D. Fell's copy

Our Lady of Fatima
Dominican Convent School

MEMO

VERITAS

MATHEMATICS PAPER 2
TRIAL EXAMINATION

Grade 12 August 2017

Time: 3 hours Marks: 150

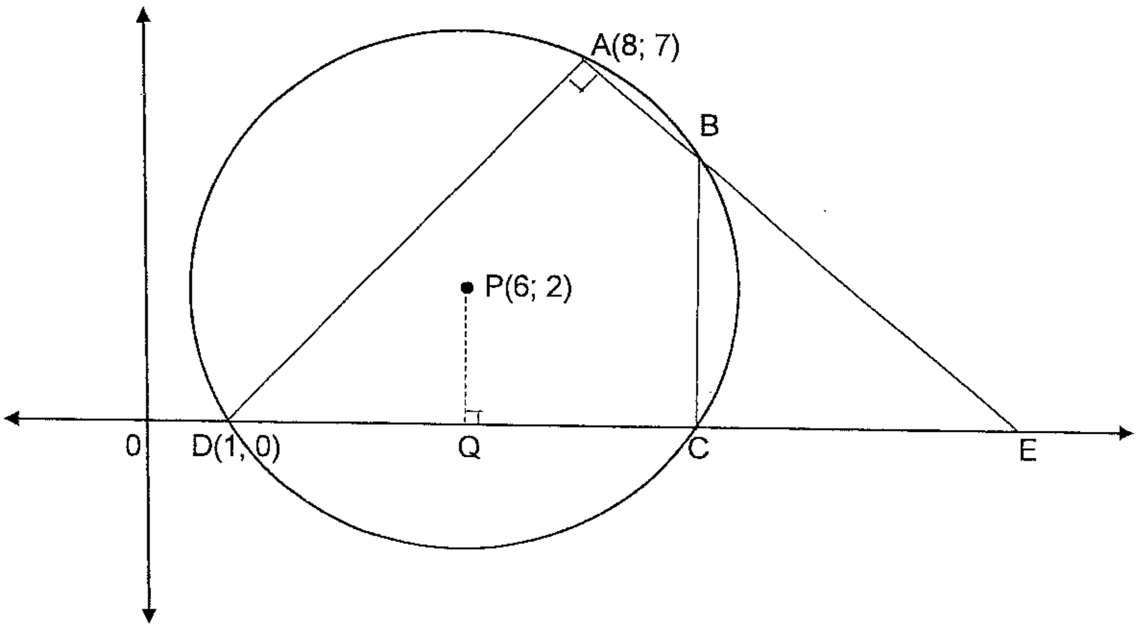
EXAMINER: Mrs. D. Fell **MODERATOR: Mrs. S. Moodley**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This examination paper consists of 26 pages. All questions must be attempted.
2. Answer the questions ON THE QUESTION PAPER in the space provided.
3. Any extra lined or blank ~~paper~~ pages in this booklet may be used if extra space is needed.
4. Number your answers exactly as the questions are numbered.
5. Please refer to the 2 page information sheet supplied.
6. Round off to **2 decimal places** unless otherwise stated.
7. Calculators may be used. Make sure that they are in DEGREE MODE.
8. Diagrams are **not** drawn to scale.
9. When you refer to angles ensure that you name them accurately.
10. Write only in black or blue ink.
11. All necessary working as per mark allocation must be clearly shown.
12. It is in your own interest to **write legibly** and to present your work neatly.
13. Read the questions CAREFULLY.
    YOU CAN DO THIS!

SECTION A

1. The Cartesian plane below shows a circle ABCD with centre P(6;2).
   AB produced intersects the X axis at E.
   D = (1; 0) and A = (8; 7) .



1.1 Why is $B\hat{C}D = 90^0$? (1) L1

opp. ∠'s of cyclic quad. supplementary ✓

1.2 Prove that $A\hat{D}C = 45^0$ (4) L2

$\tan A\hat{D}C = ✓m \; m_{AD} = \frac{7-0}{8-1} ✓a = 1 ✓ca$

$\therefore A\hat{D}C = 45^\circ$. ✓a

1.3

1.3.1 Calculate the radius of the circle. (2) L1

$r = DP = \sqrt{(6-1)^2+(2-0)^2}$ ✓ $P(6;2)$
$= \sqrt{29}$ $D(1;0)$
$\approx 5,39$ ✓ (2d.pl.)

1.3.2 Hence write down the equation of the circle ABCD. (2) L1

$(x-6)^2 + (y-2)^2 = 29$ ✓ ✓

1.4.1 Why is DQ = QC? (1) L1

line from centre ⊙ ⊥ chord, bisects chord. ✓

1.4.2 **State** the co-ordinates of (it is not necessary to show working):

a.) C. (2) L2

$C = (11;0)$ ✓m ✓a

b.) B (2) L3

$B = (11;4)$ ✓ca ✓a

1.5 Find the equation of the straight line ABE. (3) L2

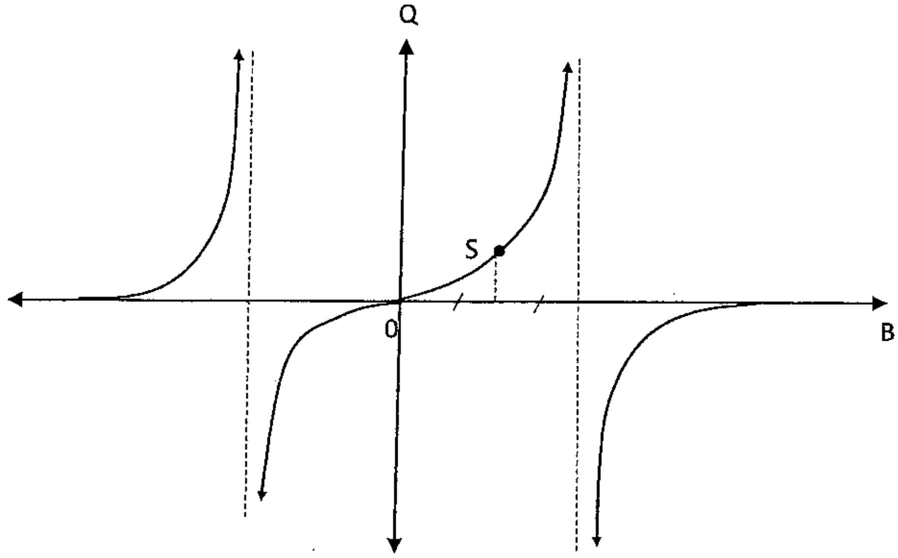
$m_{ABE} = -\frac{1}{m_{AD}} = \frac{-1}{1} = -1$ ✓ $\therefore y = -x + c$
sub $(8;7)$ $\therefore$ ✓m $7 = -8 + c$

$15 = c$ ✓ca

$\therefore y = -x + 15$

[17]

2. The function $Q = \tan 2B$ is sketched above for $\hat{B} \in [-90^0; 90^0]$.



2.1 Give the co-ordinates of the point S. $S = (22{,}5^\circ ; 1)$ (2)

2.2 Write down the equation of the asymptote to the right of the Y axis. (1)

$x = 45^\circ$ (not just 45°)

2.3 If $P = \frac{\sin B - 2\sin^3 B}{2\sin^2 B . \cos B}$, write P in terms of Q. (5)

$P = \frac{\sin B(1 - 2\sin^2 B)}{\sin B . 2\sin B . \cos B} = \frac{\cos 2B}{\sin 2B} = \frac{1}{\tan 2B}$

$= \frac{1}{Q}$

2.4 Hence, state for which value/s of $\hat{B} \in [-90^0; 90^0]$, P is undefined. (2)

For $\hat{B} = -90^\circ ; 0^\circ ; 90^\circ$ (ie. when Q or $\tan 2B = 0$)

all answers correct

[10]

3.1 Prove that $2\sin(\theta - 45^0) - \sqrt{2}.\cos(180^0 - \theta) = \sqrt{2}.sin\theta$ (4)

$LHS = 2[\sin\theta\cos45° - \sin45°.\cos\theta] - \sqrt{2}.(-\cos\theta)$

$= 2[\frac{\sqrt{2}}{2}\sin\theta - \frac{\sqrt{2}}{2}\cos\theta] + \sqrt{2}\cos\theta$

$= \sqrt{2}\sin\theta - \sqrt{2}\cos\theta + \sqrt{2}\cos\theta$

$= \sqrt{2}\sin\theta$

$= RHS.$

3.2 Hence, find the general solution for θ rounded to the nearest degree if:

$2\sin(\theta - 45^0) - \sqrt{2}.\cos(180^0 - \theta) = cos\theta$ (5)

$\therefore \sqrt{2}\sin\theta = \cos\theta$

$\therefore \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{1}{\sqrt{2}}$ $\therefore \theta$ quads 1 & 3

$\therefore$ key $\angle \approx 35°$

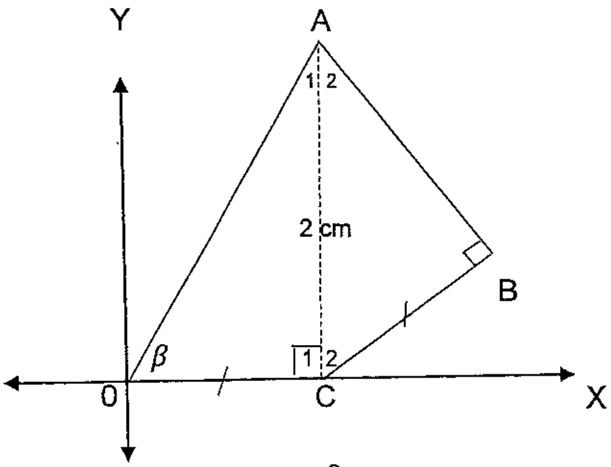
$\therefore \theta = 35° + k180°, k \in I$

[9]

alternative solution:

$\theta = 35° + k360°$ or $215° + k360°, k \in I.$

4.



In the diagram above, $sin\beta = \frac{2}{\sqrt{5}}$,

4.1 What is the length of OA?

accept either.

$\boxed{\sqrt{5}\,cm \approx 2{,}24cm\ (2dp)}$ (1)

4.2 Calculate the length of BC. (2)

$BC = OC = \sqrt{\sqrt{5}^2 - 2^2} = 1cm$

4.3 Prove that OA is **not** parallel to CB. (5)

$\triangle ABC, \quad \hat{B} = 90^\circ \quad \therefore \quad \cos\hat{C}_2 = \frac{CB}{2cm} = \frac{1cm}{2cm} = 0{,}5$

$\therefore \hat{C}_2 = 60^\circ.$

$\therefore A\hat{O}C + O\hat{C}B = \beta + 90^\circ + 60^\circ.$

$\left.\begin{array}{l} \sin\beta = \frac{2}{\sqrt{5}} \\ \therefore \beta = 63{,}43^\circ \end{array}\right\}$

$\therefore A\hat{O}C + O\hat{C}B = 63{,}43^\circ + 150^\circ$

$\neq 180^\circ.$

But these are co-interior ∠s

$\therefore$ OA $\not\parallel$ CB.

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4.4 Determine the equation of circle ABC. (4)

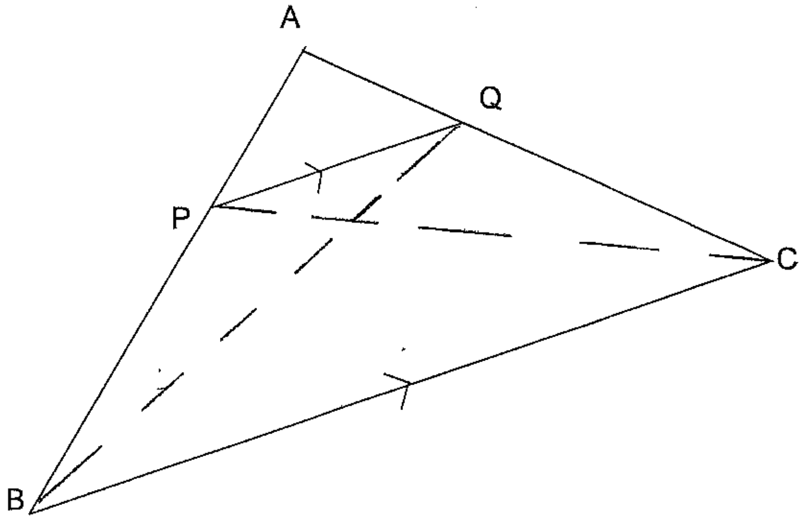
$\hat{B} = 90°$ ∴ AC diameter. (converse of ∠ in semi-⊙)
∴ $r = \frac{1}{2}$ cm = 1cm.

centre = midpt$_{AC}$ = $(1 ; 1)$. ✓ca ✓ca

[12]

∴ $(x-1)^2 + (y-1)^2 = 1^2 = 1$ ✓ca ✓a

5.a) Use the diagram below to prove the theorem that states that the line drawn parallel to one side of a triangle, divides the other two sides in the same proportion. (6)



Given: $\triangle ABC$ with $PQ \parallel BC$

R.T.P: $\dfrac{AP}{PB} = \dfrac{AQ}{QC}$ (or other forms that match proof.)

Constr: Join BQ & PC.

Proof: $\dfrac{\triangle APQ}{\triangle BPQ} = \dfrac{AP}{PB}$ ($\triangle$'s with common vertex Q, bases in same line APB)

Sim, $\dfrac{\triangle APQ}{\triangle CPQ} = \dfrac{AQ}{QC}$

$PQ \parallel BC$ (given)

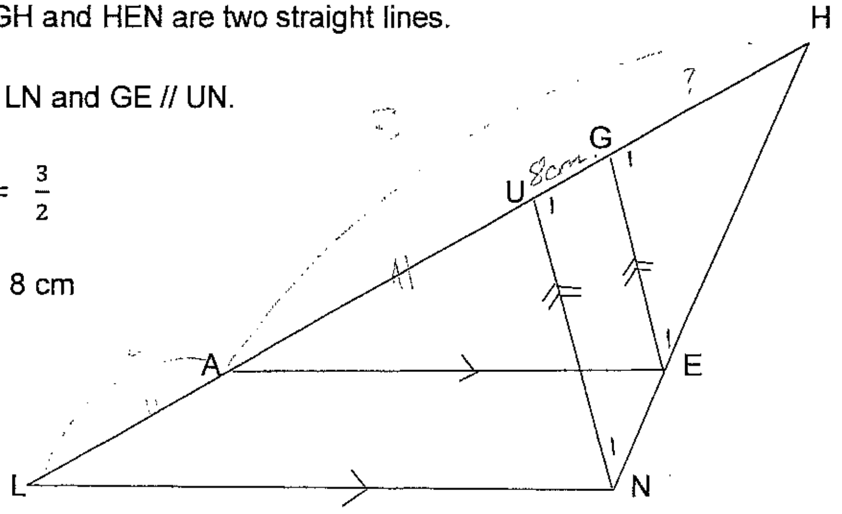
$\therefore \triangle BPQ = \triangle CPQ$ ($\triangle$s bet. same $\parallel$'s on same base)

$\therefore \dfrac{\triangle APQ}{\triangle BPQ} = \dfrac{\triangle APQ}{\triangle CPQ}$

$\therefore \dfrac{AP}{PB} = \dfrac{AQ}{QC}$

5.b) In the diagram below:

* LAUGH and HEN are two straight lines.
* AE // LN and GE // UN.
* $\frac{AH}{AL} = \frac{3}{2}$
* UG = 8 cm



(1.) Calculate the length of HG. (4)

$\triangle HLN$, $AE \parallel LN$

$\therefore \frac{HE}{EN} = \frac{AH}{AL} = \frac{3}{2}$ (proportion theorem)

$\triangle HUN$, $\frac{HG}{UG} = \frac{HE}{EN} = \frac{3}{2}$.

$\therefore \frac{HG}{8cm} = \frac{3}{2}$

$\therefore HG = \frac{3}{\cancel{2}} \cdot \overset{4}{\cancel{8}}cm. = 12cm$

(2.) Determine the value of $\frac{GE}{UN}$, giving reasons. (4)

In Δs HUN & HGE,
- $\hat{H}$ common
- $\hat{G}_1 = \hat{U}_1$ (corres. ∠'s. GE//UN)
- Sim, $\hat{E}_1 = \hat{N}_1$

$\therefore \Delta HUN \,|||\, \Delta HGE$ (AAA)

$\therefore \frac{GE}{UN} = \frac{HG}{HU} = \frac{12}{20}$

$\therefore \frac{GE}{UN} = \frac{3}{5}$

(3.) If, in addition to the above information, LA = AU, find the length of LH in cm. (3)

$AU = LA = \frac{2}{5} LH$ $\left(\frac{LA}{AH} = \frac{2}{3} \text{ given}\right)$.

$\therefore LH - AL - AU = UH = 20\text{cm}$.

$\therefore LH - \frac{2}{5}LH - \frac{2}{5}LH = 20\text{cm}$

$\therefore \frac{1}{5}LH = 20\text{cm}$

$\therefore LH = 5.(20\text{cm})$

$= 100\text{cm}$

6.

a.) A team of environmentalists conducted a survey on 14 different countries, recording each country's population as well as their eco-friendly rating (EFR) which was on a scale of 0 to 10 (0 being bad and 10 being outstanding).

Population in millions	5	20	10	45	5	10	50	45	23	25	15	30	40	45
EFR	9	6	7	6	8	8	3	2	7	7	1	5	4	4

1.) Determine the equation of the line of best fit. (4 decimal places) (2)

$y = 7{,}9364 - 0{,}0927x$

2.) If the correlation coefficient, $r = r_1$ for this sample, calculate r_1 and hence discuss the strength of the correlation, relating to the context. (3)

$r_1 = -0{,}6248 \Rightarrow$ moderate, negative, linear relationship bet. population + ~~EFR~~ EFR of country.

b.) The only outlier in this data set is (15 million ; 1).
If this outlier is ignored, then $y = 8{,}9187 - 0{,}1132x$ and $r = r_2 = -0{,}8862$.
If the team was approached by a 15th country, with a population of 15 million, to predict what their EFR would be, based on their survey,

1.) Why would the team use the line of best fit calculated without the outlier? (2)

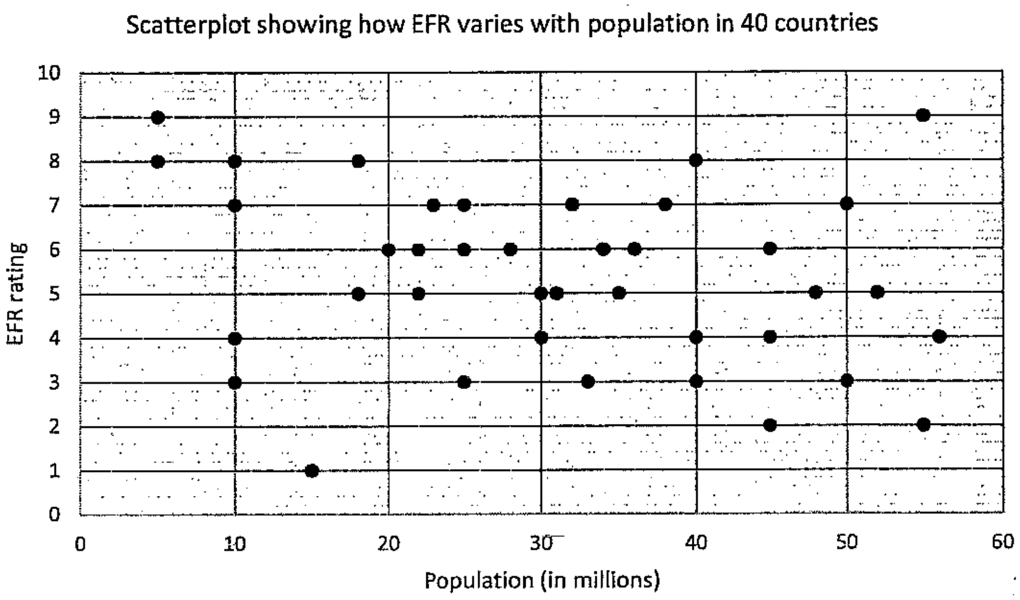
$r_2 = -0{,}8862$ stronger correl. coeff. than $r_1 = -0{,}6248$
$\therefore$ stronger rel. bet pop + EFR
$\therefore$ prediction will be more reliable.

2.) Calculate the predicted EFR for this 15th country. (to nearest whole number) (2)

$\therefore y = 8{,}9187 - 0{,}1132(15)$ ✓m
$= 7{,}2207 \approx 7$ = EFR. ✓a

(if no rounding here, don't penalise)

c.) The team decided to increase their original sample size by conducting EFR assessments on 26 more countries and their resulting scatterplot for the 40 countries appeared as follows:



In which one of the following intervals is this data set's correlation coefficient most likely to be? (There is no need to use your calculator here)

(1.) $-1 < r < -0{,}25$
(2.) $-0{,}25 < r < 0{,}25$
(3.) $0 < r < 0{,}65$
(4.) $r < -1$ OR $r > 1$ (1)

(2.)

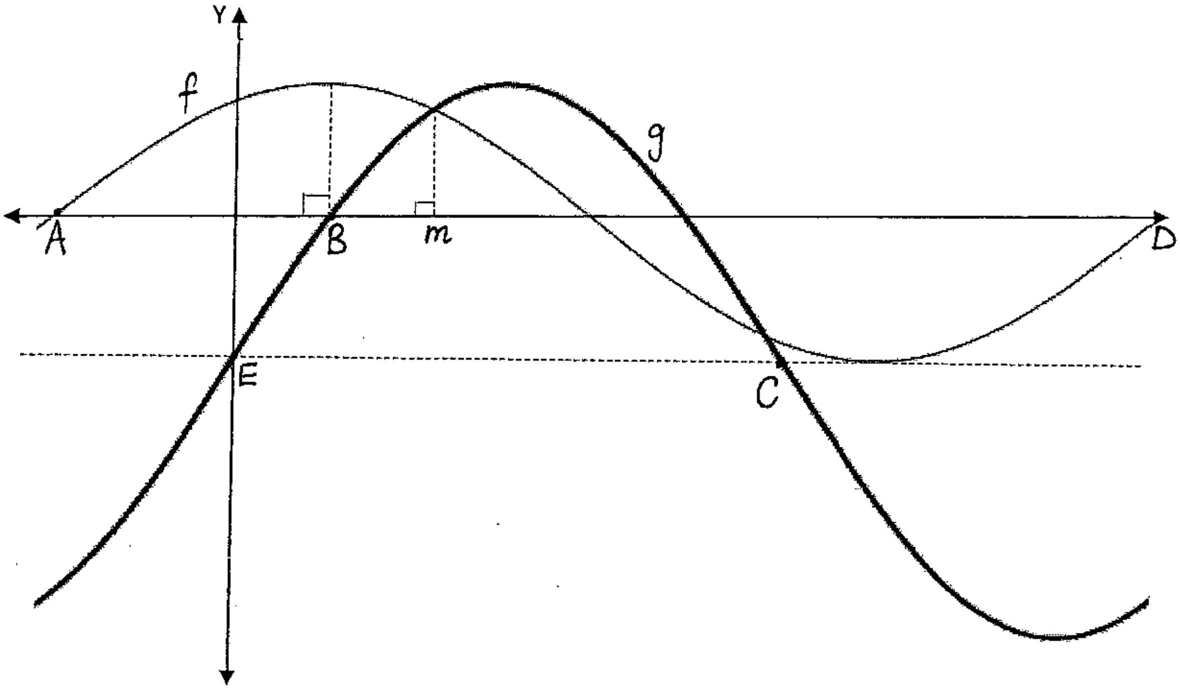
[10]

SUB-TOTAL SECTION A: [75]

SECTION B

7. The equations of the two graphs sketched below are

$f(x) = \cos(x - 30^0)$ and $g(x) = 2sinx - 1$



7.1 Determine the co-ordinates of the following points:

a.) A $\boxed{(-60^\circ ; 0)}$ ✓

b.) B $\boxed{(30^\circ ; 0)}$ ✓

c.) C $\boxed{(180^\circ ; -1)}$ ✓

d.) D $\boxed{(300^\circ ; 0)}$ ✓

(4)

7.2 Give the equation of the straight line passing through E and C. (1)

$\boxed{y = -1}$ ✓

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7.3 Use the graphs above to find the value/s of x if $f(x) = g(30^0)$. (3)

ie. $f(x) = 0$, $\therefore$ $x = -60°$; $120°$; $300°$.

✓ca. ✓ca.

7.4 From your graph it looks like $m = 60^0$.
Show by calculations whether this is true or false. (3)

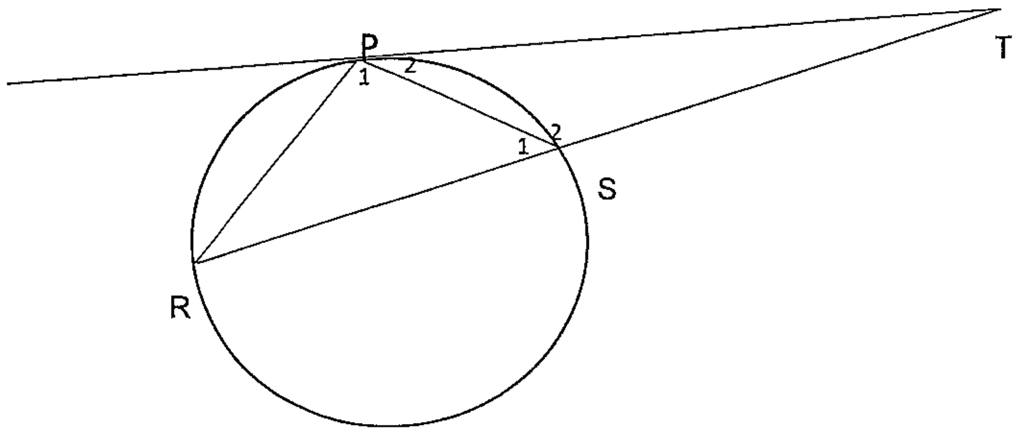
pt. int. at $x = m$, $\therefore$ Does $\cos(m - 30°) = 2\sin(m) - 1$ if $m = 60$
$g(60°) = 2\sin 60° - 1 = 2\frac{\sqrt{3}}{2} - 1 = 0{,}71$. ✓
$f(60°) = \cos(60° - 30°) = \cos 30° = \frac{\sqrt{3}}{2} = 0{,}866$ ✓

[11]

$\therefore$ $g(60°) \neq f(60°)$.

$\therefore$ $m \neq 60°$ $\therefore$ it is false ✓

8.



In the above diagram, PT is a tangent to circle PRS.

Prove that PT^2 = TS. RT

[5]

think to use ||| Δ's ✓

In Δ's PTS & RTP,

• $\hat{P}_2 = \hat{R}$ (tan/chord theorem) ✓

• $\hat{T}$ common

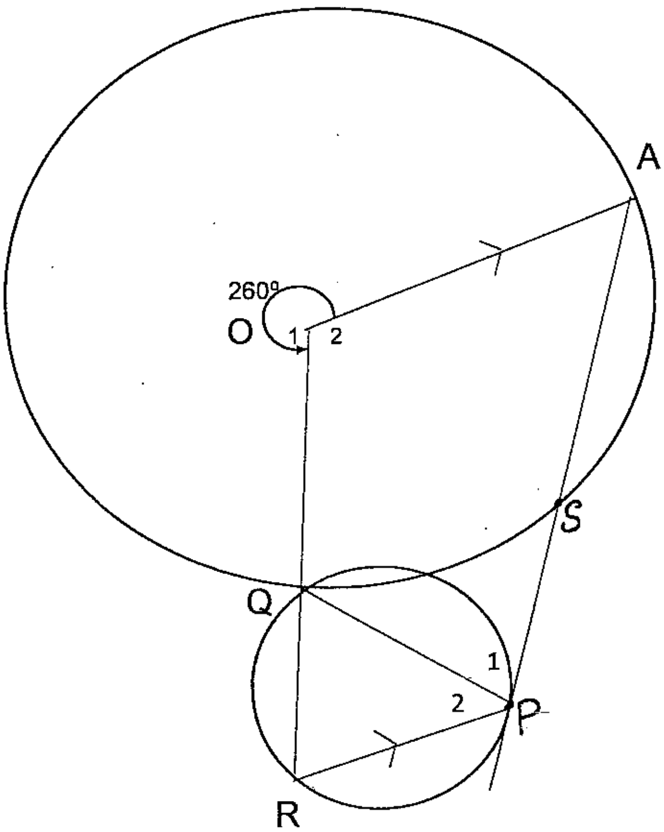
• $\therefore \hat{S}_2 = R\hat{P}T$ ($\angle$ sum Δ) ✓

$\therefore \Delta PTS \; ||| \; \Delta RTP$ (AAA) ✓ correct order.

$\therefore \dfrac{PT}{RT} = \dfrac{TS}{TP}$ ✓ NB!!

$\therefore PT^2 = TS.RT$

9.



In the diagram above:

- Circle, centre O, has reflex $Q\hat{O}A = 260^0$.
- A smaller circle PQR intersects the bigger circle at Q.
- OQ is produced to intersect the smaller circle at R.
- AP, a tangent to the smaller circle at P, intersects the bigger circle at S.
- RP // OA.

9.1 Prove that OAPQ is cyclic. (5)

$\hat{O}_2 = 360° - \hat{O}_1$ (∠'s in rev.) ✓

$= 100°$

$\hat{R} = 180° - \hat{O}_2$ (co-int. ∠'s. OA // RP) ✓

$= 180° - 100°$

$= 80°$

no reasons at all −1

$\hat{P}_1 = \hat{R} = 80°$ (tan/chord theorem) ✓

$\therefore \hat{O}_2 + \hat{P}_1 = 100° + 80° = 180°$ ✓

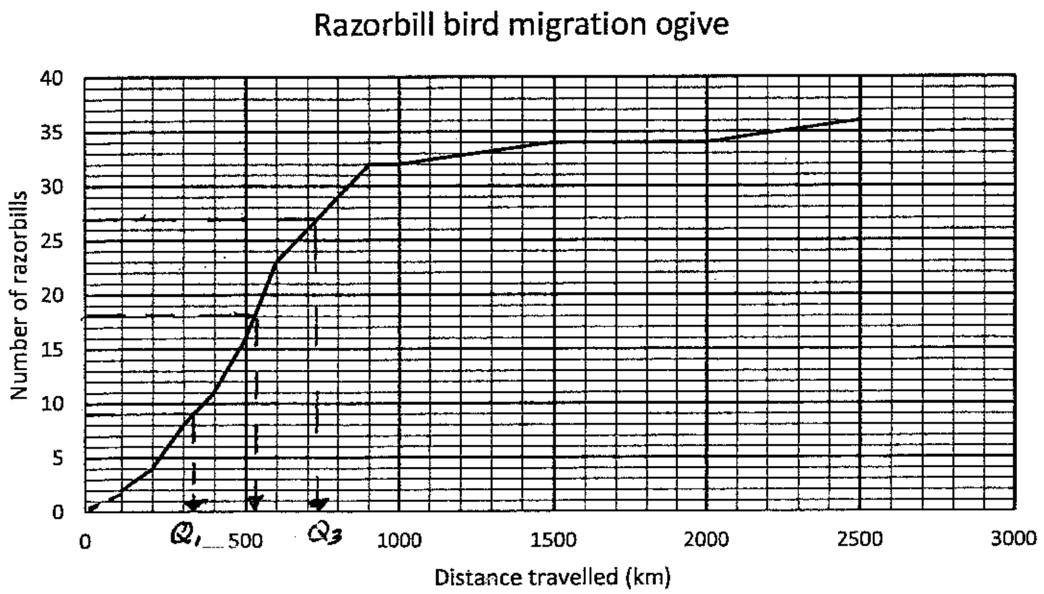
$\therefore$ OAPQ is cyclic (opp. ∠'s are supp) ✓

9.2 Use a simple construction to prove that PS = PQ. (6)

Join QS. ✓
$A\hat{S}Q = \frac{1}{2}\hat{O}_1 = \frac{1}{2}(260^\circ)$ (∠ at centre ⊙ = 2 ∠ on circum) ✓
$= 130^\circ$ ✓
$\therefore Q\hat{S}P = 180^\circ - 130^\circ = 50^\circ$ (adj. ∠'s on str. line) ✓
$\therefore S\hat{Q}P = 180^\circ - (50^\circ + \hat{P}_1)$ (∠ sum △QPS)
$= 180^\circ - (50^\circ + 80^\circ) = 50^\circ$ ✓
$\therefore Q\hat{S}P = S\hat{Q}P$ $(= 50^\circ)$ ✓
$\therefore PS = PQ$ (sides. opp. eq. ∠'s)

[11]

10. The data shown in the ogive (cumulative frequency graph) below, deals with a bird migration survey conducted with **36 razorbills.**



10.1 Use the graph to estimate the distance exceeded by 50% of these razorbills during their migration.

(2)

✓m on graph. ≈ 530 km ✓a

- ***The estimate for the mean distance travelled by razorbills was calculated to be 611,11 km:***

10.2 Is this data normally distributed or left skewed or right skewed? Give a reason. (2)

mean − median = 611,11 − 530 = 81,11 km > 0 ✓m
∴ (+ve skew) i.e. right skew ✓a

A similar survey is conducted with a different type of bird and :

- ***the median is 610,5 km***
- ***the IQR (interquartile range) is 100 km less than the IQR of the razorbill sample***
- ***the data is normally distributed.***

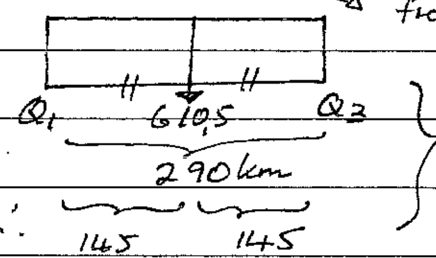
10.3 Calculate an estimate for the upper and the lower quartiles for this second data set. (5)

$IQR = IQR_{RB} - 100$ ✓m

$= (Q_3 - Q_1) - 100$

$\approx (720 - 330) - 100 = 290\text{ km.}$ ✓a from graph.

$\therefore$



✓m

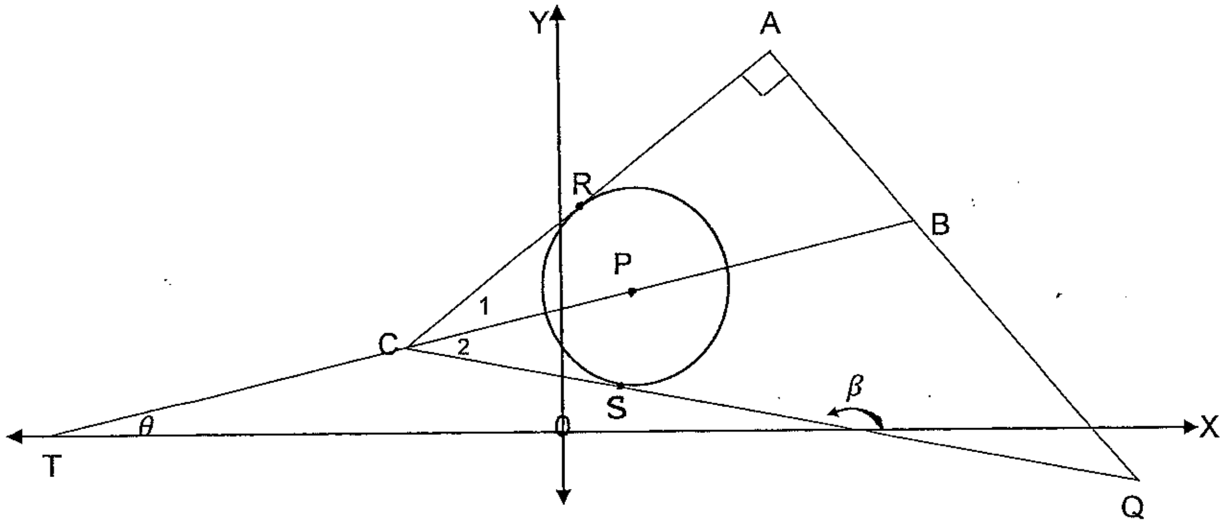
$\therefore Q_1 = \text{lower quartile} = 610,5 - 145 = 465,5\text{ km}$ ✓ca

$Q_3 = \text{upper quartile} = 610,5 + 145 = 755,5\text{ km}$ ✓ca

[9]

11. In the diagram below,

- CSQ and CRA are tangents to the circle, centre P, at S and R respectively.
- PC is produced on both sides to intersect the X axis at T, and AQ at B.
- $C\hat{A}Q = 90^0$



- The equation of the circle is $x^2 - 4x + y^2 - 6y + 8\frac{3}{4} = 0$:

11.1 Find the co-ordinates of P. (3)

complete sq.

$(x-2)^2 - 4 + (y-3)^2 - 9 + 8\frac{3}{4} = 0$

$\therefore (x-2)^2 + (y-3)^2 = 4\frac{1}{4} = \frac{17}{4}$

$\therefore P = (2; 3)$

11.2 What is the length of RP? (leave your answer in surd form if necessary) (1)

$RP = r = \frac{\sqrt{17}}{2}$

- Q = (12; $-1\frac{5}{8}$) and C = (- 4; a) and
- The equation of CQ is $8y + 2x = 11$.

11.3 Find the value of a. (2)

sub $x = -4$ $\therefore$ $8y - 8 = 11$

$8y = 19$

$y = \frac{19}{8} = 2\frac{3}{8}$

11.4 Find the gradient of BC. (2)

$m_{BC} = \frac{y_P - y_C}{x_P - x_C} = \frac{3 - 2\frac{3}{8}}{2+4} = \frac{5}{48}$

11.5 Prove that $\widehat{C_2} = 20^0$ to the nearest degree. (4)

$\hat{C}_2 = \overset{180^\circ -}{\cancel{1}}\beta + \theta$ (ext. ∠ Δ = sum int. opp. ∠'s)

$\tan\beta = \frac{-2}{8} = -\frac{1}{4}$ $\tan\theta = m_{BC} = \frac{5}{48}$

Key ∠ = $14,04^\circ$ $\therefore \theta = 5,95^\circ$

$\therefore \beta = 180^\circ - 14,04^\circ$ $\therefore \hat{C}_2 = \overset{180^\circ -}{\cancel{1}}165,96^\circ + 5,95^\circ$

$= 165,96^\circ$ $= 14,04^\circ + 5,95^\circ \approx 20^\circ$

11.6.1 Name two congruent triangles that make $\widehat{C_1} = \widehat{C_2}$. $\boxed{\Delta CRP \equiv \Delta CSP}$

(1)

11.6.2 Hence show that $\frac{AB}{BQ} = sin50^0$ (4)

ΔABC, $\hat{A} = 90^\circ$ $\therefore$ $\frac{AB}{BC} = \sin\hat{C}_1 = \sin 20^\circ$

$\therefore AB = BC.\sin 20^\circ$

P.T.O for more lines

ΔCBQ, $\frac{BQ}{\sin\hat{C}_2} = \frac{BC}{\sin Q}$ $\hat{Q} = 180^\circ - (90^\circ + 20^\circ + 20^\circ)$

$= 50^\circ$ ΔACQ

$\therefore$ $BQ = \frac{BC.\sin 20^\circ}{\sin 50^\circ}$ $\therefore \frac{AB}{BQ} = \frac{BC\sin 20^\circ}{\left(\frac{BC\sin 20^\circ}{\sin 50^\circ}\right)} = \sin 50^\circ$

[17]

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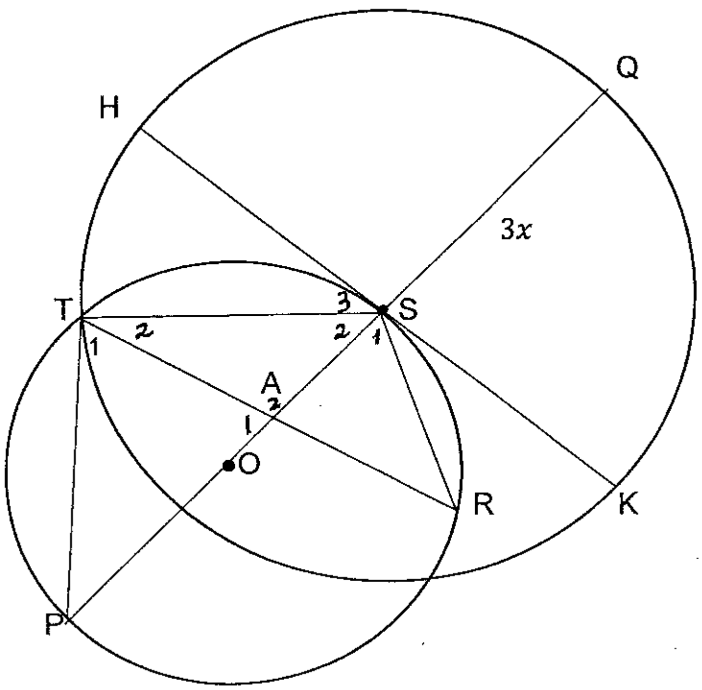
12.



In the diagram above,

* HSK is a tangent to circle PTSR, centre O, at S.
* S is the centre of the bigger circle.
* SQ = $3x$.

12.1 Prove that $\widehat{A_2} = \widehat{S_3} + \widehat{S_1}$. (4)

$\hat{P} = \hat{S}_3$ (tan/chord theorem)
$\hat{S}_1 = \hat{T}_1$ ($\angle$'s in same seg, subt. by same arc PR)
$\therefore \hat{A}_2 = \hat{T}_1 + \hat{P}$ (ext. $\angle$ ΔATP = sum int. opp. $\angle$'s)
$= \hat{S}_1 + \hat{S}_3$

12.2 If $r_1 : r_2 = 5 : 6$ where r_1 = the radius of the smaller circle and r_2 = the radius of the bigger circle, find the length of TP in terms of x, showing reasoning. (7)

$TS = SO = r_2 = 3x$ ✓a (radii)

$PS = 2r_1$ ✓m and $r_1 = \frac{5}{6} r_2 = \frac{5}{6} \cdot 3x$

$= 2{,}5x$ ✓a

$\therefore PS = 2.(2{,}5x) = 5x$ ✓ca

In ΔPTS, $P\hat{T}S = 90°$ ($\angle$ in semi-$\odot$) ✓

$\therefore TP^2 = PS^2 - TS^2$ (Pythagoras) ✓m

$= (5x)^2 - (3x)^2$

$= 16x^2$

$\therefore TP = 4x$ ✓ca

[11]

13. Part of a table decoration at a function involves folding a circular piece of filter paper, as shown below to line a cocktail glass in the shape of a cone.
The filter paper is folded such that OA meets up with OB where $A\hat{O}B = 60^0$, to form the three dimensional cone which lines the cocktail glass.
AP (or BP) is the diameter of the cocktail glass.

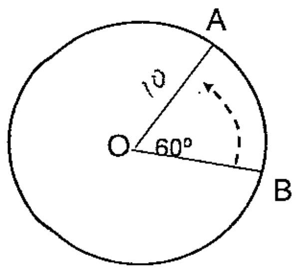


Figure 1. Filter paper

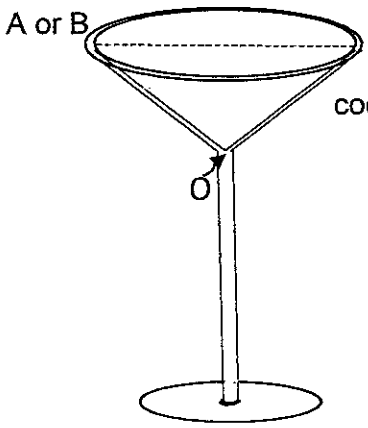


Figure 2. cocktail glass

13.1 If the circumference of filter paper in figure 1 = C_1 , and
the circumference of the top of the cocktail glass in figure 2 = C_2 ,
Show that $C_2 = \frac{5}{6} C_1$ (2)

m a

$C_2 = \frac{300^\circ}{360^\circ} C_1$ (reflex $\hat{O} = 360^\circ - 60^\circ$)
$= \frac{5}{6} C_1.$

Hence:

13.2.1 If the radius of the filter paper in figure 1 is 10 cm, show that the diameter of the top of the cocktail glass in figure 2, AP, is $16\frac{2}{3}$ cm. (2)

$C_2 = \frac{5}{6} C_1 = \frac{5}{6_3}(2\pi(10)) = \frac{50}{3}\pi.$
$\therefore d.\pi = \frac{50}{3}\pi$

$\therefore AP = \frac{\frac{50}{3}\pi}{\pi} = \frac{50}{3} = 16\frac{2}{3}$ cm

13.2.2 Calculate the size of $A\hat{O}P$ in degrees (to the nearest degree) (4)

ΔAOP, $o^2 = a^2 + p^2 - 2ap\cos A\hat{O}P$ ✓m

$\therefore AP^2 = 10^2 + 10^2 - 2(10)(10)\cos A\hat{O}P$ ✓m

$(16\frac{2}{3})^2 = 200 - 200\cos A\hat{O}P$

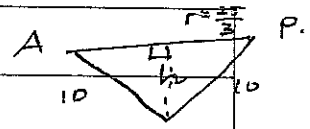
$\cos A\hat{O}P = \frac{200 - (16\frac{2}{3})^2}{200} = -\frac{7}{18}$ ✓ca

Key ∠ = 67,11°

$\therefore A\hat{O}P = 180° - 67,11° = 112,88° \approx 113°$ ✓ca.

13.3 Calculate the volume of the cocktail glass in cubic cm to 1 decimal place.
(*Volume of a cone* = $\frac{1}{3}\pi . r^2 . h$) (3)

$V = \frac{1}{3}\pi . \left(\frac{25}{3}\right)^2 . h.$



$h = \sqrt{10^2 - \left(\frac{25}{3}\right)^2}$ ✓m

$= 5,527....$ ($\approx 5,53$ ok)

$\therefore V = \frac{1}{3}\pi . \left(\frac{25}{3}\right)^2 . 5,527...$ ✓ca

$= 401,986...$ ca ✓

$\approx 402\ cm^3$ (1d.p) - might vary slightly - ok! [11]

SUB-TOTAL-SECTION B : [75]