PRELIM PAPER I – 2017

MATHS

QUESTION 1

a. $\left(\frac{3^{-3}\cdot x^{-3}}{27\cdot x^{-3}}\right)^{-2/3}$

$= (3^{-6})^{-2/3}$ (3)

$= 3^4$

b. 1. $2x^2 - x = 0$

$x = 0$ or $x = \frac{1}{2}$ (2)

2. $3x^2 + 5x - 2 \geq 0$ (4)

CV: $x = \frac{1}{3}$ $x = -2$

$x \leq -2$ or $x \geq \frac{1}{3}$

3. $20 - 2x = (6-x)^2$

$20 - 2x = 36 - 12x + x^2$

$x^2 - 10x + 16 = 0$

$x \neq 8$ or $x = 2$ (5)

4. $7^x - 8 + \frac{7}{7^x} = 0$

$7^{2x} - 8\cdot 7^x + 7 = 0$

$(7^x - 7)(7^x - 1) = 0$

$7^x = 7$ or $7^x = 1$ (6)

$x = 1$ $x = 0$

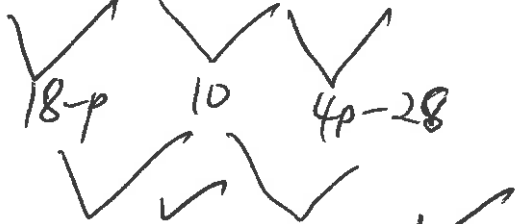
QUESTION 2:

91. $26^2 - 25^2 + 24^2 - 23^2 = 98$ (2)

2. $T_n = (n+3)^2 - (n+2)^2 + (n+1)^2 - n^2$

$T_n = 4n + 6$ (6)

b. P: 18 : 28 4p



(6)

$-8 + p = 4p - 38$

$30 = 3p$

$p = 10$

QUESTION 3:

a) $\lim_{x\to 0} \frac{f(x+h) - f(x)}{h}$

$= \frac{f(h) - f(0)}{h}$ (2)

b. 1. F
2. T
3. F
4. F

c) $f(x+h) = -(x+h)^2 + \beta(x+h)$

$= -x^2 - 2xh - h^2 + \beta x + \beta h$ ✓

$f'(x) = \lim_{h\to 0} \frac{-x^2 - 2xh - h^2 + \beta x + \beta h + x^2 - \beta x}{h}$ ✓

$= \lim_{h\to 0} \frac{h(-2x - h + \beta)}{h}$ ✓

$= -2x + \beta$ ✓ (5)

d. $y = x^{2/n} - \frac{3}{2}x^{-1}$

$\frac{dy}{dx} = \frac{2}{n}x^{\frac{2}{n}-1} + \frac{3}{2}x^{-2}$ ✓ (4)

$= \frac{2}{n}x^{\frac{2}{n}-1} + \frac{3}{2x^2}$ ✓

e. $f'(x) = 3x^2 + 2ax + q$ ✓

$f''(x) = 6x + 2a > 0$ ✓✓ (4)

$18 + 2a > 0$

$2a > -18$

$a > -9$ ✓

QUESTION 4:

a. $q = 2$ ✓

$p = -1$ ✓

$-4 = \frac{a}{3/2 - 1} - 2$ ✓

$a = -1$ ✓ (4)

b. $D: x \in \mathbb{R} : x \neq 1$ ✓

$R: y \in \mathbb{R} : y \neq -2$ ✓ (2)

c. $m(x) = \frac{1}{x+1} - 2$ ✓✓✓ (3)

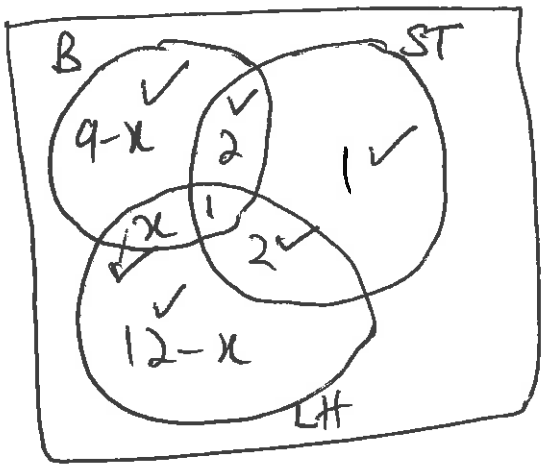
QUESTION 5:

a) $0,8 = 0,6 + 0,3 - P(A \cap B)$ ✓ ✓

$P(A \cap B) = 0,1 \neq 0$ ✓ (2)

∴ not mutually exclusive.

b)



$9 - x + 2 + 1 + x + 2 + 1 + 12 - x = 24$ ✓

$27 - x = 24$ (7)

$x = 3$ ✓

QUESTION 6:

a) $\frac{20!}{8!6!4!2!} = 1745944200$ ✓ ✓ (4)

b) R . . . R (4)

$\frac{18!}{6!6!4!2!} = \frac{14}{45}$ (0,15) ✓ ✓

1745944200

QUESTION 7:

a.1. $BO = 350000\left(1 + \frac{9\%}{2}\right)^6 - 21000\left(\frac{\left(1 + \frac{9\%}{2}\right)^6 - 1}{\frac{9\%}{2}}\right)$ ✓ ✓ (4)

$= R314\,736,32$ ✓

2. $Interest = 21000 \times 6 - (350000 - BO)$ ✓ ✓

$= R90\,736,82$ ✓ (3)

b. $320000\left(1 + \frac{9,5\%}{12}\right)^2$ ✓ =

$35000\left[\frac{1 - \left(1 + \frac{9,5\%}{12}\right)^{-n}}{\frac{9,5\%}{12}}\right]$ ✓ ✓

$n = 168,56\ldots$

$14,05$ ✓ years

QUESTION 8:

a1. $k+(n-1)k = 200$

$n = \frac{200}{k}$ ✓✓ (2)

2. $S_{\frac{200}{k}} = \frac{200}{2k}(k+200)$ ✓ (3)

$= \frac{100}{k}(k+200)$ ✓

b. $a = 5 \quad r = 4/5$

1. $T_{20} = 5\left(\frac{4}{5}\right)^{20-1}$ ✓✓ (2)

$= 0,07.$

2. $S_\infty = \frac{5}{1-4/5} = 25$ ✓✓ (2)

3. $S_k = \frac{5\left(\left(\frac{4}{5}\right)^k - 1\right)}{\left(\frac{4}{5}\right)-1}$ ✓ $> 24,95$

$-25\left(\left(\frac{4}{5}\right)^k - 1\right)$ ✓ $> 24,95$

$\left(\frac{4}{5}\right)^k$ ✓ $< -0,998+1$ (4)

$\left(\frac{4}{5}\right)^k < 0,002$

$k \log\left(\frac{4}{5}\right) < \log(0,002)$

$k > \frac{\log(0,002)}{\log(0,8)}$ ✓

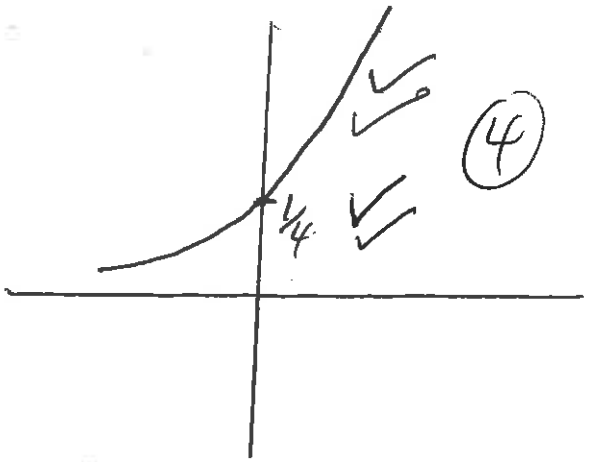
4. $k = 28$

QUESTION 9:

a1. $b = 2$ ✓✓ (2)

2. $f^{-1}(x) = 2^x$

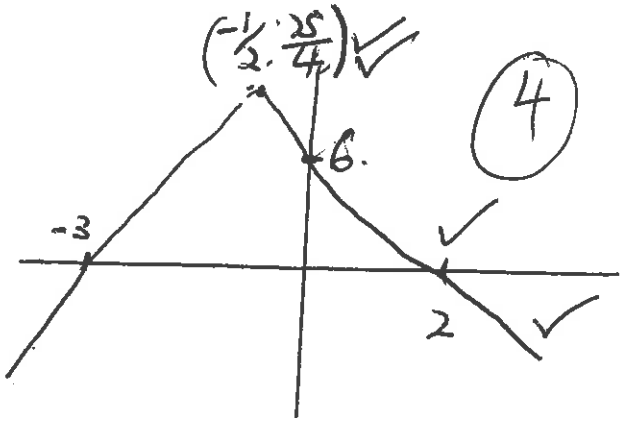
$f^{-1}(x-2) = 2^{x-2}$



(4)

3. $x > 0$. ✓ (2)

b. 1.



(4)

2. Turning points

$(-1; 21/4)$ ✓✓ (2)

QUESTION 10:

a) $d = 0$ ✓

$y = -(x)(x-6)^2$

$= -x^3 + 12x^2 - 36x$ (4)

$b = 12$ ✓ $c = 36$ ✓

b. $f'(x) = -3x^2 + 24x - 36 = 0$ ✓

$x = 2 \quad x = 6$ (3)

$\therefore A(2; -32)$ ✓

c. $T(3; -27)$ ✓

$f'(3) = 9$ ✓

$-27 = 9(3) + c$ ✓ (5)

$y = 9x - 54$ ✓

QUESTION 11:

a) 1. $P'(t) = 45 - 1,2t^2 = 0$ ✓

$t^2 = 37,5$ (4)

$t = \frac{5\sqrt{6}}{2}$ $(6,12)$ ✓

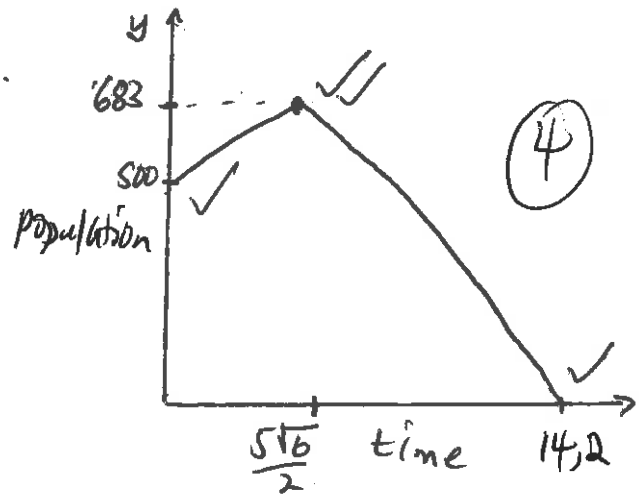
$P\left(\frac{5\sqrt{6}}{2}\right) = 683,71$ ✓

$\therefore$ max population $= 683$

2. $45t - 0,4t^3 + 500 = 0$ ✓

$t = 14,2$ ✓ (3)

3.



(4)

b) $V = k\pi r^2 - kr^3$

$V'(r) = 2k\pi r - 3kr^2 = 0$ ✓

$r \neq 0$ ✓ $2\pi = 3r$ (5)

$r = \frac{2\pi}{3}$ ✓