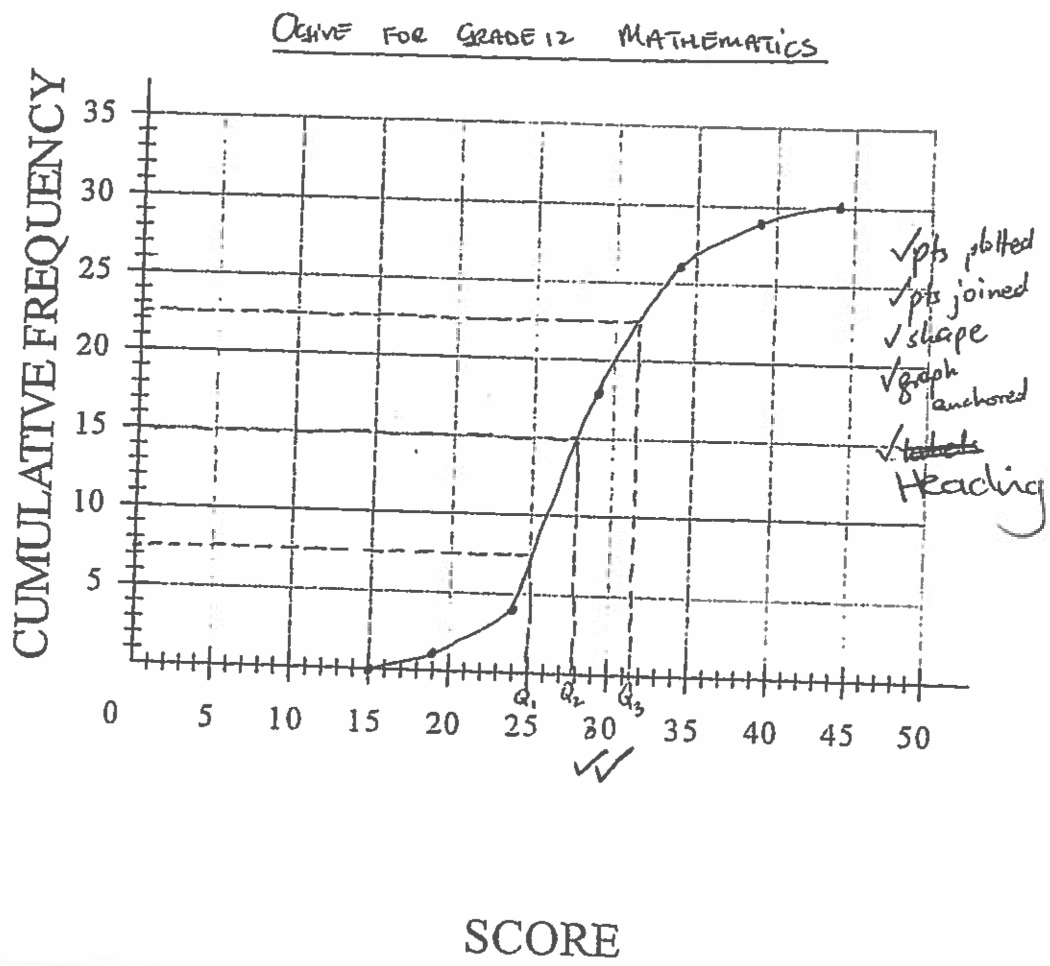
b. Describe the distribution: normal, positively skewed or negatively skewed. (1)

positively skewed OR skewed to the right

c. Use the graph paper to sketch the ogive for this information. Indicate clearly where the first quartile, median and third quartile can be read off. (8)

Tally	SCORE	CUMULATIVE FREQUENCY
1	$15 < x \leq 20$	1
3	$20 < x \leq 25$	4
14	$25 < x \leq 30$	18
8	$30 < x \leq 35$	26
3	$35 < x \leq 40$	29
1	$40 < x \leq 45$	30

✓



Question 9

a. If $\tan 15° = \frac{a}{b}$ and $a^2 + b^2 = c^2$, prove without a calculator, that $\frac{2ab}{c^2} = \frac{1}{2}$. (4) P

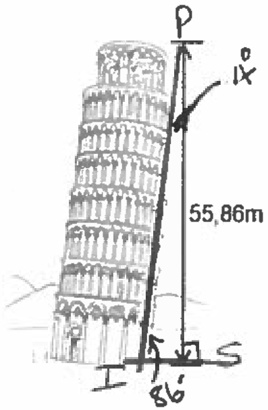
$\sin 15 = \frac{a}{c}$ $\cos 15 = \frac{b}{c}$

$\sin 15 \times \cos 15 = \frac{a}{c} \times \frac{b}{c}$ ✓

$2 \sin 15 \cos 15 = \frac{2ab}{c^2}$ ✓

$\sin 30 = \frac{2ab}{c^2}$ ✓ $\frac{1}{2} = \frac{2ab}{c^2}$ ✓

b. The leaning tower of Pisa currently "leans" at an angle of 4° and has a perpendicular height of 55,8 m. Determine how tall the tower was when it was originally built. (4)



Construction ✓
and identifying ✓
where 4° is

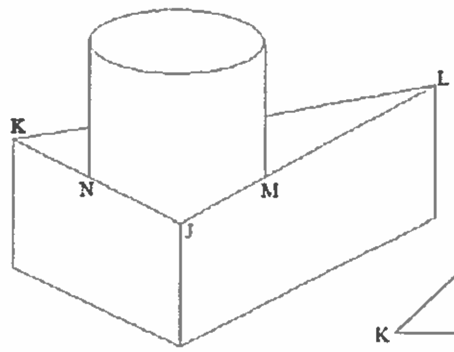
$\sin 4° = \frac{55{,}86}{PI}$ ✓

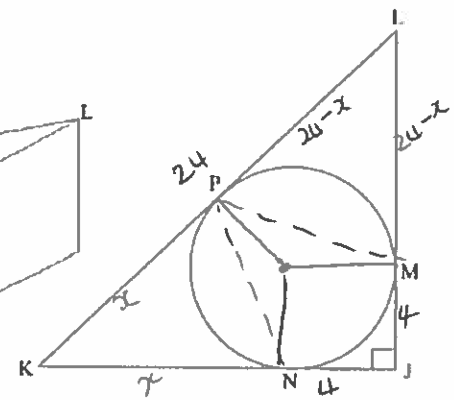
$PI = 56m$ ✓

[8]

Question 10

A cylindrical peg with radius 4 units fits snugly into a box, the base of which is a right-angled triangle. If $KP = x; K\hat{J}L = 90°$ and the hypotenuse is 24 units, determine with reasons, the perimeter of ΔJKL. (5) P





$KP = x$ given

$LP = 24 - x$

$KN = x$ tan same pnt (KL & KJ) ✓

$LM = 24 - x$ tan same pnt (KL & LJ) ✓

$JM = JN = 4$ tan same pnt (LJ & KJ) ✓
rad ⊥ tan (opp sides sq)

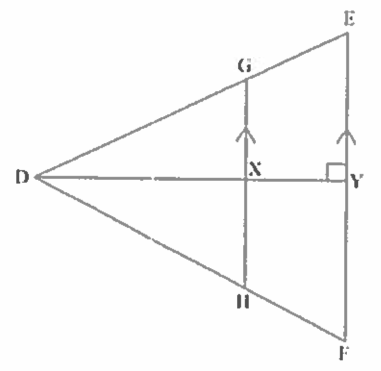
$Per = 24 + (x + 4) + (4 + 24 - x)$ ✓
$= 56$ units ✓

[5]

Reasons to be given unless otherwise stated in Question 11 – 12

Question 11

In the figure, GH is drawn parallel to EF. DY is perpendicular to EF and cuts GH at X.



a. Prove:
1. $\Delta DGH\ ///\ \Delta DEF$ (3) R

In $\triangle$ DGH and DEF

D = D common ✓
DGH = DEF corr GH//EF ✓
H = F sum ∠s $\triangle$

$\therefore \triangle DGH /// \triangle DEF$ AAA ✓

2. $\frac{DX}{DY} = \frac{GH}{EF}$ (4) R

$\frac{Dx}{Dy} = \frac{DG}{DE}$ ✓ line // 1 side $\triangle$ ✓

~~$\frac{Dx}{Dy} = \frac{GH}{EF}$~~ (AAA) ✓ $\frac{DG}{DE} = \frac{GH}{EF}$

$\therefore \frac{Dx}{Dy} = \frac{GH}{EF}$ ✓

b. If the area of $\Delta\, GHD$ is equal to the area of quadrilateral $GHFE$:
1. Express the area of $\Delta\, DEF$ in terms of $\Delta\, GHD$ and $GHFE$. (1) R

area $\triangle$ DEF = area $\triangle$ DGH + area GHFE

2. Hence, or otherwise, prove that $\frac{1}{2}EF.DY = GH.DX$ (4) R

$\frac{1}{2}$ EF × Dy = $\frac{1}{2}$GH × Dx ✓ + $\frac{1}{2}$GH × Dx ✓

$\frac{1}{2}$ EF Dy = GH × Dx ✓

3. Prove that $\frac{DG}{DE} = \frac{1}{\sqrt{2}}$. (4) C

$\frac{Dx}{Dy} = \frac{1}{2}\left(\frac{EF}{GH}\right)$ ✓

$\frac{Dx}{Dy} = \frac{1}{2}\frac{Dy}{Dx}$ ✓

$\left(\frac{Dx}{Dy}\right)^2 = \frac{1}{2}$ ✓

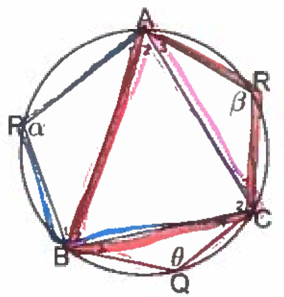
$\frac{Dx}{Dy} = \frac{1}{\sqrt{2}}$ ✓

$\frac{DG}{DE} = \frac{1}{\sqrt{2}}$

[16]

Question 12

a. In the diagram below A, R, C, Q, B and P are points on the circumference of the circle. Angles α, β and θ are shown.



Prove that: $\alpha + \beta + \theta = 360°$ (5) C

cyclic Quad APBC $\hat{P} + \hat{C}_2 = 180°$
$\alpha + \hat{C}_2 = 180°$ ✓
cyclic Quad ABQC
$A_2 + Q = 180°$
$A_2 + \theta = 180$ ✓
cyclic Quad ABCR $B_2 + R = 180$
$B_2 + \beta = 180°$ ✓
But $\hat{A}_2 + \hat{B}_2 + \hat{C}_2 = 180$ sum ∠s △ ✓
$\therefore \alpha + \beta + \theta = 360°$ ✓

[5]

Additional Working Pages