

EXAMINATION NUMBER:

Teacher:

St. Anne's Diocesan College



Form 6

Core Mathematics: Paper II

September 2017

Time: 3 hours

Marks: 150

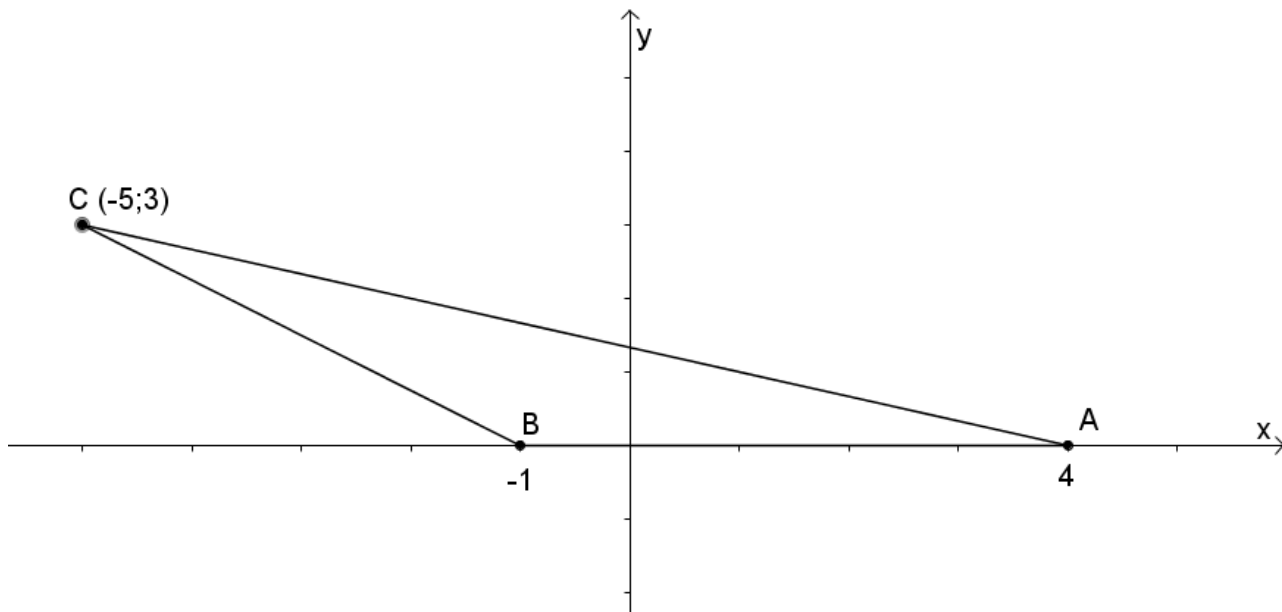
Please read the following instructions carefully:

1. This question paper consists of 22 pages, an Information sheet, and an **APPENDIX A** which relates to **SECTION B: Question 7d**. Please check that you have all 3 handouts.
2. Write your examination number and teacher's name in the space provided on this question paper.
3. **Answer all the questions on the question paper and hand this in at the end of the examination.**
4. Read the questions carefully.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off answers to **1 decimal digit** where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. Diagrams are not drawn to scale.
10. It is in your own interest to present your work neatly.
11. The last page can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

	SECTION A (75 marks)					SECTION B (75 marks)				
Question	1	2	3	4	5	6	7	8	9	10
Mark										
Total	20	23	11	9	12	14	16	18	17	10
	150					100%				

SECTION A
QUESTION 1

Given $\triangle ABC$ with A and B on the x -axis as shown, and $C(-5;3)$.



(a) Determine the length of BC .

(2)

(b) (1) Calculate the size of \widehat{ABC} .

(3)

(2) Hence, or otherwise, determine the area of $\triangle ABC$.

(3)

(c) Determine **M**, the mid-point of **AC**.

(2)

(d) If **B** is the centre of the circle which passes through the points **A** and **C**,

(1) Write down the equation of this circle in the form $(x - a)^2 + (y - b)^2 = r^2$.

(3)

(2) What relationship will **BM** have with **AC**? Give a reason for your answer.

(2)

(e) If **BM** is extended to meet the point **D**, $(x; y)$, such that **ABCD** is a parallelogram,

(1) Determine the coordinates of **D**, showing all relevant working and giving reason(s).

(3)

(2) **ABCD** is also one of the quadrilaterals listed below. **Highlight/ circle** the correct answer and give a reason for your choice.

Rectangle

Kite

Rhombus

Square

Cyclic Quadrilateral

(2)

[20]

QUESTION 2

- (a) Complete: $\cos 2\theta =$ _____
or $=$ _____
or $=$ _____ (1)

- (b) (1) Prove that $\frac{\sin 2\theta}{\cos 2\theta + 1} = \tan \theta$.
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____ (3)

- (2) For which values of θ is $\frac{\sin 2\theta}{\cos 2\theta + 1}$ undefined if $\theta \in [-180^\circ; 180^\circ]$.
- _____
- _____
- _____
- _____
- _____ (3)

- (3) Hence, solve for θ if $\frac{\sin 2\theta}{\cos 2\theta + 1} = -2$ and $\theta \in [-180^\circ; 180^\circ]$.
- _____
- _____
- _____
- _____
- _____ (3)

(c) If $\cos 2\beta = \frac{1}{9}$ and $360^\circ \leq 2\beta \leq 720^\circ$, determine the value of $\sin \beta$, without the use of a calculator.

(3)

(d) Simplify:
$$\frac{\cos(-\theta) \cdot \sin(\theta - 180^\circ)}{\sin(90^\circ - \theta) \cdot \cos(360^\circ + \theta) \cdot \tan \theta}$$

(3)

(e) (1) Show that $\cos(A + 45^\circ) = \cos 45^\circ(\cos A - \sin A)$.

(3)

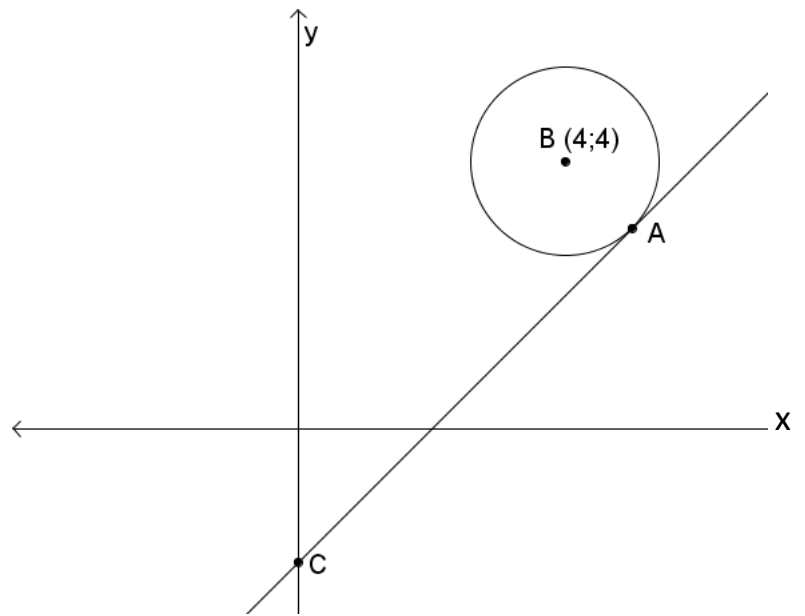
(2) Hence, determine the general solution of $\cos A - \sin A = \sqrt{2}$.

(4)

QUESTION 3

A circle with centre $B(4; 4)$ has a tangent AC at point A .

The equation of the tangent is $y - x = -2$, and C lies on the y -axis.



(a) What is the gradient of AC ?

_____ (1)

(b) Determine the equation of AB .

_____ (3)

(c) Show that A is the point $(5; 3)$.

_____ (2)

(d) Determine the equation of the circle centre B .

_____ (3)

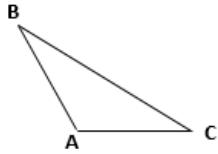
(e) If the circle is reflected across the y -axis and the radius is tripled, write down the new circle equation.

_____ (2)

[11]

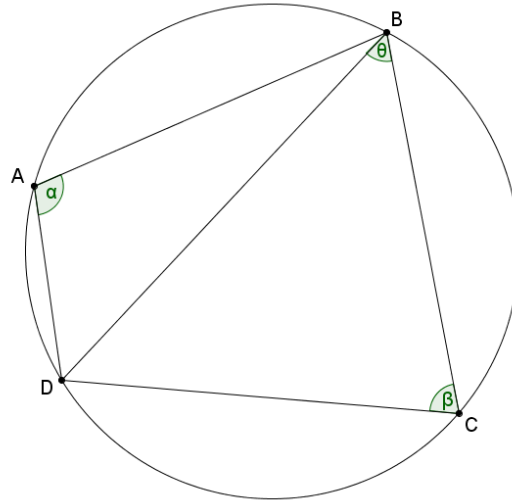
QUESTION 4

- (a) Complete the following: (1) Opposite angles of a cyclic quadrilateral are _____ (1)



- (2) In any $\triangle ABC$: $\cos A =$ _____
 _____ (1)

- (b) ABCD is a cyclic quadrilateral with $AD = 6\text{cm}$, $AB = 9\text{cm}$, $BD = 12\text{cm}$ and $DC = 8\text{cm}$.
 Diagram NOT drawn to scale.



Answer with reasons, if applicable:

- (1) Show that $\alpha = 104,5^\circ$

- (2) Determine the value of β

- (3) Determine the value of θ

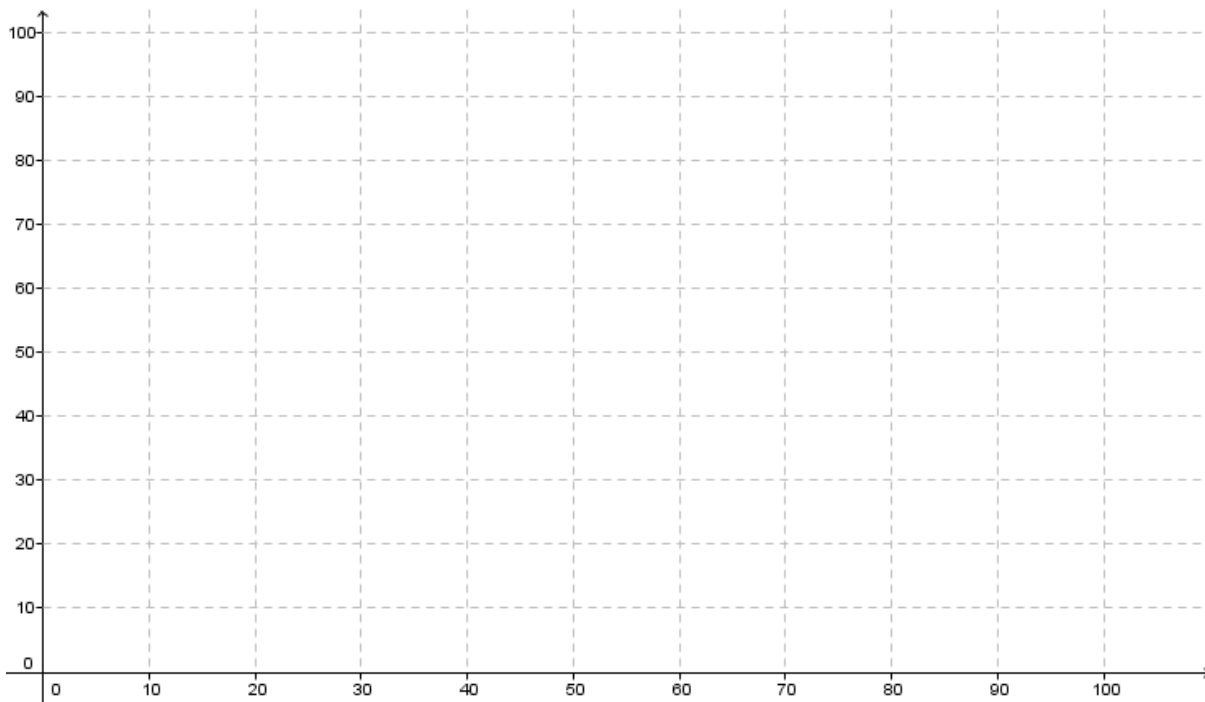
[9]

QUESTION 5

The following data consists of marks obtained in Mathematics by a group of learners in their Trial examinations and in their Final examinations.

Trials % (x)	72	65	47	85	53	37	69	36	90	74
Finals % (y)	80	71	43	87	59	43	65	40	93	79

- (a) Use the data supplied in the table to draw a scatter-plot in the space provided below. (3)



- (b) Calculate the correlation coefficient (to 3 decimal places) for the data above and then comment on the strength of the relationship.

(2)

- (c) Find the equation for the line of best fit in the form $y = \dots\dots\dots$
(round your variables off to ONE decimal place).

(2)

(d) Hence, sketch this regression line from (c) onto your scatter plot. Remember to show the y -intercept and the mean coordinate. (2)

(e) Thobile scored 55% overall in her Core Maths Trial exams, but was **unable to write her Final Maths Exams**. Her teacher uses the above regression line to predict an appropriate mark for Thobile to include in her Portfolio Assessment for her Final Maths Assessment. Determine her Final Exam mark, showing all working.

(2)

(f) What is the name given to the process of predicting Thobile's mark using the regression line. (1)

[12]

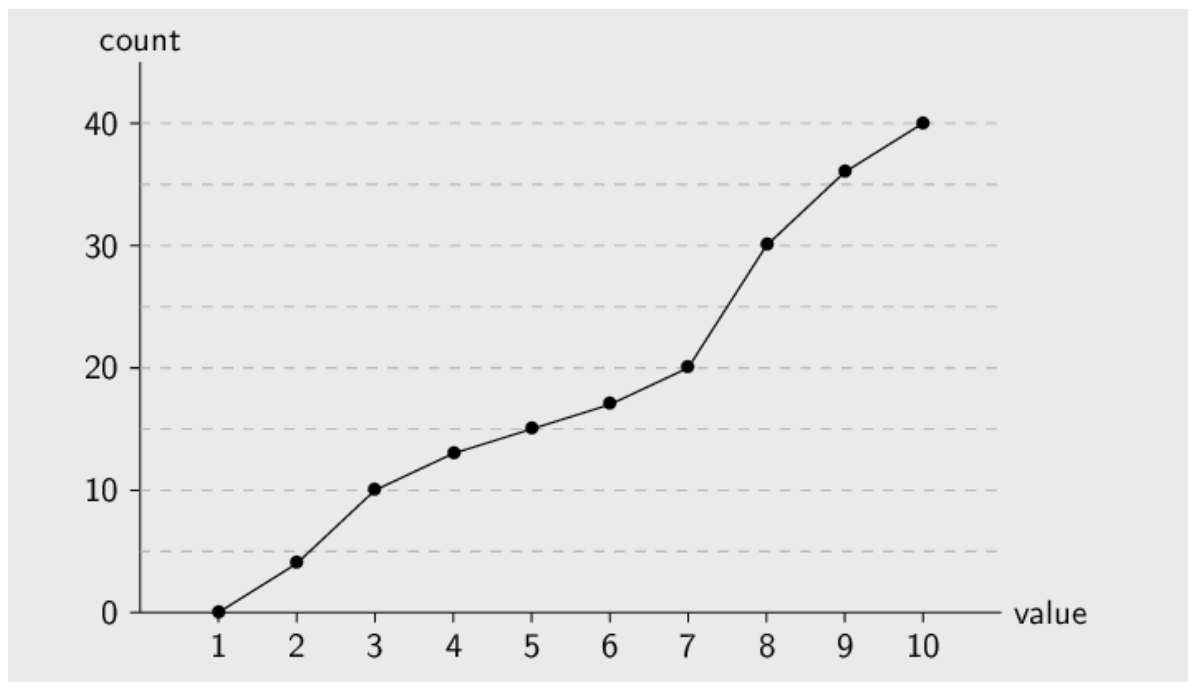
Total for Section A: 75 marks

Sometimes the questions are complicated and the answers are simple – Dr Seuss

SECTION B

QUESTION 6

(a) A cumulative frequency graph is plotted below.



Use the graph to answer the following questions:

- (1) determine the five number summary of this data and represent it in a box and whisker diagram in the space provided below. (4)



- (2) describe the distribution of data. (1)

- (3) what percentage of people scored at least a value of 5? (2)

(b) The monthly income of ten sales assistants are listed below:

R 10 000; R 14 500; R 9 500; R 15 000; R 12 000; R 13 800; R 12 250; R 14 000; R 12 500; R 13 000

(1) Use your calculator to determine:

(i) the mean _____ (1)

(ii) the standard deviation _____ (1)

(2) What percentage of salaries are less than 1 standard deviation away from the mean?

(3)

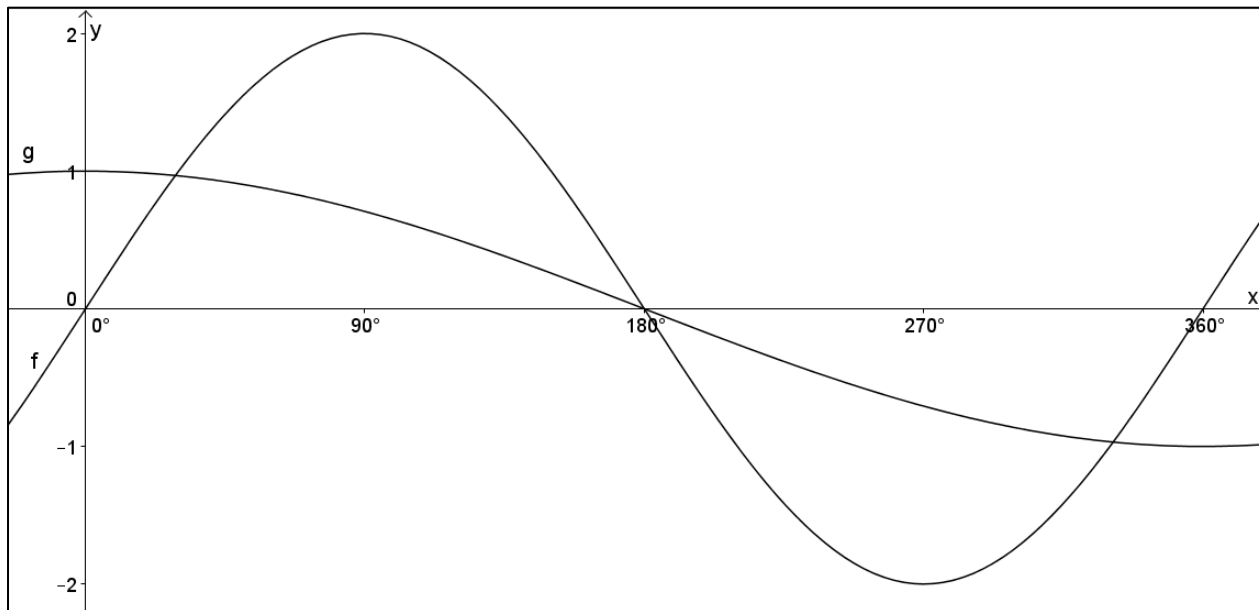
(3) In the month of September, the sales assistants are offered a bonus of either R500 added to their pay or 10% of their salary, added to their pay. Which option should they choose. Show all necessary working.

(2)

[14]

QUESTION 7

The following graphs are drawn below: $f(x) = a\sin x$ and $g(x) = \cos bx$



(a) Why is $a = 2$ and $b = \frac{1}{2}$ for the given graph equations?

(2)

(b) Use the graph to solve $g(360^\circ) - f(90^\circ)$.

(2)

(c) For what x -values in the interval $[0^\circ; 360^\circ]$, will $f(x) \cdot g(x) > 0$?

(2)

(d) The class was asked to solve $a\sin x = \cos bx$ for $x \in [0^\circ; 360^\circ]$.

Jenny's answer is copied below.

PLEASE REFER TO THE COLOUR VERSION ON APPENDIX A - A SEPARATE SHEET OF PAPER.

<p><u>Working</u></p> $1x = 2 \times \frac{1}{2}x$ <p style="background-color: yellow;">$\sin 2A = 2\sin A \cdot \cos A$</p> $\div \text{ by } \cos \frac{1}{2}x$	$2\sin x = \cos \frac{1}{2}x$ $2\sin(2 \times \frac{1}{2}x) = \cos \frac{1}{2}x$ $2\sin \frac{1}{2}x \cdot \cos \frac{1}{2}x = \cos \frac{1}{2}x$ $\frac{2\sin \frac{1}{2}x \cdot \cancel{\cos \frac{1}{2}x}}{\cancel{\cos \frac{1}{2}x}} = \frac{\cancel{\cos \frac{1}{2}x}}{\cancel{\cos \frac{1}{2}x}}$	<p><u>Working</u></p> $\div \text{ by } 2$ $\times \text{ by } 2$	$2\sin \frac{1}{2}x = 1$ $\sin \frac{1}{2}x = \frac{1}{2}$ <p style="border: 1px solid red; border-radius: 50%; padding: 2px; display: inline-block;">$\sin x = 1$</p> <div style="text-align: center; margin-top: 10px;"> </div> <p style="background-color: cyan; display: inline-block; padding: 2px;">$\therefore \text{Soln: } x = 90^\circ$ ✗</p>
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- (1) The teacher has circled, in red, 3 non-related mistakes that Jenny has made. Explain briefly why each mistake is mathematically INCORRECT.

(3)

- (2) Jenny realised her answer was wrong when she looked at the graph and saw that the two graphs intersected THREE times – not once and not at 90° ! She asks you for some help. ☺

Look at the given graphs and estimate the three x -values of the points of intersection of f and g .

Label them A, B and C on the graph on the previous page. (1)

A: $x \approx$	B: $x \approx$	C: $x \approx$
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- (3) Now algebraically solve the equation correctly for Jenny, for $x \in [0^\circ; 360^\circ]$.

Round your answers to the nearest whole number where applicable.

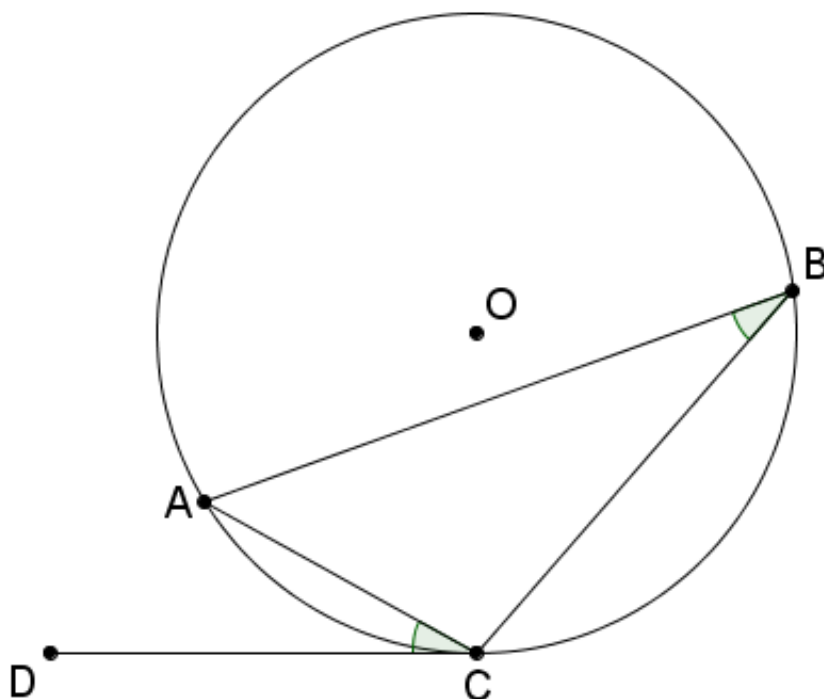
(*HINT: Jenny's first working line, in BLUE, is correct!*)

(6)

[16]

QUESTION 8

- (a) In the diagram A, B and C are points on the circle with centre O. DC is a tangent to the circle at C. Use the diagram to prove the theorem which states that : $\hat{ACD} = \hat{ABC}$.



(5)

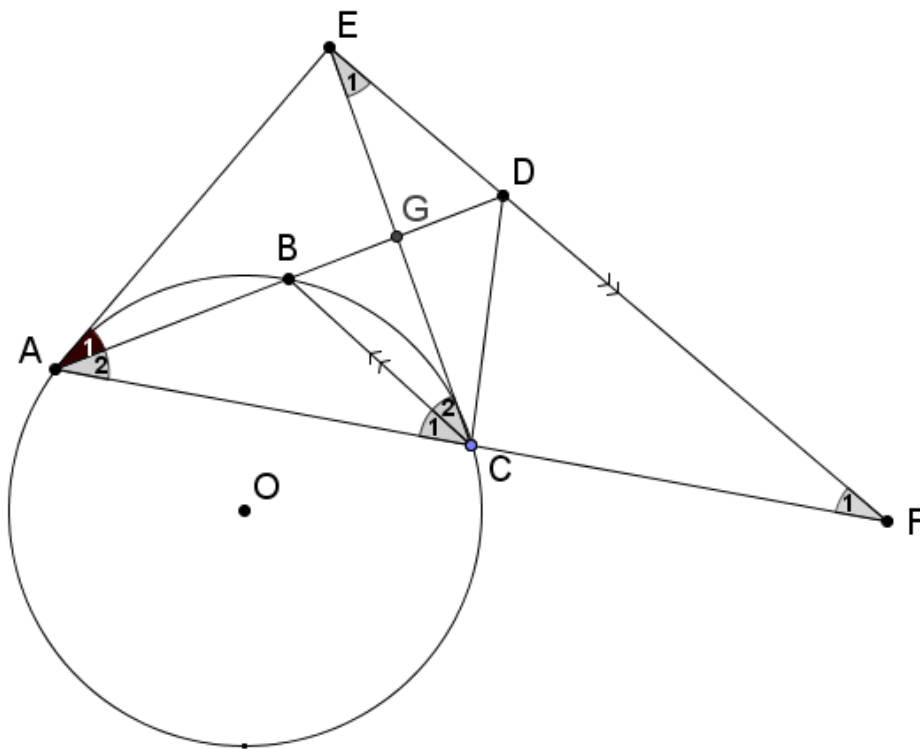
- (b) AE and EC are tangents to the circle with centre O .
 AB and BC are equal chords.
 Chord AC is produced to F such that $EF \parallel BC$.

Let $\hat{A}_1 = x$

- (1) The class had to find 5 other angles that were equal in size to x .

Indira found all 5 correct angles but left out the reasons.

Fill them in for her, in the order that she wrote them down. (5)



	Angle	Reason
1	$\hat{C}_1 = x$	
2	$\hat{F}_1 = x$	
3	$\hat{A}_2 = x$	
4	$\hat{C}_2 = x$	
5	$\hat{E}_1 = x$	

IF YOU NEED MORE LINES FOR THE FOLLOWING QUESTIONS THE NEXT PAGE IS BLANK

- (2) Prove that $ACDE$ is a cyclic quadrilateral.

_____ (2)

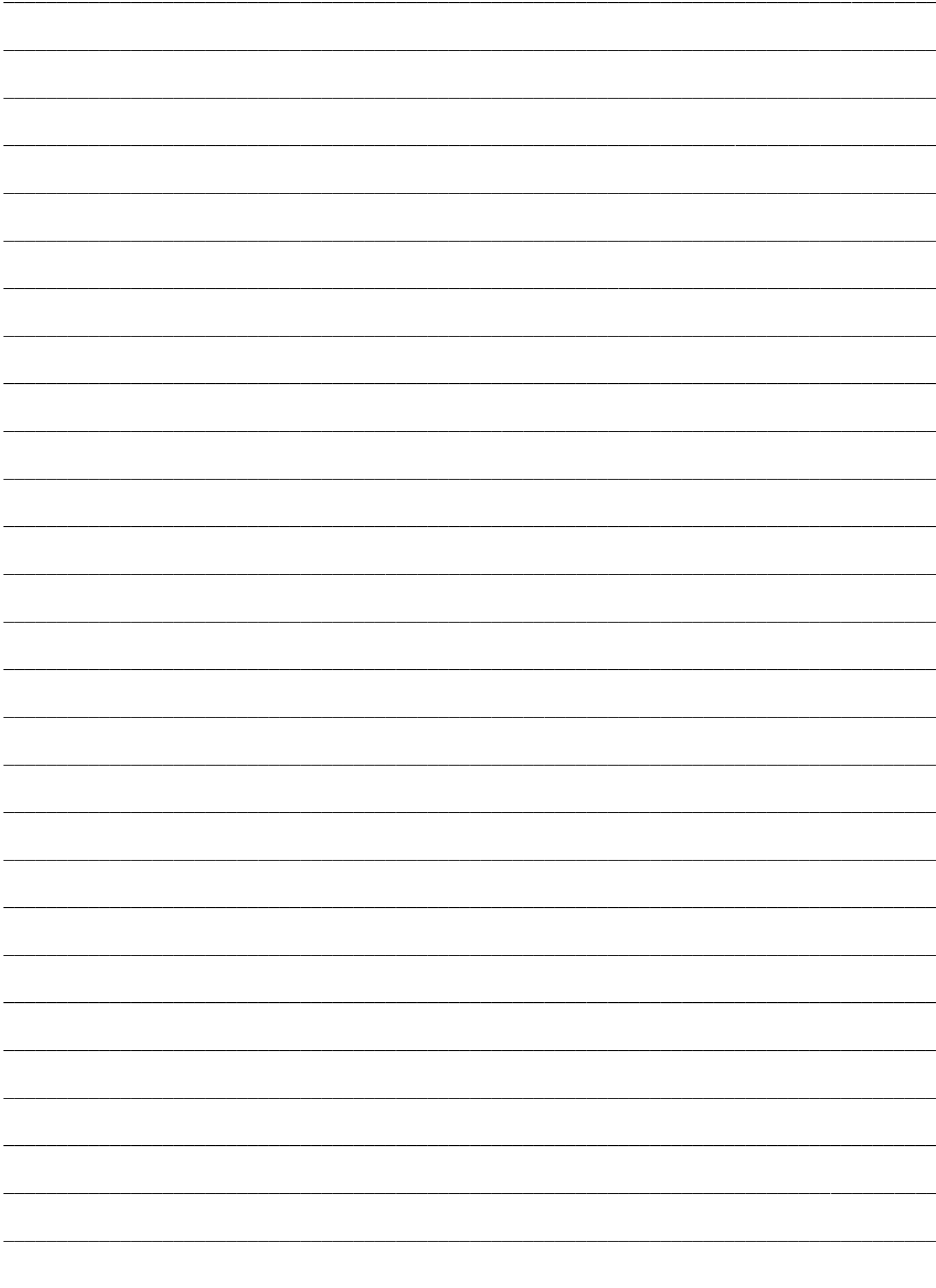
- (3) Prove that $AE = CF$.

_____ (2)

- (4) Prove that EDF is a tangent to the circle BCD at point D .

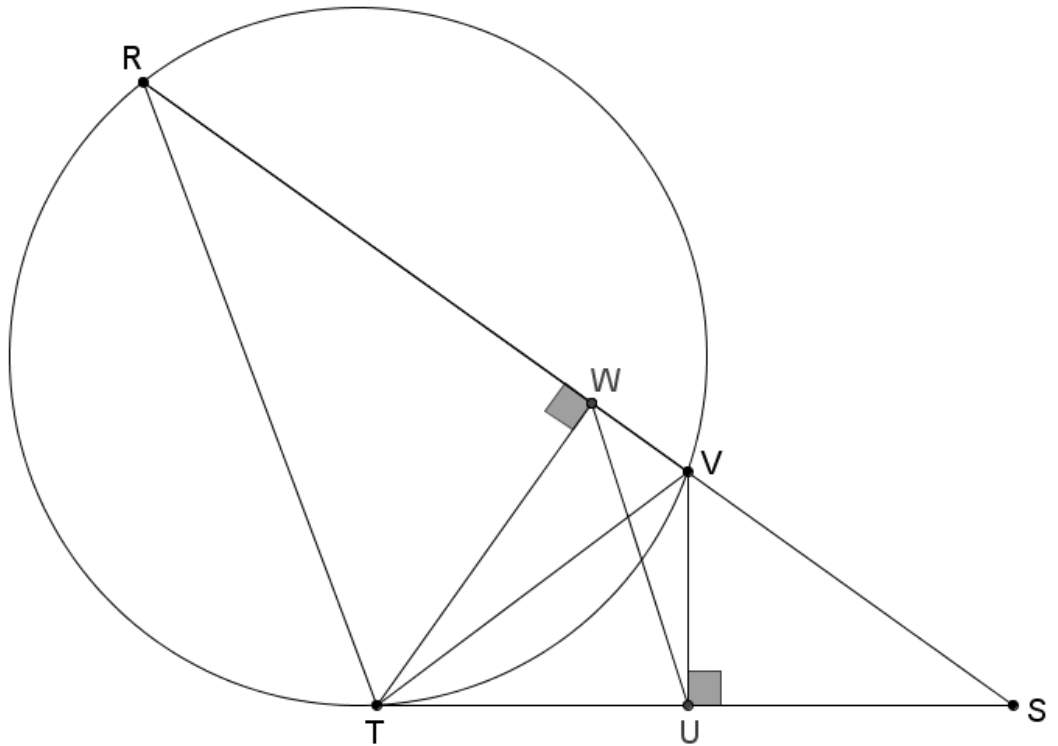
 _____ (4)

[18]



QUESTION 9

- (a) (1) The Proportion- Intercept Theorem states that the line drawn parallel to one side of a triangle _____ (1)
- (2) R, V and T are points on the given circle. W is a point on RV such that $TW \perp RS$. RW produced meets the tangent at T at S. U is a point on the tangent such that $UV \perp TS$.



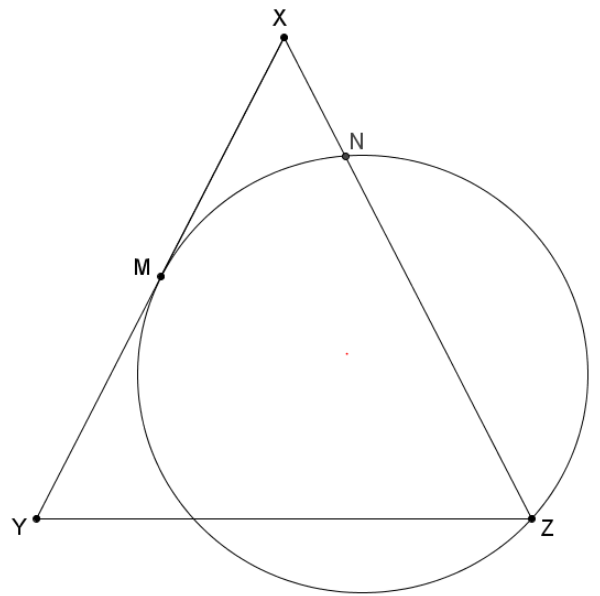
- (i) Why is TWVU a cyclic quadrilateral?

(1)

- (ii) Prove that $\frac{RW}{WS} = \frac{TU}{US}$.

(4)

- (b) XYZ is an isosceles triangle with $XY = XZ$.
 M is the mid-point of XY .
 XY is a tangent to the circle at M .



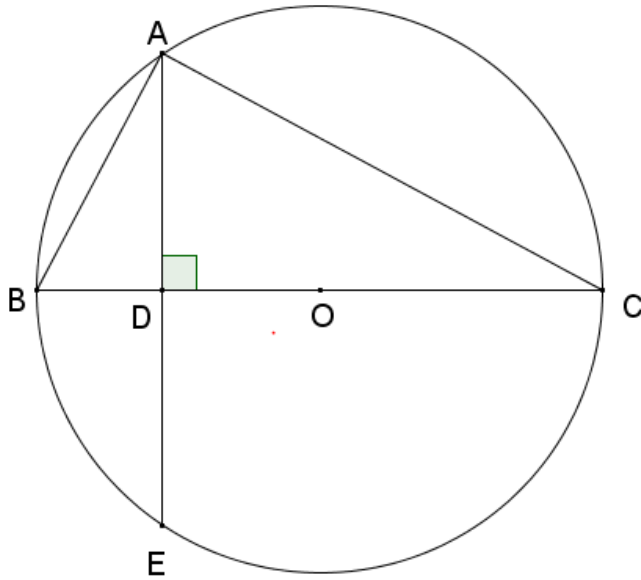
- (1) Prove that $\triangle XMN \sim \triangle XZM$. *You will need to construct.*

(3)

- (2) Prove that $XZ = 4XN$.

(3)

(c)



Given circle, centre O , with points A, C, E and B on the circumference. The straight line $BDOC$ is the diameter which is perpendicular to chord ADE .

(1) Why is $\hat{A} = 90^\circ$? _____ (1)

(2) Give a reason why $\triangle ABD \sim \triangle CBA$? _____

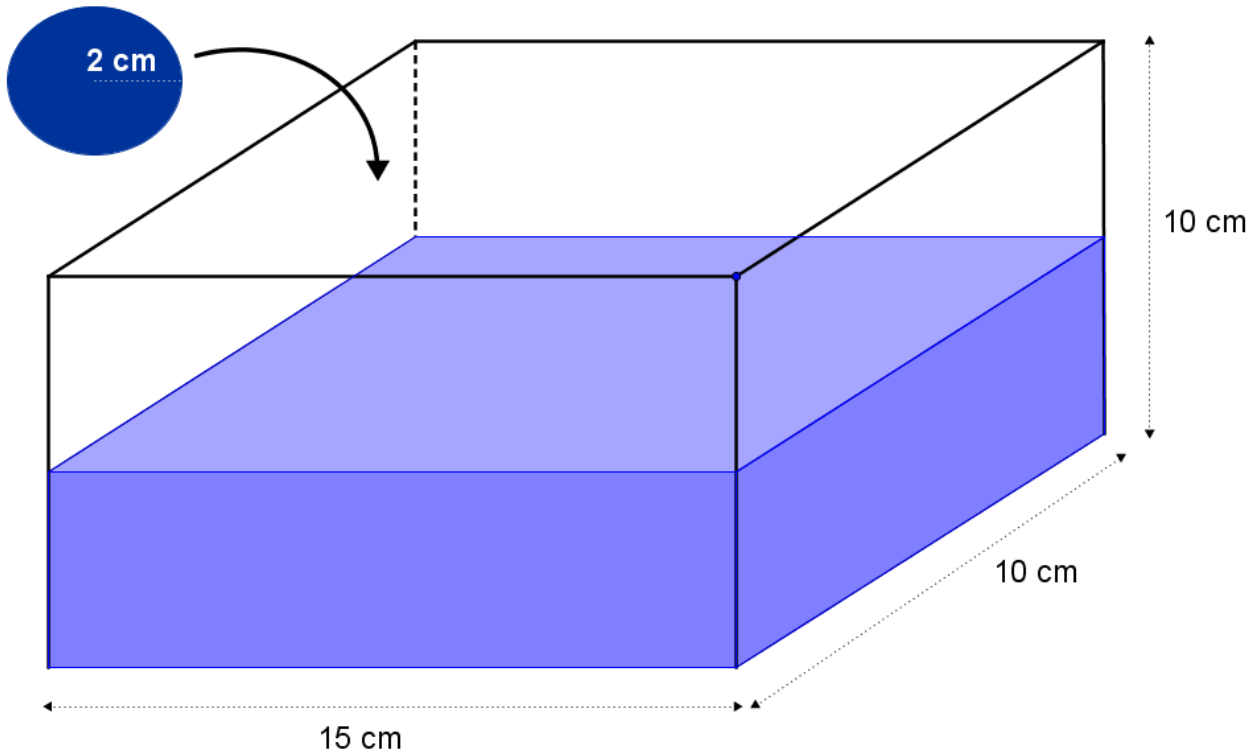
_____ (1)

(3) If $AB = \sqrt{20}$ units and $DO = 3$ units, prove that the radius of the circle is 5 units, showing all relevant working.

_____ (3)

[17]

QUESTION 10



- (a) A rectangular glass vase measures 15cm by 10 cm by 10 cm as shown in the picture above. The vase is half full with water.

A solid, decorative glass marble, with a radius of 2 cm, is placed into the vase and drops to the bottom.

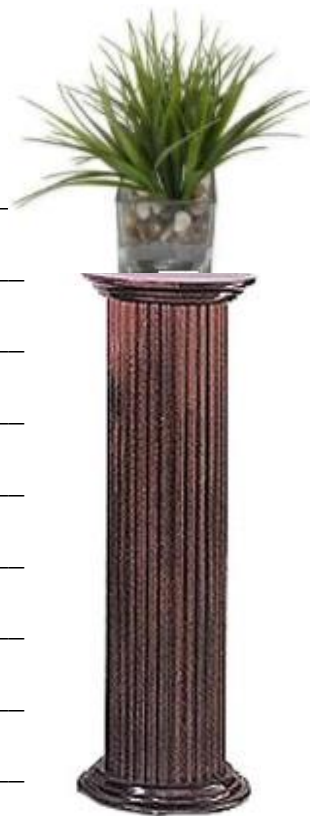
Useful Formulae:	$V = l \times b \times h$	$V = \frac{4}{3}\pi r^3$
	$S.A = 2lb + 2bh + 2bh$	$S.A = 4\pi r^2$

- (1) Assuming that the marble sinks to the bottom, and stays there, calculate the amount by which the water level in the vase rises.

- (2) Based on your calculation in (a), use Maths to determine the most number of marbles you could drop into the vase without overflowing any water?

- (b) The vase is going to sit on top of a cylindrical pillar.
If the **curved surface area** of the cylindrical pillar is $6\,283,20\text{ cm}^2$ and its volume is $31\,416\text{ cm}^3$, find the ratio of its diameter to its height.

Useful Formulae: $V = \pi r^2 h$ $S.A = 2\pi r^2 + 2\pi r h$



(4)

[10]

Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding. – William Paul Thurston

