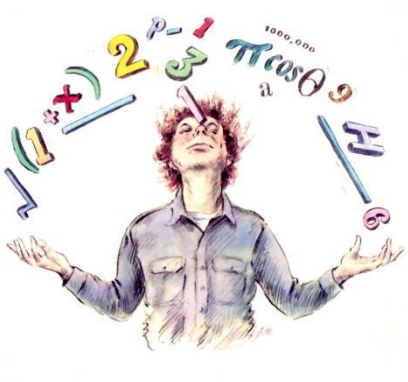
ST ANDREW'S COLLEGE

THE DIOCESAN SCHOOL FOR GIRLS

ST ANDREW'S PREPARATORY



Department of Mathematics

TIME: **3 Hours**	**DATE: 24 July 2017**	**Total marks:** **150**
Setter: **JH/CF**	**GRADE 12 PRELIM EXAMINATION** **MATHEMATICS: PAPER I**	**Moderator:** **DAS**

Name of student:

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 7 pages and an Information Sheet of 2 pages (i – ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions on the A4 paper provided. Number your answers clearly.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL
23	**21**	**14**	**17**	**10**	**21**	**12**	**19**	**13**	**150**

SECTION A

QUESTION 1

(a) Solve for x:

(1) $3x^2 = x + 5$ (4)

(2) $\sqrt{5-x} - x = 1$ (4)

(3) $\frac{x}{b} - b = \frac{x}{a} - a$ (3)

(4) $5x \geq 2(x^2 + 1)$ (4)

(b) Simplify:

(1) $\frac{27^{3x}.9^{2x}}{3^{13x-2}}$ (3)

(2) Hence solve for p if $\frac{27^{3x}.9^{2x}}{3^{13x-2}} = 5^p$ (2)

(c) If $\sqrt{2} = p$ and $\sqrt{3} = q$, express the following in terms of p and q: $\frac{\sqrt{8} - \sqrt{18}}{\sqrt{27} + \sqrt{3}}$ (3)

[23]

QUESTION 2

(a) The geometric series $18 \; ; \; x + 4 \; ; \; x$ is given, where $x < 3$.

(1) Determine the value of x. (4)

(2) Explain why the series is convergent. (1)

(3) Determine S_∞. (2)

(b) Determine $\sum_{k=-1}^{20} (3k + 2)$. (4)

(c) The sum of the first *n* terms of a series is given by the formula:

$$S_n = n^2 - 2n$$

Determine:

(1) the sum of the first 6 terms. (1)

(2) the first 3 terms of the sequence. (3)

(d) A quadratic pattern has $T_1 = T_3 = 0$ and $T_4 = -3$. Determine:

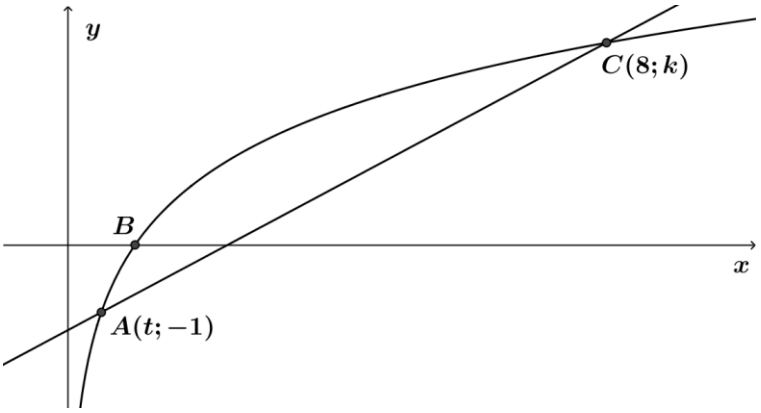
(1) the value of the 2nd difference of this pattern. (4)

(2) T_5. (2)

[21]

QUESTION 3

The graphs of $g(x) = mx + c$ and $h(x) = \log_2 x$ are drawn below. B is the x-intercept of h. $A(t;-1)$ and $C(8;k)$ are the points of intersection of graphs g and h.



(a) Write down the coordinates of B. (2)

(b) Calculate the value of k. (2)

(c) Calculate the value of t. (2)

(d) Write down the values of x for which $g(x) \geq h(x)$. (2)

(e) Write down the equation defining the inverse of h in the form $y = \ldots$ (2)

(f) Write down the range of h^{-1}. (2)

(g) Determine the value(s) of x for which $h(x) \leq 3$. (2)

[14]

QUESTION 4

(a) Determine $\lim\limits_{x\to 1}\frac{2x^2-7x+5}{4x^2-7x+3}$ (3)

(b) Determine $f'(x)$ **from first principles** if $f(x)=-2x^2+6x$. (5)

(c) Determine $\frac{dy}{dx}$ for each of the following:

(1) $y=\sqrt[3]{x}-\frac{x}{2}$ (3)

(2) $y=\frac{3x^4+7x^2-5x}{2x^2}$ (3)

(d) Given $f(x)=7+\frac{2}{x}-3x^3$, evaluate $f(1)-f'(1)$. (3)

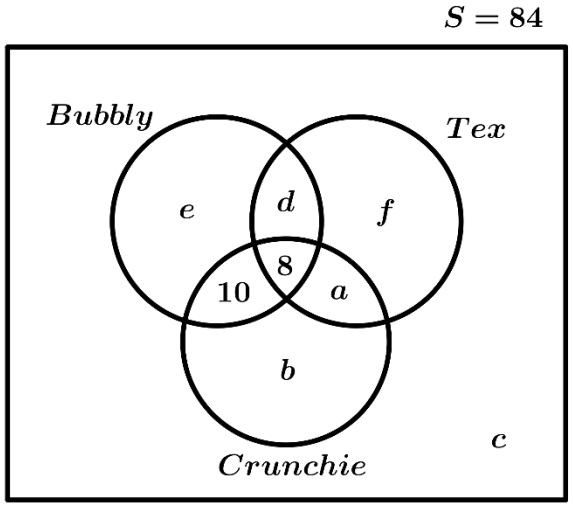
[17]

QUESTION 5

A survey regarding pupils' favourite chocolates was conducted among 84 high school pupils. Three chocolates, namely Bubbly (B), Crunchie (C) and Tex (T), were used in the survey. The results are as follows:

- 41 pupils favour Bubbly.
- 34 pupils favour Tex.
- 40 pupils favour Crunchie.
- 18 pupils favour both Bubbly and Crunchie.
- 8 pupils favour all three chocolates.
- 75 pupils favour at least one chocolate.
- n (T and C) = 17.

The Venn diagram alongside shows the above information:



(a) Determine the values of a, b, c, d, e and f. (8)

(b) Determine the probability that a randomly selected pupil favours at least two of the three chocolates. (2)

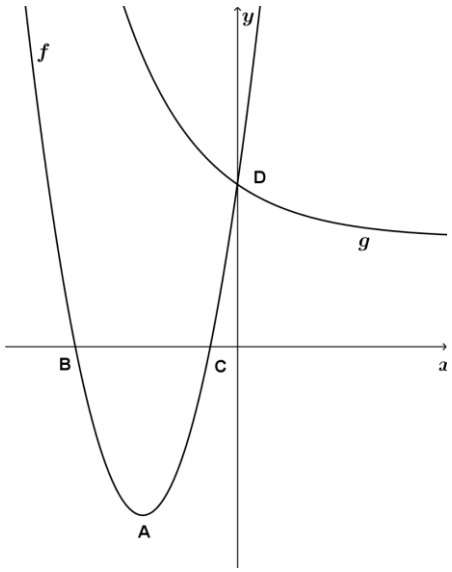
[10]

SECTION B

QUESTION 6

(a) In the diagram below:

- $f(x) = 2x^2 + 7x + 3$
- $g(x) = 2^{-x} + 2$
- Point A is the turning point of f
- Points B and C are the x-intercepts of f
- Point D is the y-intercept of f and g



(1) Determine the coordinates of points A, B, C and D (6)

(2) Determine the values of x for which $f(x).g(x) \leq 0$. (2)

(3) Determine the equation of :

(i) $h(x)$, if $h(x)$ is obtained by translating $f(x)$ so that point B lies on point D. (4)

(ii) $p(x)$, if $p(x)$ is the reflection of $g(x)$ across the line $y = x$.
Give your answer in the form $p(x) = ...$ (3)

(b) Given: $f(x) = \dfrac{-2x-6}{x+1}$

(1) Draw a sketch graph of f. Show all asymptotes as well as any intercepts with the axes. (4)

(2) Write down the equation of $k(x)$ if k is the reflection of f about the x-axis. (2)

[21]

QUESTION 7

(a) P(A) = $\frac{1}{4}$ and P(A or B) = $\frac{1}{3}$. Determine P(B) if:

(1) A and B are mutually exclusive events. (3)

(2) A and B are independent events. (4)

(b) Consider the word: TARADIDDLE

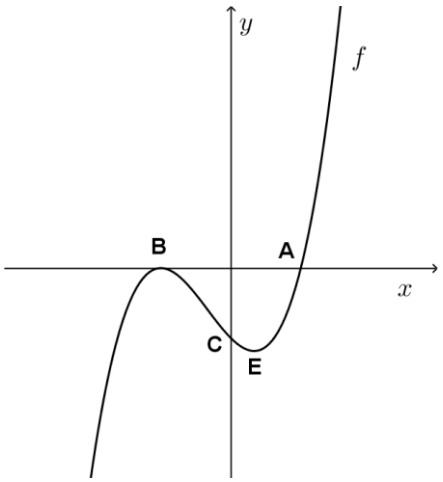
(1) How many different 'words' are possible if all the letters are used? (2)

(2) What is the probability that a 'word' will start with an I and end with a D? (3)

[12]

QUESTION 8

The figure below shows the graph of: $f(x) = (x-1)(x+1)^2 = x^3 + x^2 - x - 1$.
B is a turning point of f and A and B are points of intersection of f with the *x*-axis.



(a) Write down the length of AB. (2)

(b) Determine the coordinates of E, a turning point of f. (4)

(c) Calculate the values of *x* for which the tangent to f is parallel to $g(x) = \frac{5}{3}x + 2$. (4)

(d) Determine the equation of the tangent to f at the point where $x = 3$. (4)

(e) For which value(s) of *k* will $f(x) = k$ have one real root? (2)

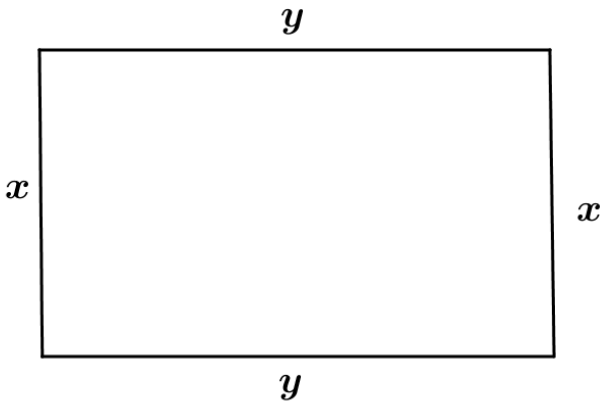
(f) For which value(s) of x will $\frac{f'(x)}{f(x)} \geq 0$? (3)

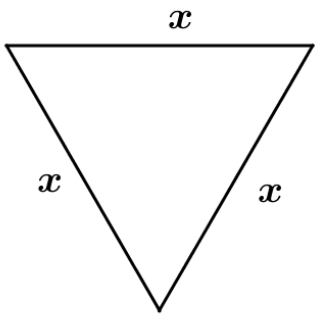
[19]

QUESTION 9

(a) A cubic function $y = x^3 + bx^2 + cx + 22$ has a local minimum at the point (3;-5).
Determine the values of b and c. (5)

(b) A length of wire 200 meters long is cut into two pieces. One piece is used to form a rectangle with sides x and y. The other piece is used to form an equilateral triangle with sides equal to x.





Determine the maximum area of the rectangle. (5)

(c) The profit made by a company is dependent on how many items are sold in a week.
The profit (P) in rands per hour is calculated using the formula:

$$P = x\left(4900 - \frac{x^2}{3}\right)$$

where x is the number of articles sold in a week.

Determine how many articles need to be sold in a week in order to maximise the profit. (3)

[13]

TOTAL: 150