NAME: MARKING GUIDELINE

SAHETI SCHOOL

PRELIMINARY EXAMINATION 2017

MATHEMATICS GRADE 12

PAPER 2

Time: 3 hours

Total: 150

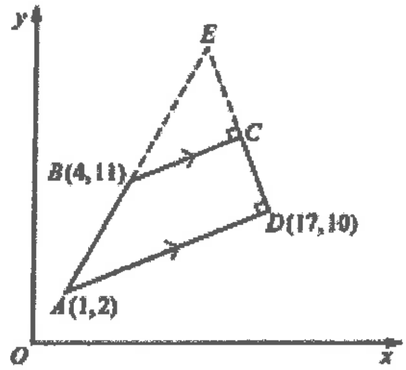
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 21 pages, graph paper, and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1:

The diagram, which is not drawn to scale, shows a trapezium *ABCD* in which *BC* is parallel to *AD*. The side $AD \perp DC$. Point $A(1;2)$, $B(4;11)$ and $D(17;10)$.



Calculate:

a) The gradient of line BC (2)

$$= \frac{10-2}{17-1} = \frac{1}{2}$$

b) The equation of line EC (3)

$m_{EC} = -2$

$10 = -2(17) + c$

$y = -2x + 44$

c) The coordinates of C (4)

$BC: \quad 11 = \frac{1}{2}(4) + c$

$y = \frac{1}{2}x + 9$

$-2x + 44 = \frac{1}{2}x + 9$

$x = 14 \quad y = 16 \quad C(14;16)$

d) The angle $E\hat{A}D$ (4)

$= \tan^{-1}\left(\frac{11-2}{4-1}\right) - \tan^{-1}\left(\frac{1}{2}\right)$

$= \underline{\underline{45}}$

The lines AB and DC are extended to meet at E. Find

e) The coordinates of E. (4)

AE: $2 = 3(1) + c$

$y = 3x - 1$

$3x - 1 = -2x + 44$

$x = 9$

$y = 26$ $\quad \underline{\underline{E(9;26)}}$

f) The ratio of the area of triangle EBC to the area of trapezium ABCD. (4)

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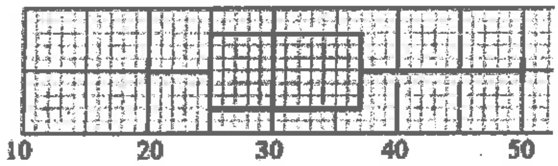
$\frac{EB \cdot EC}{EA \cdot ED - EB \cdot EC}$ OR $\frac{\frac{1}{2} \cdot 5\sqrt{5} \cdot 5\sqrt{5}}{\frac{1}{2} \cdot 8\sqrt{5} \cdot 8\sqrt{5} - \frac{1}{2} 5\sqrt{5} \cdot 5\sqrt{5}}$

$= \frac{5\sqrt{10} \cdot 5\sqrt{5}}{8\sqrt{10} \cdot 8\sqrt{5} - 5\sqrt{10} \cdot 5\sqrt{5}}$

$= \underline{\underline{\frac{25}{39}}}$ $\qquad = \underline{\underline{\frac{25}{39}}}$

QUESTION 2:

a) Children from school A took part in a fun run for charity. The times, to the nearest minute, taken by the children from school A are summarised in the diagram below.



1) Write down the time by which 75% of the children in school A had completed the run. (1)

37 ✓

2) State the name given to this value. (1)

Upper quartile ✓

3) Briefly comment on the box and whisker plot. (2)

- positively skewed ✓
- any other meaningful related explanation ✓

b) Sunita and Shelley talk to each other once a week on the telephone. Over many weeks they recorded, to the nearest minute, the number of minutes spent in conversation on each occasion. The following table summarises their results.

Time (to the nearest minute)	Number of conversations
5–9	2
10–14	9
15-19	20
20-24	13
25-29	8
30-34	3

1) Two of the conversations were chosen at random. Find the probability that both of them were longer than 24.5 minutes. (2)

$11/55 = 1/5$ 2

2) Calculate an estimate of the mean time spent on their conversations. (2)

19,3 2

During the following 25 weeks, they monitored their weekly conversation and found that at the end of the 80 weeks their overall mean length of conversation was 21 minutes.

3) Find the mean time spent in conversation during these 25 weeks (4)

$$21 = \frac{55 \times 19{,}3 + 25 \times \bar{x}}{80}$$

$$1680 = 1060 + 25\bar{x}$$ 4

$$\bar{x} = 24{,}8$$

4) Comment on these two mean values. (2)

- Spent more time on the phone during the last 25 weeks as the mean is higher than that of 55 weeks.

2

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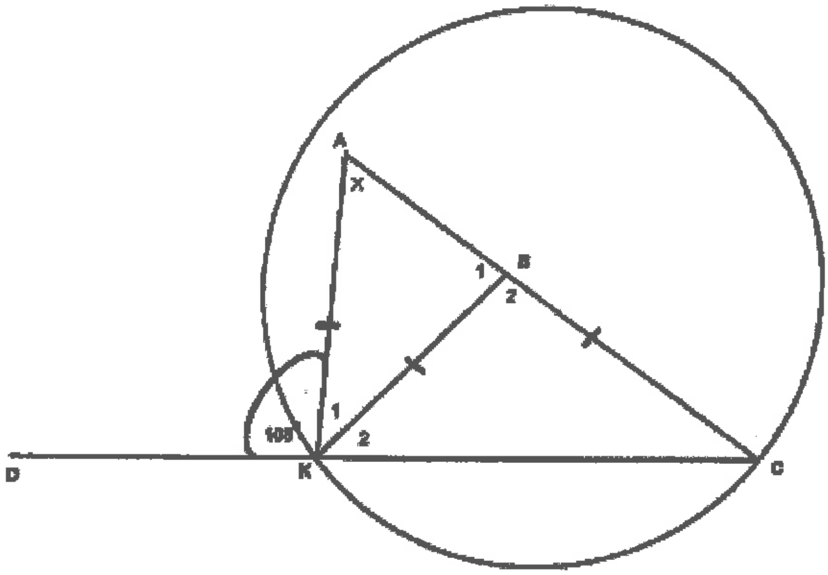
QUESTION 3:

B is the centre of the circle. $AK = KB = BC$. $A\hat{K}D = 108^o$. $\hat{A} = x$



Give reasons for your answers:

a) Express $\hat{B}_1$ in terms of x. (2)

x (∠s opp = sides)

b) Show that $\hat{C} = \frac{x}{2}$ (3)

$B_2 = 180 - x$ (∠s on a st line)

$\therefore \hat{C} = \frac{180-(180-x)}{2}$ (∠s opp = sides)

$= x/2.$

c) Solve for x. (3)

$x + \frac{x}{2} = 108$ (Sum of opp int ∠s = ext ∠ of a Δ)

$x = 72^\circ$

d) Hence, calculate the value of $\hat{R}_1$ (2)

$\hat{R}_1 = 180 - 2(72)$ (Int ∠s of △ = 180)

$= 36$

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QUESTION 4:

a) If $f(x) = -cos(40^o - 2x)$ and $g(x) = tan(-x)$.

1) Write down the period of $g(x)$ (1)

$180°$

1

2) Write down the amplitude of $f(x)$ (1)

1

1

3) Write down the period of $f\left(\frac{2x}{3}\right)$ (2)

270

2

4) Write down the equation of $h(x)$ if $h(x) = -f(x - 10)$ (3)

$= \cos(40 - 2(x - 10))$

$= \cos(60 - 2x)$

3

b) If $\cos 22^o = m$, express the following in terms of m.

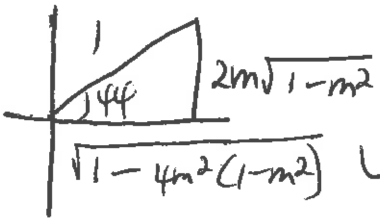
1) $\sin 22^o$ $= \sqrt{1 - m^2}$ (2)

2

2) $sin44^o$ $= 2\sin 22 \cos 22$ (3)

$= 2.\sqrt{1-m^2} . m$

3) $tan44^o$ (4)



$$= \frac{2m\sqrt{1-m^2}}{\sqrt{1-4m^2(1-m^2)}}$$

4

$$\left(\frac{2m\sqrt{1-m^2}}{2m^2-1}\right)$$

c) Simplify as far as possible without the use of a calculator.

1) $\frac{tan200^o.sin300^o.cos110^o}{cos135^o.sin225^o.sin160^o}$ (6)

$$\frac{\tan(180+20)\cdot\sin(360-60).\cos(180-70)}{\cos(180-45)\cdot\sin(180+45)\cdot\sin(180-20)}$$

$$= \frac{\tan 20.(-\sin 60)\cdot(-\cos 70)}{(-\cos 45)\cdot(-\sin 45)\cdot(\sin 20)}$$

6

$$= \frac{\tan 20 \cdot \sqrt{3}/2}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}$$

$$= \sqrt{3}\cdot\tan 20$$

2) $\frac{sin(180-x).tan(180+x).cos(-x).sin(90-x)}{cos(x-360).cos(90+x).tan(-x)}$ (6)

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$= \frac{\sin x \cdot \tan x \cdot \cos x \cos x}{\cos x \cdot (-\sin x)(-\tan x)}$

$= \underline{\underline{\cos x}}$

SECTION B

QUESTION 5:

a) Three airport management trainees, Ryan, Sunil and Tim, were each instructed to select a random sample of 12 suitcases from those waiting to be loaded onto aircraft.

Each trainee also had to measure the volume, *x*, and the weight, *y*, of each of the 12 suitcases in his sample, and then calculate the value of the correlation coefficient, *r*, between *x* and *y*.

- Ryan obtained a value of -0.843
- Sunil obtained a value of 0.007

Explain why neither of these two values is likely to be correct (2)

• Ryan value indicates that as volume increases then weight decreases.
• Sunil value indicates no relationship ✓ ✓ (2)
⇒ Would expect weight to increase with volume.

b) Joe, a manager, recorded the volumes, *v*, and the weights, *w*, of a random sample of 8 suitcases as follows.

v	28.1	19.7	46.4	23.6	31.1	17.5	35.8	13.8
w	14.9	12.1	21.1	18.0	19.8	19.2	16.2	14.7

1) Calculate the correlation coefficient *r*, between *v* and *w* for this data to 4 decimal places. (2)

$r = 0{,}5419$ ✓✓ (2)

2) Interpret your value in the context of this question. (2)

moderate positive linear relationship ✓ between volumes and weights of suitcases. ✓ (2)

3) Determine the equation of the least squares regression line of *w* on *v* to 4 decimal places. (3)

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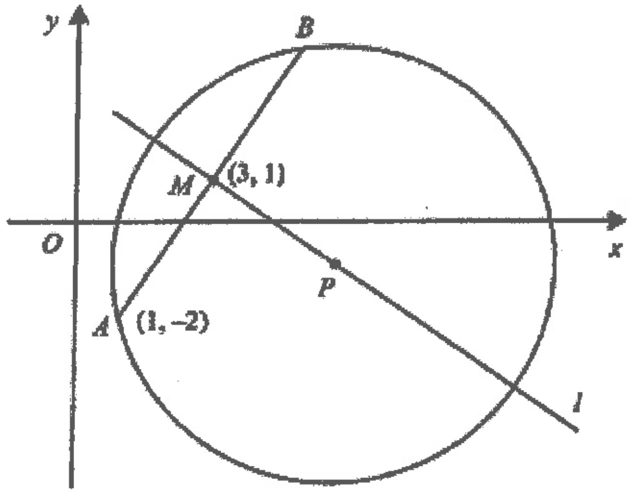
$w = 12{,}8379 + 0{,}1542v$

QUESTION 6:

a) The points $A(1;\,-2)$ and *B* lie on a circle with centre *P*, as shown in the diagram below. The midpoint *M* of *AB* has coordinates$(3; 1)$. The line l passes through the points *M* and *P*.



1) Find an equation for *MP* (3)

$m_{AM} = \frac{3}{2} \therefore m_{MP} = -\frac{2}{3}$

$1 = -\frac{2}{3}(3) + c$

$y = -\frac{2}{3}x + 3$

(3)

2) Given that the *x*-coordinate of *P* is 6, calculate the *y*-coordinate. (1)

$y = -\frac{2}{3}(6) + 3$

$= -1$

(1)

3) Find an equation for the circle. (4)

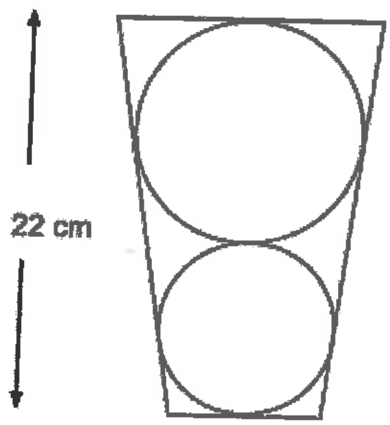
$(x-6)^2+(y+1)^2=r^2$

$(1-6)^2+(-2+1)^2=26$

$\therefore (x-6)^2+(y+1)^2=26$

(4)

b) A new company sign has a logo in the shape of two circles, one sitting on top of the other as shown and 22*cm* high.



1) The equation of the smaller circle is $x^2+y^2-12x-26y+189=0$ and the line of centres is parallel to the y-axis. Find the equation of the larger circle. (6)

$(x-6)^2+(y-13)^2=16$

$C(6;13)$ $r=4$.

$\therefore$ Diameter of larger circle $=22-8=14$

$\therefore r=7$.

$\therefore$ centre $(6;24)$

$\therefore (x-6)^2+(y-24)^2=49$

(6)

2) Calculate the equation of the common tangent to the two circles parallel to x-axis. (2)

$\underline{\underline{y=17}}$ ✓✓ 2

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QUESTION 7:

a) Given that $\frac{\sin x}{1+\cos x}+\frac{1}{\tan x}=\frac{1}{\tan x.\cos x}$

1) Prove the above identity (5)

RHS $= \frac{1}{\sin x}$ ✓

LHS $= \frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x}$ ✓

$= \frac{\sin^2 x + \cos x + \cos^2 x}{\sin x(1+\cos x)}$ ✓✓

$= \frac{1+\cos x}{\sin x(1+\cos x)}$ ✓ $= \frac{1}{\sin x} =$ RHS

2) For which value(s) of x, in the interval $0^o \leq x \leq 180^o$, is the identity undefined (2)

$x = 90 + 180k \quad k \in \mathbb{Z}$.

$x = 90; 180$ ✓✓ $; 0^\circ$

b) Solve for x if $sin2x - cos2x = -1$ (6)

$\sin 2x = \cos 2x - 1$

$2\sin x \cos x = -2\sin^2 x$ ✓

$2\sin x(\cos x + \sin x) = 0$ ✓ ✓

$2\sin x = 0$ ✓

$\underline{x = 180k}$

$\sin x = -\cos x$

$\tan x = -1$ ✓

✓ $\underline{x = -45 + 180k \quad k \in \mathbb{Z}}$

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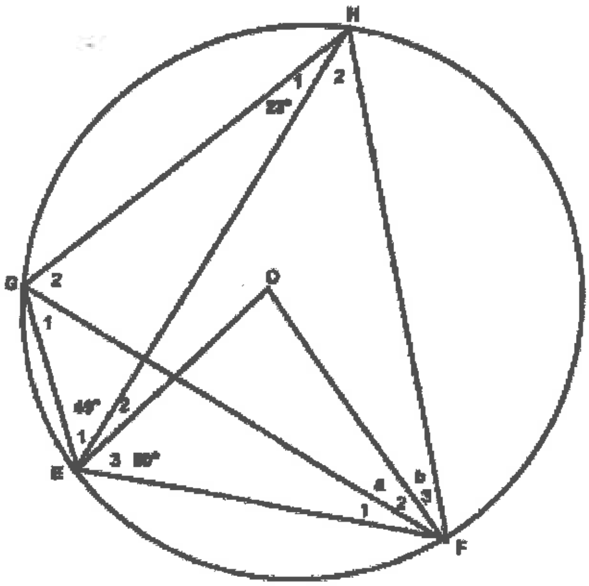
QUESTION 8:

a) In the diagram below, E, F, H and G are points on the circle with centre O.



Calculate the sizes of *a* and *b*, give reasons. (5)

$a + b = 49°$ ✓ (∠s in same segment)

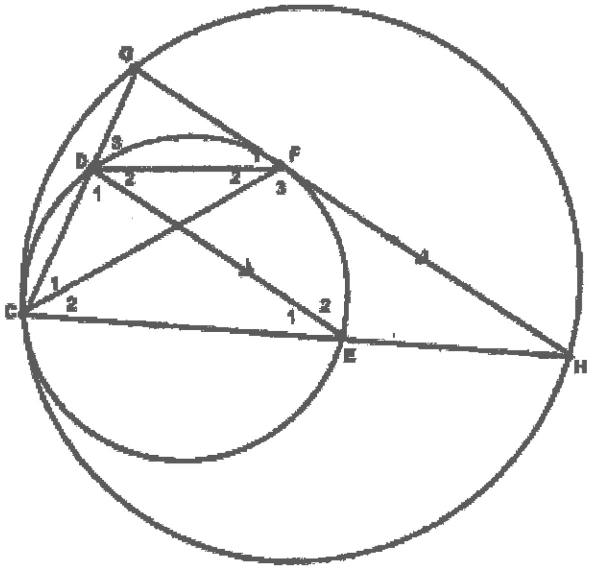
$F_1 = 23°$ ✓ (∠s in same segment)

✓ $\therefore a = 50 - 23°$ (∠s opp = sides)

$= \underline{\underline{27°}}$ ✓

$\therefore b = 49 - 27$
$= \underline{\underline{22^\circ}}$

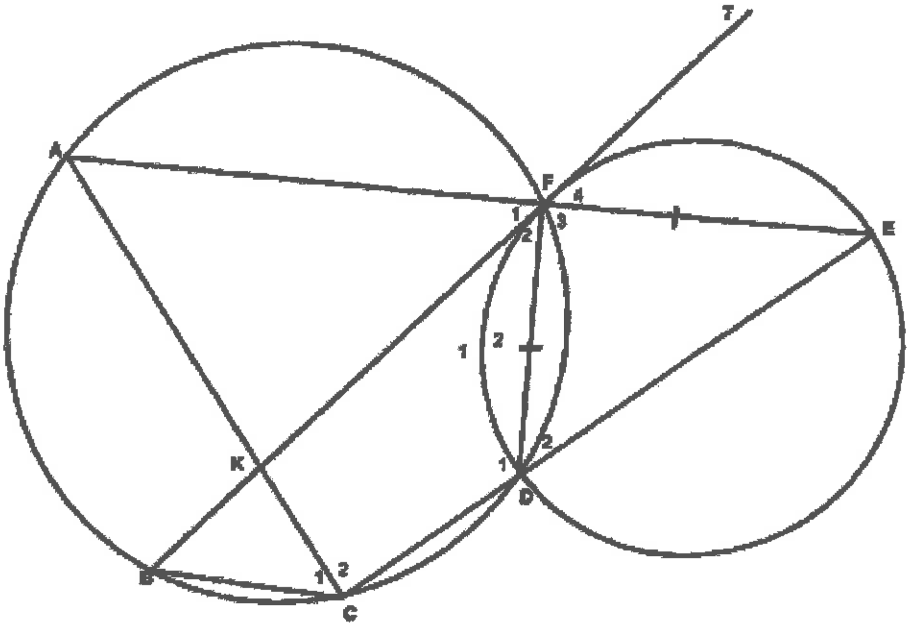
b) In the diagram below, $GH \parallel DE$ and GH is a tangent to the smaller circle.



Prove with reasons that, $\hat{C}_1 = \hat{C}_2$ (5)

Construct chord FE:
$\hat{D}_2 = \hat{C}_2$ (∠s in same segment)
$\hat{F}_1 = \hat{C}_1$ (tan chord thm)
$\hat{F}_1 = \hat{D}_2$ (Alt ∠s $GH \parallel DE$)
$\therefore \hat{C}_1 = \hat{D}_2$
$\therefore \hat{C}_1 = \hat{C}_2$

c) The two circles shown intersect at points F and D. BFT is a tangent to the smaller circle at F. Straight line AFE is drawn such that $DF = EF$. CDE is a straight line and chord AC and BF cut at K.



Prove, with reasons:

1) $BT \parallel CE$. (4)

$\hat{E} = \hat{D}_2$ (∠s opp = sides)
$\hat{F}_4 = \hat{F}_1$ (VOA)
and $\hat{F}_4 = \hat{D}_2$ (tan chord thm)
$\therefore \hat{F}_1 = \hat{D}_2$
$\therefore BT \parallel CE$ (corr ∠s)

2) $BCEF$ is a parallelogram. (4)

$BT \parallel CE$ (proven)
$\therefore \hat{F}_1 = \hat{C}_1$ (∠s same segment)
$\therefore AE \parallel BC$ (Alt ∠s
$\therefore BCEF$ is parallelogram (both opp sides ∥)

3) $AC = BF$ (4)

$\hat{F}_1 = \hat{C}_1$ (proven)

$\hat{F}_1 = \hat{E}$ (proven)

$\hat{A} = \hat{C}_1$ (Alt ∠s $AE \parallel BC$)

$\therefore \hat{A} = \hat{E}$

$\therefore AC = CE$ (∠s opp = sides)

$BF = CE$ (∠s opp sides parm =)

$\therefore AC = BF$

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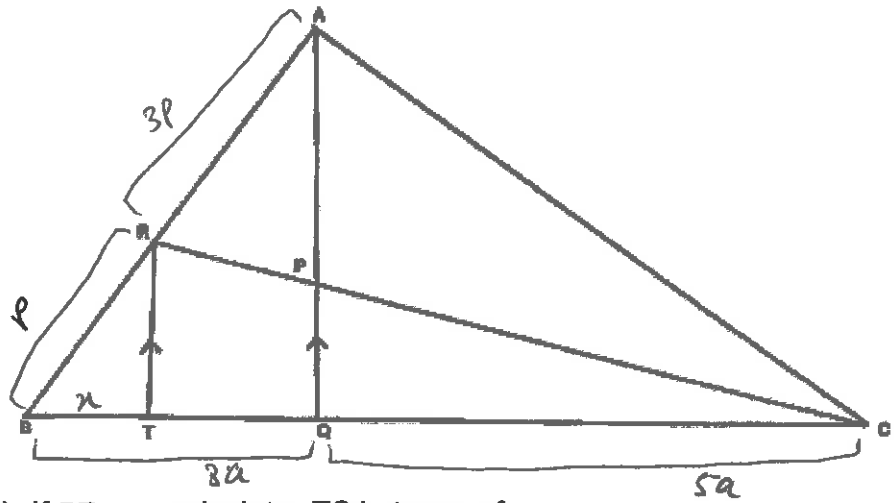
QUESTION 9:

If $AQ \parallel RT$, , $\frac{BQ}{QC} = \frac{3}{5}$ and , $\frac{BR}{RA} = \frac{1}{3}$



a) If $BT = x$, calculate, TQ in terms of x (2)

$TQ = 3x$ (lines ∥ to one side of the Δ)

b) Calculate the numerical value of $\frac{CP}{PR}$ (3)

$x+3x = 3a$

$4x = 3a$

$x = \frac{3a}{4}$ ✓

$\therefore QT = \frac{9a}{4}$ ✓

$\therefore \frac{CP}{PR} = \frac{5}{9/4}$ (lines ∥ to one side of △)

$= 20/9$ ✓

c) Calculate the numerical value of $\frac{Area\ of\ \triangle RCT}{Area\ of\ \triangle BRT}$ (4)

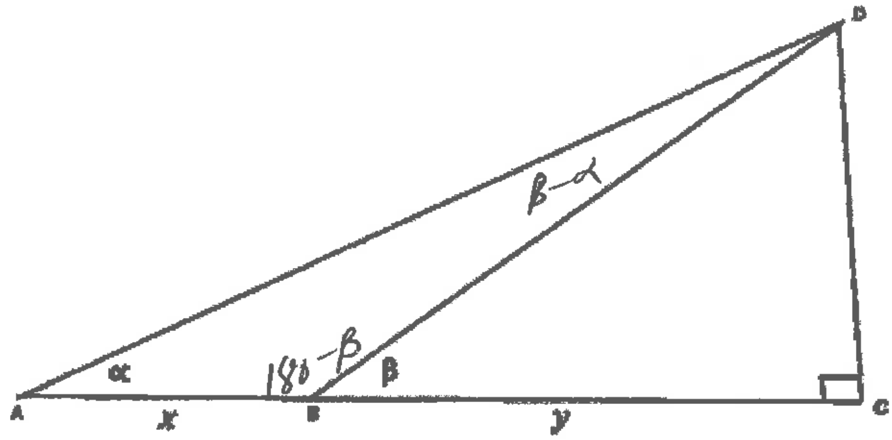
$= \frac{\frac{29a}{4}}{3a/4}$ ✓✓

$= \frac{29}{3}$ ✓✓

[9]

QUESTION 10

In the diagram below $\hat{A} = \alpha$, $C\hat{B}D = \beta$, $\hat{C} = 90^o$ $AB = x$ and $BC = y$.



a) Prove that: $y = \frac{x.sin\alpha.cos\beta}{sin(\beta - \alpha)}$ (5)

$\cos\beta = \frac{y}{BD}$

$BD = \frac{y}{\cos\beta}$ ✓✓

$\frac{x}{\sin(\beta-\alpha)} = \frac{BD}{\sin\alpha}$ ✓✓

$\frac{x\sin.\alpha}{\sin(\beta-\alpha)} = \frac{y}{\cos\beta}$ ✓

$\frac{x\sin\alpha\cos\beta}{\sin(\beta-\alpha)} = y$

b) If $\alpha = 42^o$, $\beta = 68^o$ and $y = 4$. Calculate AC (3)

$4 = \frac{x\sin 42 . \cos 68}{\sin(68-42)}$ ✓

$6,995 = x$

$7 = x$ ✓

$\therefore AC = 11$ ✓

[8]