ST. DAVID'S MARIST INANDA

MATHEMATICS
PRELIMINARY EXAMINATION
PAPER 2

GRADE 12
13 September 2017

EXAMINER: MRS S RICHARD **MODERATOR: MRS C KENNEDY**	**MARKS: 150** **TIME: 3 hours**

NAME: Memo.

PLEASE PUT A CROSS NEXT TO YOUR TEACHER'S NAME:

Mrs Kennedy	Mrs Nagy	Mr Vicente	Mrs Richard	Mrs Black

INSTRUCTIONS:

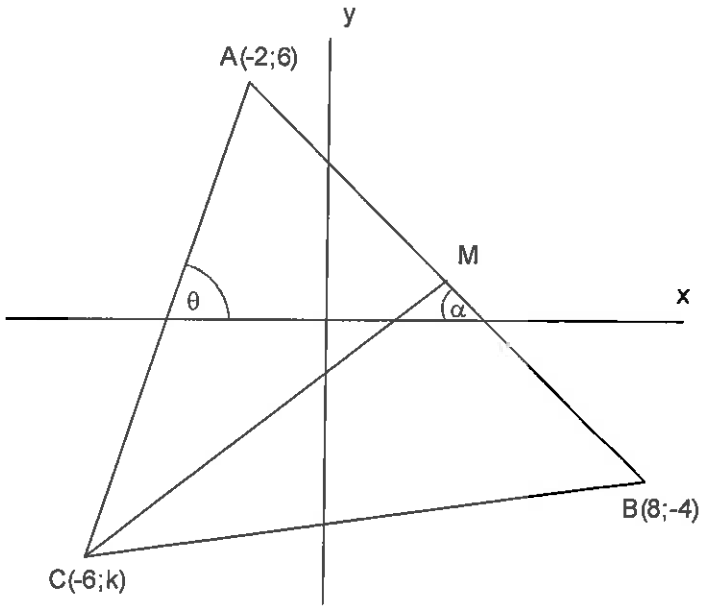
- ✓ This paper consists of 24 pages and a separate 2 page information sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ It is in your interest to show all your working details and give valid reasons where necessary.
- ✓ Reasons for all Geometry must be clearly stated.
- ✓ Work neatly. Do NOT answer in pencil.
- ✓ Diagrams are not drawn to scale.

SECTION A	Q1 [17]	Q2 [21]	Q3 [19]	Q4 [9]	Q5 [9]		SUB-TOTAL [75]
LEARNER'S MARKS							
SECTION B	Q6 [13]	Q7 [11]	Q8 [13]	Q9 [19]	Q10 [4]	Q11 [10]	Q12 [5]	SUB-TOTAL [75]
LEARNER'S MARKS								

SECTION A

QUESTION 1

In the diagram below, $A(-2;6), B(8;-4)$ and $C(-6;k)$ are three points in the Cartesian plane with point M the midpoint of AB and $AB = BC$.



a) Determine the coordinates of M. (2)

$$M\left(\frac{-2+8}{2};\ \frac{6-4}{2}\right) = (3;1)$$

b) Determine the value(s) of k. (6)

$$AB = BC$$

$$(6-(-4))^2 + (-2-8)^2 = (k+4)^2 + (-6-8)^2$$

$$10^2 + 10^2 = k^2+8k+16 + 14^2$$

$$0 = k^2 + 8k + 12$$

$$0 = (k+6)(k+2)$$

$$k = -6 \text{ or } k = -2$$

c) Hence, determine the equation of the line CM if $k = -6$. (3)

$$m_{cm} = \frac{1-(-6)}{3-(-6)}$$
$$= \frac{7}{9} ✓$$
$$y = \frac{7}{9}x + c$$
$$1 = \frac{7}{9}(3)✓ + c$$
$$1 = \frac{7}{3} + c$$
$$y = \frac{7}{9}x - \frac{4}{3} ✓$$

d) Determine the value of:

i) θ if $k = -6$ (2)

$$m_{AC} = \frac{6-(-6)}{-2-(-6)}$$
$$= 3 ✓$$
$$\tan\theta = 3$$
$$\theta = 71,57° ✓$$

ii) α (2)

$$m_{AB} = \frac{6-(-4)}{-2-8} ✓$$
$$= -1$$
$$\tan\alpha = -1$$
$$\alpha = 45° ✓$$

iii) $C\hat{A}B$ (2)

$$C\hat{A}B = 63,43° ✓✓ \quad <'s \text{ of } \Delta$$

[17]

QUESTION 2

a) Show that

$$\frac{\sin(90° + x).\,cosx.\tan(-x).\sin(x - 180°)}{\cos(180° + x).\sin(540° + x)} = sinx$$

(6)

$$= \frac{\cos x . \cos x\ (-\tan x)(-\sin x)}{(-\cos x)(-\sin x)}$$

$$= \cos x . \frac{+\sin x}{\cos x}$$

$$= \sin x \quad \text{given}$$

b) i) Given: $sinx = cos2x - 1$

Show that $2sin^2x + sinx = 0$ (1)

$\sin x = 1 - 2\sin^2 x - 1$

$2\sin^2 x + \sin x = 0$

ii) Determine the general solution of $sinx = cos2x - 1$ (6)

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -\frac{1}{2}$

ref $\angle = 30°$

$x = 0° + k.360°$ both

or

$x = 180° + k.360°$

$k \in \mathbb{Z}$

$x = 180° + 30° + k.360°$

$= 210° + k.360°$

or both

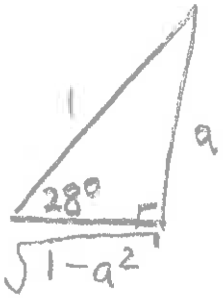
$x = 360° - 30° + k.360°$

$= 330° + k.360°$

$k \in \mathbb{Z}$

c) If $sin28° = a$ and $cos32° = b$, determine the following in terms of a and b.

i) $cos28°$ $= \sqrt{1-a^2}$ ✓✓ (2)



ii) $cos64°$ $= 2\cos^2 32° - 1$ ✓

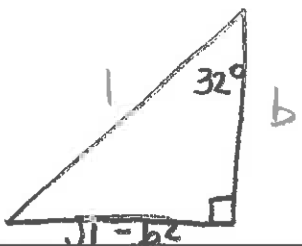
$= 2b^2 - 1$ ✓ (2)

iii) $sin4°$ $= \sin(32° - 28°)$ ✓ (4)

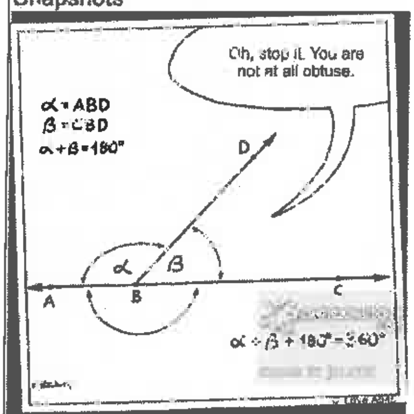
$= \sin 32° \cos 28° - \cos 32° \sin 28°$ ✓

$= \sqrt{1-b^2}\sqrt{1-a^2} - b \cdot a$

✓ use i) ✓



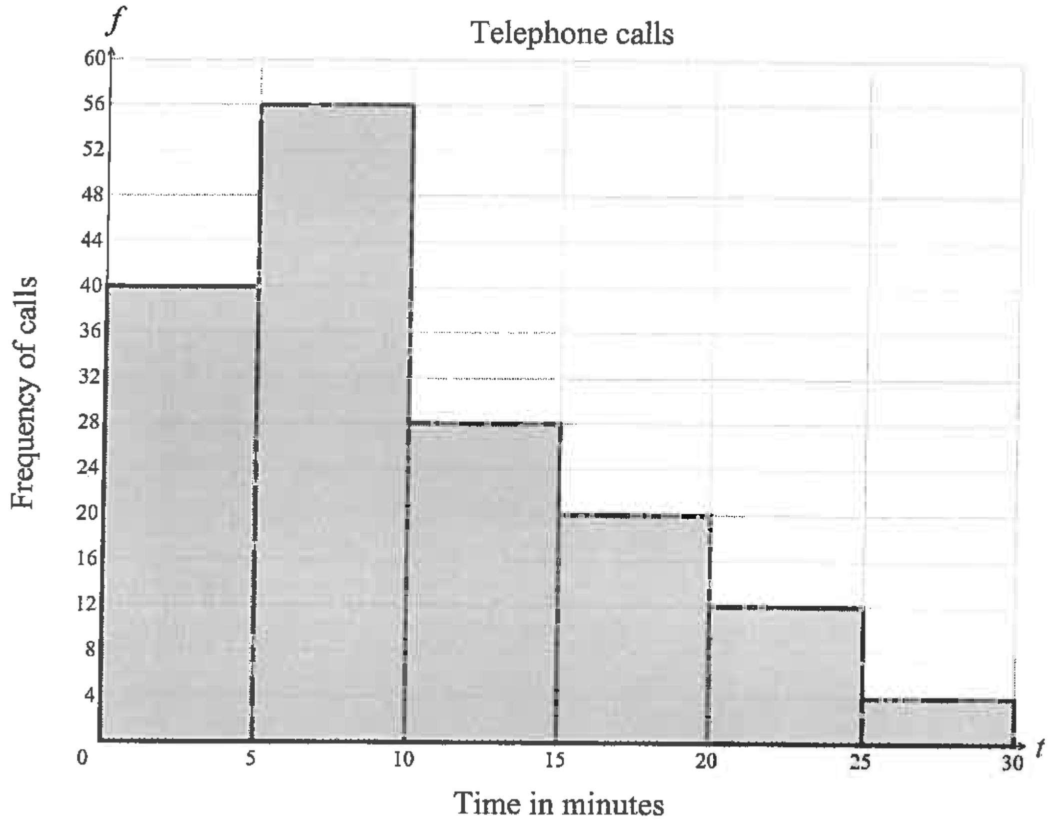
Snapshots



Complimentary angles make the other angles feel good about themselves.

[21]

QUESTION 3



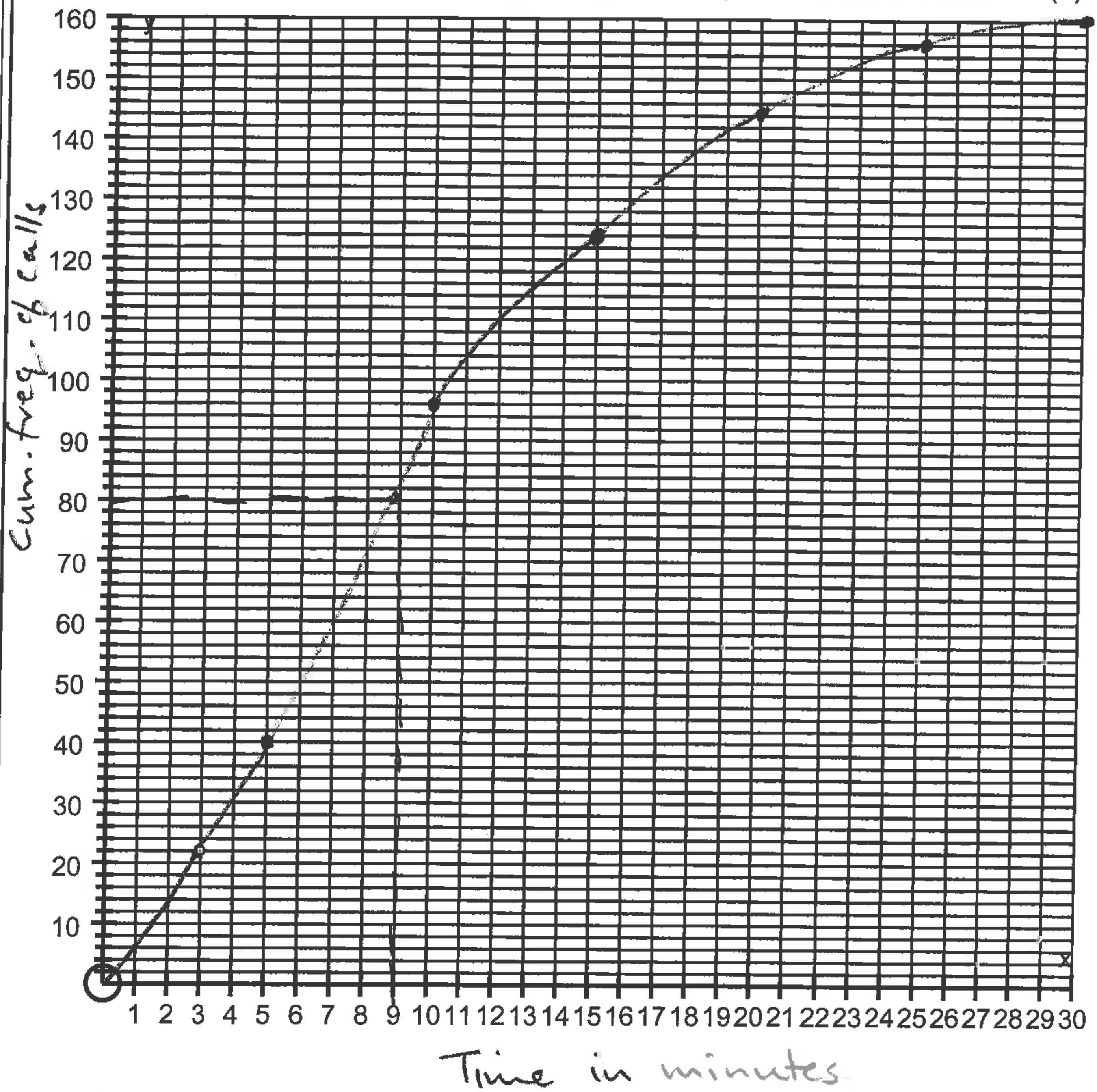
The diagram above shows a histogram for the lengths of telephone calls.

a) **Complete the frequency table for the data.** (3)

Time in minutes	Frequency	Class midpoint	Cumulative frequency
$0 \leq x < 5$	**40**	**2.5**	**40**
$5 \leq x < 10$	56	7,5	96
$10 \leq x < 15$	28	12,5	124
$15 \leq x < 20$	20	17,5	144
$20 \leq x < 25$	12	22,5	156
$25 \leq x < 30$	4	27,5	160

3 mistakes 0

b) Draw an ogive (cumulative frequency curve) to illustrate the data. (3)



c) Use your ogive (cumulative frequency curve) to answer the questions below:

i) State the median length of a phone call. (1)

8,5 minutes or 9 mins.

ii) What is the interquartile range? (3)

14 − 5 = 9 min

iii) What percentage of phone calls last for 20 minutes or longer? (2)

160 − 144 = 16

16/160 = 10%

d) Use the frequency distribution associated with your histogram to estimate

i) the mean length of a call. (2)

$\bar{x} = 10$ ✓✓

ii) the standard deviation of a call. (2)

$\sigma = 6{,}61$ ✓✓

e) Use your ogive and the your answer to d)ii) to determine the number of calls which lie within one standard deviation of the mean. (3)

$[10 - 6{,}61\ ;\ 10 + 6{,}61]$ ✓

use d)ii)

$[3{,}39\ ;\ 16{,}61]$ ✓

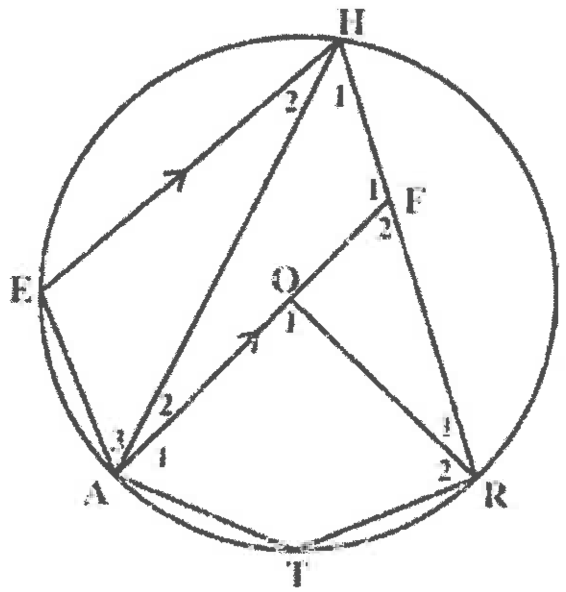
$22\ ;\ 128$

± 106 ✓ calls lie in interval

"We haven't yet found what's causing the ringing in your ears, but we were able to set them to vibrate."

[19]

QUESTION 4



In the diagram O is the centre of the circle HEATR. AOF is parallel to EH.

$\hat{F}_2 = 78°$ and $\hat{R}_1 = 22°$

Calculate, with reasons, the size of:

a) $\hat{O}_1$ $= 78° + 22°$ ext < of $\triangle$ ✓ (2)

$= 100°$ ✓

b) $\hat{H}_1$ $= 50°$ ✓ < at centre $= 2 \times$ < at circ ✓ (2)

c) $\hat{T}$ $= 130°$ ✓ opp <'s of cyclic quad ✓ (2)

d) $\hat{H}_2$ $F_1 = 102°$ adj <'s on str line ✓ (3)

$\hat{A}_2 = 28°$ <'s in $\triangle$ ✓

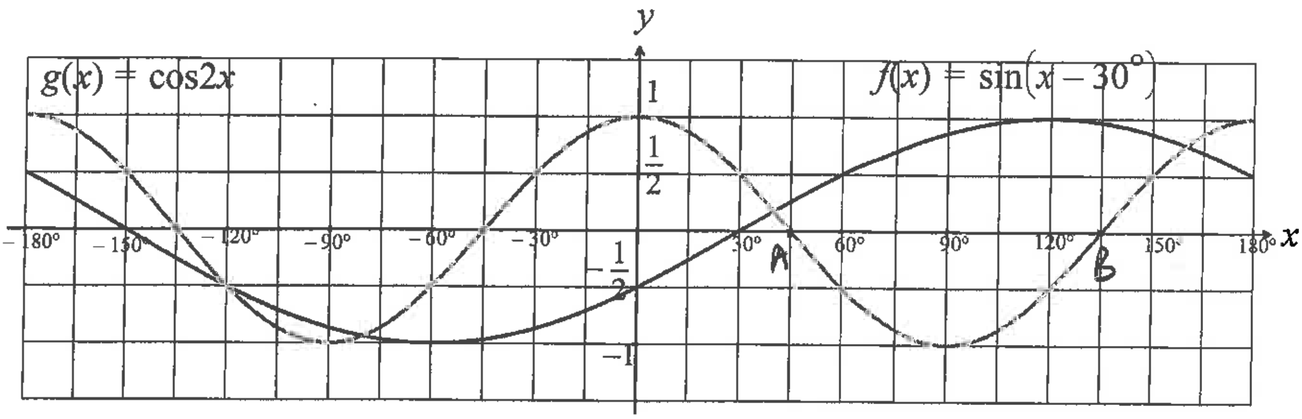
$\hat{H}_2 = 28°$ alt <'s, ‖ lines ✓

[9]

QUESTION 5

The graph shows the curves of:

$f(x) = \sin(x - 30^0)$ and $g(x) = \cos 2x$ for $x \in [-180^0; 180^0]$



Answer the following questions with the aid of the graph.

a) What is the period of the graph of $g(\frac{1}{4}x)$? (1)

$g(\frac{1}{4}x) = \cos 2(\frac{1}{4}x)$

$= \cos \frac{1}{2}x$

$\therefore 720^\circ$ ✓

b) State the amplitude of the graph of h if $h(x) = \frac{g(x)}{2}$. (1)

amplitude $= \frac{1}{2}$ ✓

c) How many solutions does the equation:

$\sin(x - 30^0) = \cos 2x$ have if $x \in [-180^0; 180^0]$? (1)

4 solutions ✓

d) For which value(s) of x is $\cos^2 x = \sin^2 x$ if $x \in [0^0; 180^0]$?

(show how the graph above can be used to answer this question) (3)

$\cos 2x = \cos^2 x - \sin^2 x$ ✓ identity.

$\cos 2x = 0$ x-intercepts ✓

$0 = \cos^2 x - \sin^2 x$

$\cos^2 x = \sin^2 x$

$x = 45°; 135°$ ✓

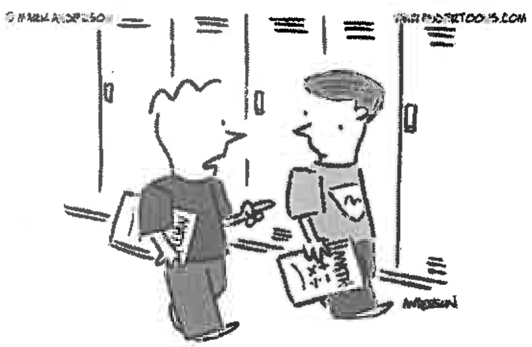
at A and B on graph

e) For which value(s) of x is $2\cos 2x.\sin(x - 30^0) \geq 0$ if $x \in [0^0; 180^0]$? (3)

$x \in [30°; 45°] \cup [135°; 180°]$ ✓ ✓

✓square brackets

[9]



"I find if you put that slash through the equal sign, the number of possible answers vastly increases."

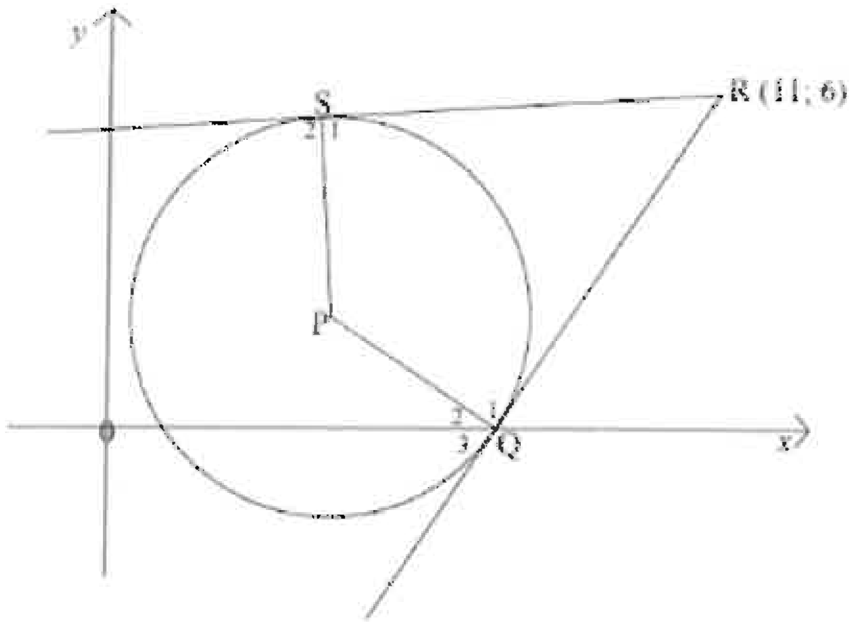
[Total Section A: 75 marks]

SECTION B

QUESTION 6

A circle centre P and radius PQ is sketched with Q on the x-axis.

QR and SR are tangents to the circle at Q and S respectively, intersecting at R(11;6)



a) If the circle defined by $x^2 - 8x + y^2 - 4y = -7$, determine the coordinates of the centre P, and the length of the radius PQ, leaving your answers in surd form. (4)

$x^2 - 8x + 16 + y^2 - 4y + 4 = -7 + 16 + 14$ ✓

$(x-4)^2 + (y-2)^2 = 13$ ✓

centre $P(4;2)$ ✓

radius $\sqrt{13}$ ✓

b) Prove that the quadrilateral PQRS is cyclic, stating reasons. (4)

$\hat{S}_1 = 90°$ tan ⊥ rad ✓

$\hat{Q}_1 = 90°$ tan ⊥ rad ✓

$\hat{S}_1 + \hat{Q}_1 = 180°$ ✓

∴ PQRS is cyclic
opp <'s suppl ✓

c) Determine the length of RS, leaving your answer in surd form. (5)

RS = RQ tan from same pt

$(11-7)^2 + (6-0)^2 = RQ^2$

$4^2 + 6^2 = RQ^2$

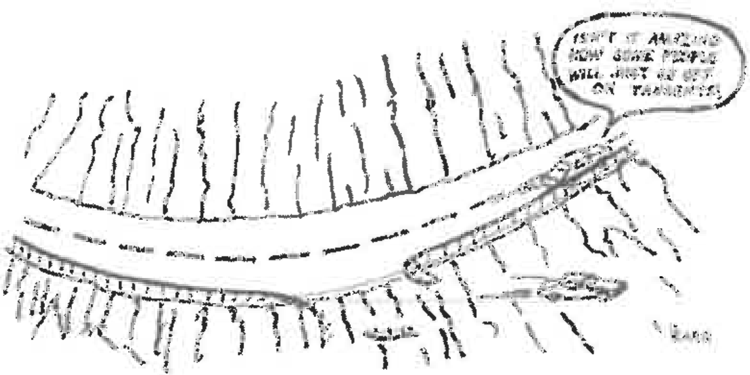
$16 + 36 = RQ^2$

$RQ^2 = 52$

$RQ = \sqrt{52}$

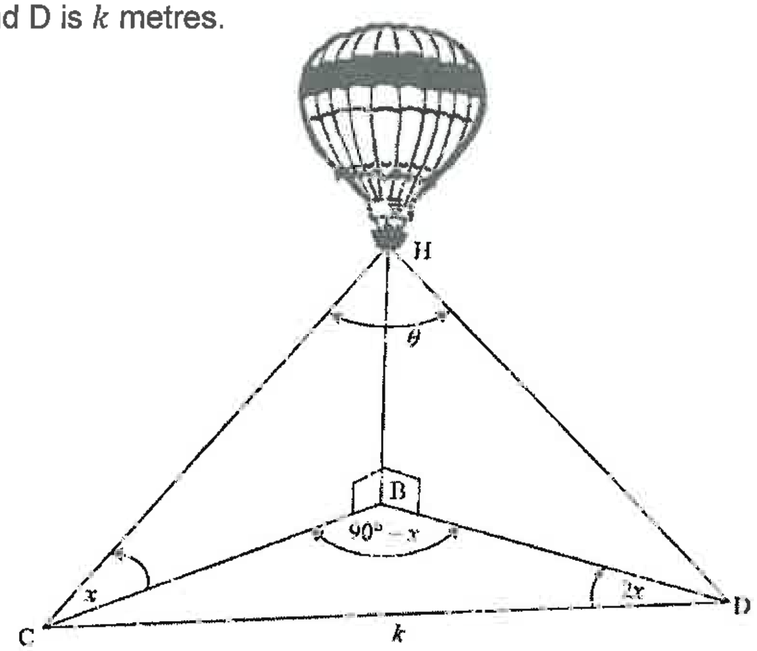
$\therefore RS = \sqrt{52}$

[13]



QUESTION 7

A hot-air balloon H is directly above point B on the ground. Two ropes are used to keep the hot-air balloon in position. The ropes are held by two people on the ground at point C and point D. B,C and D are in the same horizontal plane. The angle of elevation from C to H is x. $C\hat{D}B = 2x$ and $C\hat{B}D = 90° - x$. The distance between C and D is k metres.



a) Show that $CB = 2k\, sinx$. (4)

$$\frac{\sin(90° - x)}{k} = \frac{\sin 2x}{CB}$$

$$CB = \frac{k(2\sin x \cos x)}{\cos x}$$

$$= 2k \sin x \text{ given}$$

b) Hence, show that the length of rope HC is $2k\, tanx$. (3)

In ΔCBH

$$\cos x = \frac{CB}{HC}$$

$$HC = \frac{2k \sin x}{\cos x} = 2k \tan x \text{ given}$$

c) If $k = 40m, x = 23°$ and $HD = 31{,}8\ m$, calculate θ, the angle between the two ropes, to 2 decimal places. (4)

In Δ CHD

$$k^2 = CH^2 + HD^2 - 2\,CH\,HD\,\cos Q$$

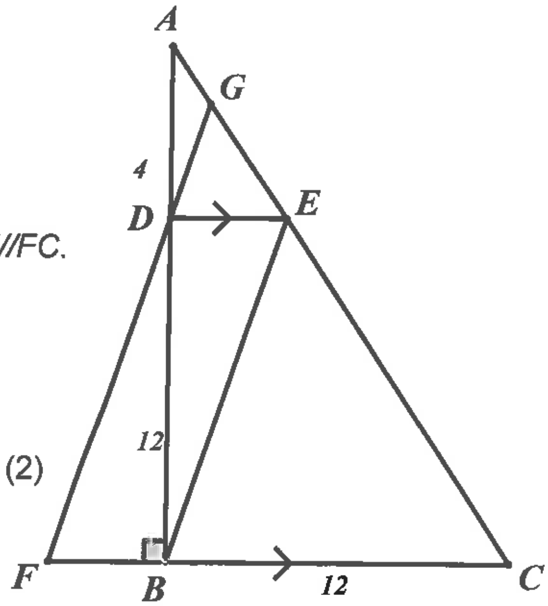
$$40^2 = (2(40)\tan 23°)^2 + 31{,}8^2 - 2(2(40)\tan 23°)(31{,}8)\cos Q$$

$$\cos Q = 0{,}26$$

$$\hat{Q} = 74{,}85°$$

[11]

QUESTION 8



Refer to the diagram (not drawn to scale):

In the diagram ΔABC is a right-angled triangle.

The point *D* lies on *AB* and *E* lies on *AC* such that *DE//FC*.

BC = 12 units, *AD* = 4 units and *DB* = 12 units.

a) Show that *AC* = 20 units (2)

$AC^2 = 12^2 + 16^2$ ✓ Pyth

$AC^2 = 400$ ✓

$AC = 20$ units given

b) Calculate, stating reasons, the **size** of:

1) *AE* (3)

DE ∥ FC given

$\frac{AE}{AC} = \frac{AD}{AB}$ ✓ line ∥ one side of △

$\frac{AE}{20} = \frac{4}{16}$ ✓

$AE = 5$ ✓ units

2) *EC* (1)

$\therefore EC = 15$ ✓ units

c) It is further given that: $GE = 3\frac{3}{4}$ units.

i) Determine the length of DE. (3)

$\triangle ABC \ ||| \ \triangle ADE$

$\frac{DE}{BC} = \frac{AD}{AB}$ ✓ $|||$ $\triangle$'s

$\therefore DE = 12 \times \frac{4}{16}$

$= 3$ units ✓

ii) Hence, or otherwise, prove that DEBF is a parallelogram. (4)

$\frac{FC}{DE} = \frac{GC}{GE}$ $|||$ $\triangle$'s ✓

$FC = \frac{3 \times 18,75}{3,75}$

$= 15$ ✓

$\therefore FB = 3$ units ✓

✓

$\therefore$ DEBF is a parallelogram (one pair of sides $=$ and $//$)

[13]

"So, for every day that your math grade stays below a B, your father will post a video of himself on YouTube."

QUESTION 9

a) Complete the statement:

A line parallel to one side of triangle .... (2)

divides the other 2 sides in proportion

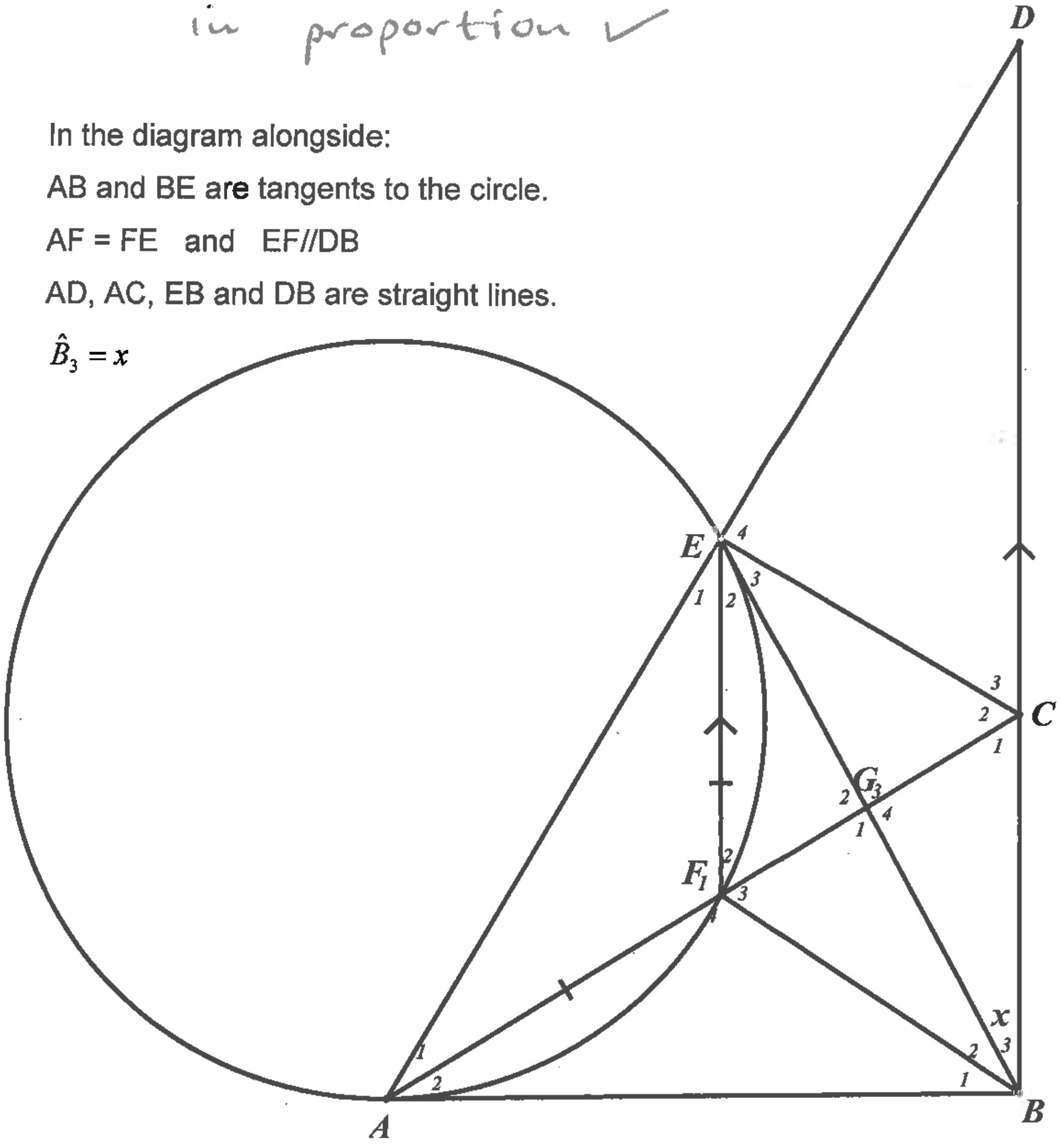
b) In the diagram alongside:

AB and BE are tangents to the circle.

AF = FE and EF//DB

AD, AC, EB and DB are straight lines.

$\hat{B}_3 = x$



i) Complete: $\frac{AF}{FC} = ...\ \frac{AE}{ED}$ (1)

ii) With reasons, write down **5** other angles equal to *x*. (5)

$\hat{E}_2 = x$ alt <'s; || lines ✓

$\hat{A}_1 = x$ tan chord thm ✓

$\hat{E}_1 = x$ AF = FE equal chords, equal sides ✓

$\hat{D} = x$ corr <'s; || lines ✓

$\hat{A}_2 = x$ tan chord thm ✓

iii) Prove that AECB is a cyclic quadrilateral. (2)

$\hat{B}_3 = \hat{A}_1 = x$ proven ✓

∴ AECD is a cyclic quad ✓ (line segment subt equal <'s)

iv) Prove that $\Delta ACB /// \Delta DAB$ (3)

$\hat{B}$ is common ✓

$\hat{A}_2 = D = x$ proven ✓

∴ $C_1 = \hat{A}_1 + \hat{A}_2$ ext < of Δ ✓

∴ ACD ||| ΔDAB equiangular

v) Hence, deduce that $AB^2 = DB.CB$ (2)

$$\frac{AC}{DA} = \frac{BC}{AB} = \frac{AB}{DB}$$ ✓ ||| Δ's ✓

∴ $AB^2 = DB \cdot BC$

vi) Is EC a tangent to the circle passing through E, G and A? Give a reason. (4)

$\hat{E}_3 = \hat{A}_2$ $\angle$'s in same segment ✓

$\hat{A}_2 = \hat{A}_1 = x$ proven. ✓

$\hat{E}_3 = \hat{A}_1$ ✓

$\therefore$ EC is a tangent to EGA

($\angle$ between line and chord ✓
$= \angle$ in alt segment)

[19]

QUESTION 10

If $sinB + cosB = 1{,}2$, evaluate, **without using a calculator** $sinBcosB$. (4)

$(\sin B + \cos B)^2 = (1{,}2)^2$ ✓

$\sin^2 B + 2\sin B\cos B + \cos^2 B = 1{,}44$

$2\sin B\cos B + 1 = 1{,}44$ ✓

$2\sin B\cos B = 0{,}44$ ✓

$\sin B\cos B = 0{,}22.$ ✓

[4]

QUESTION 11

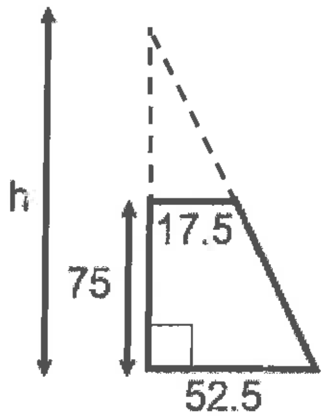
The Metropolitan Cathedral in Rio de Janeiro is a conical frustrum with height 75m, base diameter 105m and top diameter 35m.

Formulas you may need:

$$V = \frac{1}{3}\pi r^2 h$$

$$SA = \pi r l + \pi r^2$$

The diagram below shows the longitudinal cross-section of the building:



a) Show that h = 112,5m (3)

$$\frac{h}{52{,}5} = \frac{h-75}{17{,}5} \checkmark$$

$$17{,}5h = 52{,}5(h-75) \checkmark$$

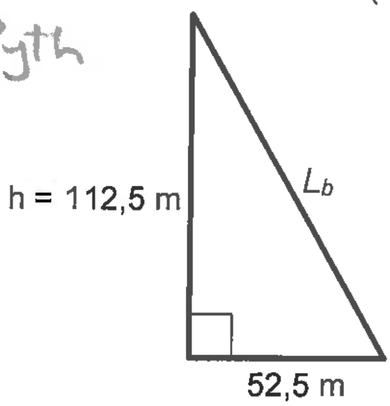
$$17{,}5h = 52{,}5h - 3937{,}5$$

$$35h = 3937{,}5 \checkmark$$

$$\therefore h = 112{,}5 \text{ m}$$

given

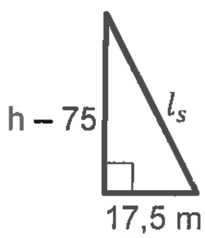
b) Calculate the slant length of the large cone, L_b, in metres. Leave your answer in surd form. (3)



$L_b^2 = 112{,}5^2 + 52{,}5^2$ Pyth

$L_b = \sqrt{15412{,}5}$ metres

c) If the slant length of the smaller cone (that has been removed), $l_s = \sqrt{1712{,}5}$ m, calculate the surface area of the frustum-shaped building in m² (do not include the circular base or roof of the building). (4)



$SA = \pi RL - \pi r l$

$= \pi(52{,}5)\sqrt{15412{,}5}$

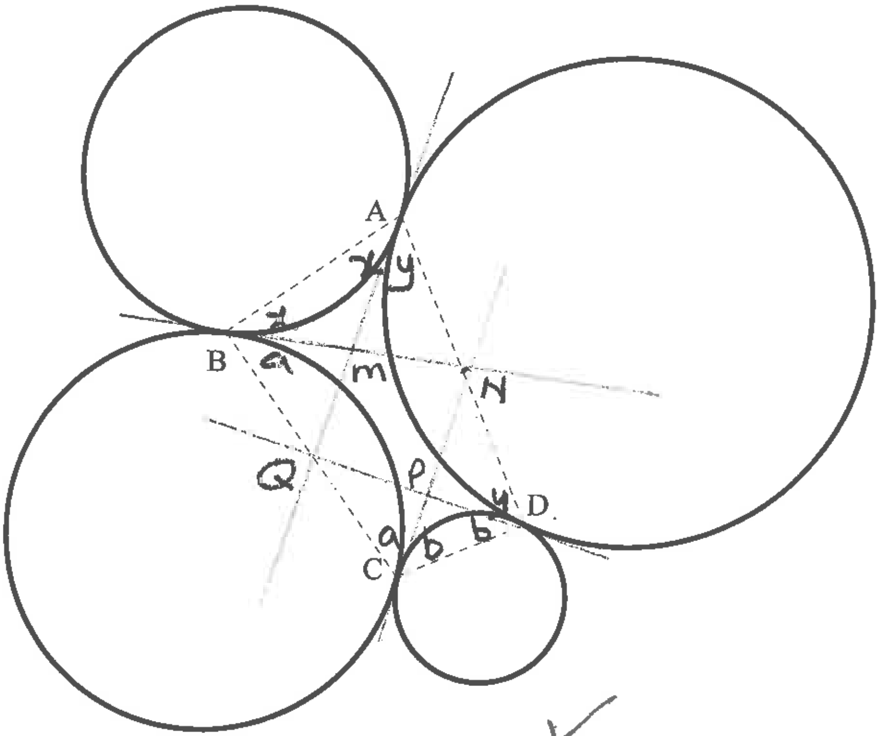
$\pi(17{,}5)(\sqrt{1712{,}5})$

$= 18\,200{,}91 \text{ m}^2$

[10]

QUESTION 12

Four circular coins of unequal sizes lie on a table so that each coin touches two, and only two, of the others. Prove that the four points of contact, ABCD are concyclic. (5)



Construct tangents and label points

MB = MA tan from same pt

NB = NC tan from same pt

PC = PD tan from same pt

AQ = QD tan from same pt

base angles of all triangles are equal ; = <'s ; = sides

label x ; y ; a ; b

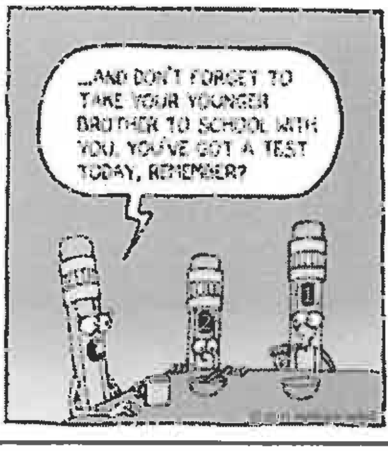
$2x + 2y + 2a + 2b = 360°$ <'s in quad

$x + y + a + b = 180°$

$\hat{A} + \hat{C} = x + y + a + b$

∴ ABCD is cyclic (opp <'s of quad suppl)

[5]



[Total Section B: 75 marks]

[Total: 150 marks]