ST. DAVID'S MARIST INANDA

MATHEMATICS
PRELIMINARY EXAMINATION
PAPER I
GRADE 12
6 September 2017

| | |
|---|---|
| EXAMINER: Mrs L. Black<br>MODERATOR: Mrs C. Kennedy | MARKS: 150<br>TIME: 3 hours |

NAME: Memo

HIGHLIGHT YOUR TEACHERS NAME:

| | | | | |
|---|---|---|---|---|
| C. KENNEDY | L. NAGY | L. VICENTE | L. BLACK | S. RICHARD |

INSTRUCTIONS:

- ✓ This paper consists of 25 pages and a separate Formula sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper and read each question carefully.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ It is in your interest to show all necessary working details.
- ✓ Work neatly. Do NOT answer in pencil.
- ✓ Diagrams are not drawn to scale.

| SECTION A | Q1 | Q2 | Q3 | Q4 | Q5 | | TOTAL |
|---|---|---|---|---|---|---|---|
| MARKS | 24 | 8 | 11 | 16 | 16 | | 75 |
| | Ch | | | | | | |
| SECTION B | Q6 | Q7 | Q8 | Q9 | Q10 | | TOTAL |
| MARKS | 19 | 25 | 10 | 17 | 4 | | 75 |
| | | | | | | | |

## QUESTION 1

a) Create a quadratic equation, in the form $ax^2 + bx + c = 0$, whose roots are $\frac{4}{3}$ and -3. (3) R

$(x+3)(3x-4)=0$

$3x^2+5x-12=0$

b) Solve for x in each of the following:

i) $2^{x+2}+2^x=20$ (2) R

$2^x.2^2+2^x=20$

$2^x(4+1)=20$

$2^x=4$

$x=2$

ii) $\sqrt{5-x}-x=1$ (4) R

$\sqrt{5-x}=1+x$

$5-x=x^2+2x+1$

$0=x^2+3x-4$

$0=(x+4)(x-1)$

$x \neq -4$ or $x=1$
N.V

b) For what values of x will $\sqrt{x^2-2x-8}$ be real? (4)

$x^2-2x-8 \geq 0$ ✓

$(x-4)(x+2) \geq 0$ ✓

$+ \quad - \quad +$

$-2 \quad 4$

$x \leq -2$ ✓ or $x \geq 4$ ✓

R

c) For the equation $3x^2+px+12=0$ determine:

i) the roots of the equation in terms of p. (2)

R

$$x = \frac{-p \pm \sqrt{p^2-4(3)(12)}}{2(3)} \quad \checkmark\text{sub}$$

$$= \frac{-p \pm \sqrt{p^2-144}}{6} \quad \checkmark$$

ii) the value(s) of p for which the roots will be equal: (2)

R

$p^2-144=0$ ✓

$p^2=144$

$p=\pm 12$ ✓

d) Solve the following simultaneously (7) R

$y+7=2x$

$x^2-xy+3y^2=15$

$y = 2x-7$ ✓

✓sub

$x^2 - x(2x-7) + 3(2x-7)^2 = 15$

$x^2 - 2x^2 + 7x + 3(4x^2 - 28x + 49) = 15$

$x^2 - 2x^2 + 7x + 12x^2 - 84x + 147 = 15$

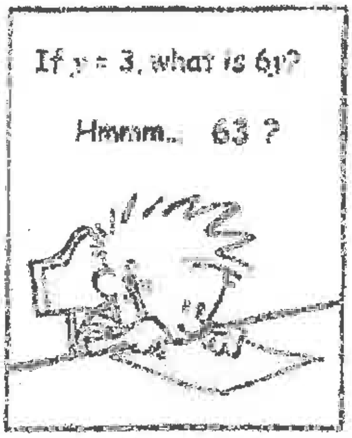
$11x^2 - 77x + 132 = 0$ ✓

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 4$ ✓ or $x = 3$ ✓

$y = 1$ ✓ $y = -1$ ✓




[24]

**QUESTION 2**

Thomas plans to start a fitness programme by going for a run each Saturday. On the first Saturday he runs 1km and plans to increase the distance by 750m each week. When Thomas reaches 10km, he will continue to run 10km each Saturday thereafter.

a) Calculate the distance that Thomas will run on the 9$^{th}$ Saturday. (2) R

$1;\ 1{,}750;\ 2{,}5 \ldots$

$\therefore T_n = 1 + (n-1)(0{,}75)$

$T_9 = 1 + 8(0{,}75)$ ✓

$= 7\text{ km}$ ✓

b) Determine on which Saturday Thomas will first run 10km. (3) R

$10 = 1 + (n-1)(0{,}75)$ ✓

$9 = 0{,}75n - 0{,}75$

$9{,}75 = 0{,}75n$ ✓

$n = 13$ ✓

c) Calculate the total distance that Thomas would have run over the first 24 Saturdays. (3) C

$S_{13} = \frac{13}{2}\left(2(1) + 12(0{,}75)\right)$

$= 71{,}5$ ✓

next 11 sats he runs 10km each.

$\therefore 11 \times 10 = 110\text{ km}$ ✓

$\therefore 110 + 71{,}5 = 181{,}5\text{ km}$ ✓

**[8]**

## QUESTION 3

Mr Vicente took a home loan of R850 000 to buy a house and was required to pay monthly instalments for 30 years. The bank offered him a loan at an effective interest rate of 8,3%.

a) Determine the nominal interest rate, compounded monthly, that he is required to pay. (3) R

$$1 + 0{,}083 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$$

$$1{,}006\ldots = 1 + \frac{i_{nom}}{12}$$

$$i_{nom} = 0{,}08$$

$$= 8\%$$

b) Using the nominal interest, show that his monthly instalment was R6 237. (3) C

$$850\,000 = \frac{x\left[1 - \left(1 + \frac{0{,}08}{12}\right)^{-360}\right]}{\frac{0{,}08}{12}}$$

$P_v$

$$x = R6\ 237$$

c) Calculate the outstanding balance on his loan at the end of the first year. (3) C

$$P_v = \frac{6\ 237\left[1 - \left(1 + \frac{0{,}08}{12}\right)^{-348}\right]}{\frac{0{,}08}{12}}$$

$$= R842899{,}56$$

(−1 rounding)

d) Hence, calculate how much of the R74 844 that he paid during the first year, was taken by the finance company as payment towards the interest charged. (2)

C

$6237 \times 12 = R74\ 844$

$850\ 000 - 842\ 899{,}56 = R7100{,}44$

$74\ 844 - 7100{,}44 = R67\ 743{,}56$


[11]

## QUESTION 4

a) Given: $p(x) = -3x^2$

Determine the equation of the inverse of p stating its domain and range. (5)

$y = -3x^2$

$x = -3y^2$

$\pm\sqrt{\frac{x}{-3}} = y$

Domain: $x \leq 0$

Range: $y \in \mathbb{R}$

R

b) Given: $f(x) = 2^x$, $g(x) = f(x-2)$ and $h(x) = f^{-1}(x)$

i) Write down the equations of g and h in the form y =... (2)

$y = 2^{x-2}$

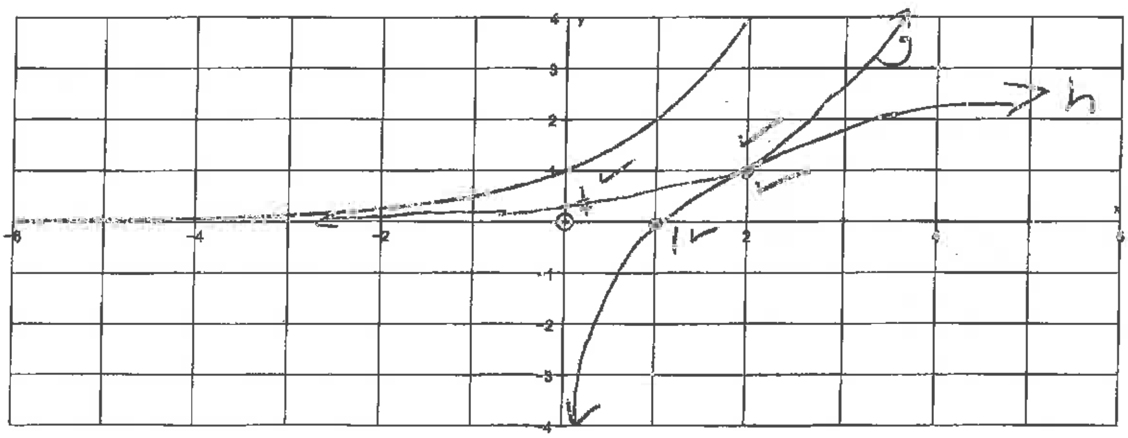
$y = 2^x$

$x = 2^y$ $y = \log_2 x$

R

ii) On the set of axes below, where f is already drawn for you, sketch the graphs for g and h. Label all the intercepts with the axes. (4)

R



iii) Using your graph, solve for x if $g(x) = h(x)$ (1) R

$x = 2$ ✓

c) Given:

| | |
|---|---|
| A | A hyperbola passing through ( 4; 4). |
| B | A circle (centre the origin) with radius 4 units. |
| C | A parabola turning at the origin and passing through ( 4; 4) |
| D | An exponential graph with a horizontal asymptote of y=1 and passing through point ( 0; 2) |

Choose one equation from the following table that matches each description above:

| | |
|---|---|
| P | $y = 2.3^x + 1$ |
| Q | $4y = x^2$ |
| R | $xy = 16$ |
| S | $\frac{x^2}{2} + \frac{y^2}{2} = 8$ |
| T | $y = \frac{4}{x}$ |
| U | $y = 4x^2$ |
| V | $x^2 + y^2 = 4$ |
| W | $x = \log_2(y - 1)$ |

Fill in your answers below: (4) R

A = ......R......

B = ......S......

C = ......Q......

D = ......W......

[16]

**QUESTION 5**

a) Given: $f(x) = 3x^2 - 4$

i) Determine $f'(x)$ from first principles. (5) R

$f(x+h) = 3(x+h)^2 - 4$

$= 3(x^2 + 2xh + h^2) - 4$

$= 3x^2 + 6xh + 3h^2 - 4$ ✓

$\therefore f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 4 - 3x^2 + 4}{h}$ ✓

$= \lim_{h \to 0} \dfrac{6xh + 3h^2}{h}$ ✓

$= \lim_{h \to 0} \dfrac{h(6x + 3h)}{h}$ ✓

$= \lim_{h \to 0} 6x + 3h$

$= 6x$ ✓

ii) $A(x;23)$, where $x > 0$, and $B(-2,y)$, are points on the graph of f. Calculate the average gradient of f between A and B. (5)

R

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$23 = 3x^2 - 4$

$27 = 3x^2$

$9 = x^2$

$x = \pm 3$

$x = 3$

$A(3;23)$

$y = 3(-2)^2 - 4$

$= 8$

$B(-2;8)$

$$m = \frac{23-8}{3+2}$$

$= 3$

b) Determine two possible functions for $y = g(x)$ such that: $\frac{dy}{dx} = 5x$ (2) C

$y = \frac{5}{2}x^2 + c$ [c can be any constant]

c) Determine $h'(x)$ given $h(x) = \frac{3x^4 + 7x^2 - 5x}{2x^2}$
Leave your answer with positive exponents. (4) R

$h(x) = \frac{3x^4}{2x^2} + \frac{7x^2}{2x^2} - \frac{5x}{2x^2}$ ✓

$= \frac{3x^2}{2} + \frac{7}{2} - \frac{5x^{-1}}{2}$ ✓

$h'(x) = 3x + \frac{5}{2x^2}$ ✓ ✓

[16]

**SECTION B**

**QUESTION 6**

a) Consider the digits 1, 2, 3, 4, 5, 6, 7, and 8 and answer the following questions:

i) How many 2-digit numbers can be formed if repetition is allowed? (2) R

$$8^2 = 64$$

ii) How many 4-digit numbers can be formed if repetition is NOT allowed? (3) R

$$8 \times 7 \times 6 \times 5 = 1680$$

OR

$$\frac{8!}{4!} = 1680$$

iii) How many numbers between 4 000 and 5 000 can be formed when repetition is allowed? (3) C

$$\underline{1}\ \underline{8}\ \underline{8}\ \underline{8} = 512$$

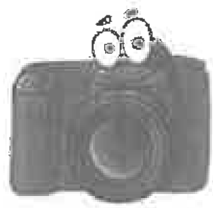
b) Three men (Johan, Clive and Steph) and 2 women (Suzette and Colleen) are to stand in a straight line to have their group photograph taken.

Determine the probability that Johan stands next to Suzette and Clive stands next to Colleen. (5) C

$\boxed{JS}$ — $\boxed{CC}$

$2!$ $\quad 2! \times 3! = 24$

$$\frac{24}{5!} = \frac{1}{5}$$

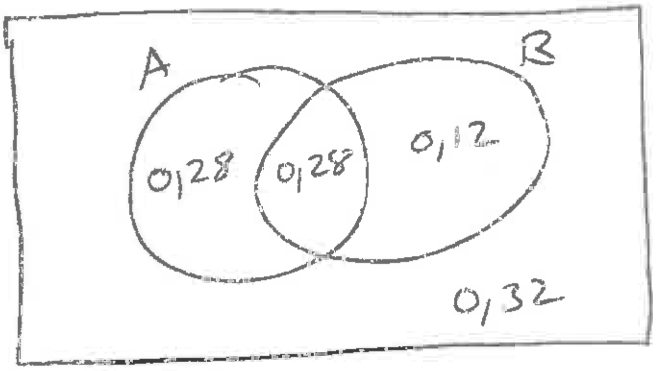


c) The probabilities for the events A and B are shown below:

$$P(A \cap B') = 0{,}28 \; , \; P(A' \cap B) = 0{,}12 \; , \; P(A \cup B) = 0{,}68$$

i) Draw a Venn diagram to illustrate the complete sample space for the events A and B. (4)

C




ii) Write down the value of P(*A*) and the value of P(*B*). (2)

R

$P(A) = 0{,}56$

$P(B) = 0{,}40$

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**QUESTION 7**

a) Given: $f(x) = 3x$, Determine the simplified expression for

$$f(x)+f\left(\frac{1}{x}\right)+\frac{1}{f(x)}+f^{-1}(x)$$ (5) C

$y = 3x$
$x = 3y$
$y = \frac{x}{3}$

$3x + \frac{3}{x} + \frac{1}{3x} + \frac{x}{3}$

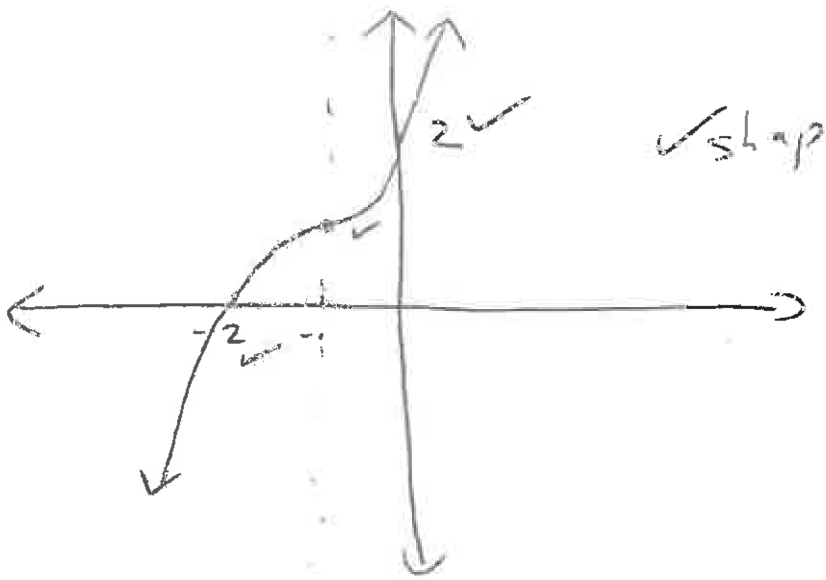
$\frac{9x^2 + 9 + 1 + x^2}{3x}$

$= \frac{10x^2+10}{3x}$

b) Given a function f that satisfies the following conditions:

$f(0) = 2, \quad f(-2) = 0, \quad f'(-1) = 0$
$f'(x) > 0 \quad for \quad x \neq -1$

Draw a rough sketch of a possible graph for f. (4) C




shape

c) Given $f(x) = x^3 + 3x^2 + x + 1$

i) Show that the tangent to the curve $y = f(x)$ at the point where $x = -2$ is $y = x + 5$. (5)

R

$f(-2) = (-2)^3 + 3(-2)^2 + (-2) + 1$

$= 3$ ✓

✓

$f'(x) = 3x^2 + 6x + 1$

$= 3(-2)^2 + 6(-2) + 1$

$= 1$ ✓

$3 = 1(-2) + c$ ✓

$5 = c$ ✓

$y = x + 5$

ii) Determine the x-coordinate of the point where this tangent intersects the curve again. (5)

R

$x+5=x^3+3x^2+x+1$

$0=x^3+3x^2-4$

~~$0=(x-1)$~~

$0=(-1)^3+3(-1)^2-4$

$=0$

$(x-1)(x^2+4x+4)=0$

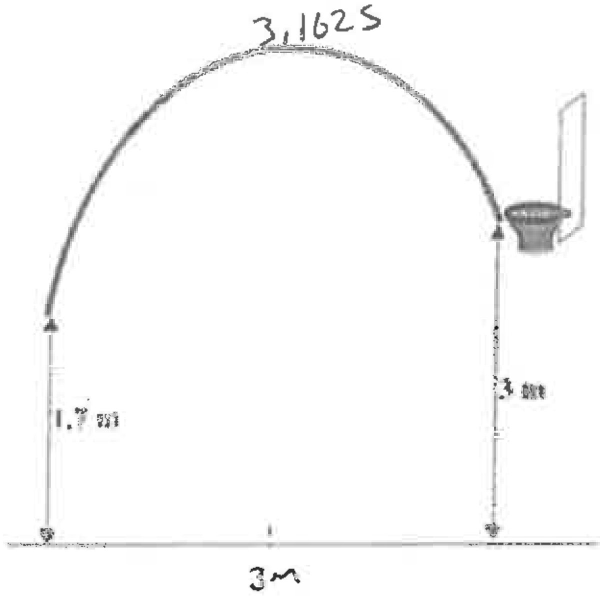
$(x-1)(x+2)(x+2)=0$

$x=1$ or $x=-2$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 0 & -4 \\ & \downarrow & -1 & -2 & 2 \\ \hline & 1 & 2 & -2 & \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & \downarrow & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

d) Braydon, an enthusiastic basketball player is practising his shooting.




He throws from a point 1,7m from the floor. Each throw follows the path of a parabola. On one of his throws, the ball reaches it's maximum height of 3,1625m when it has covered a horizontal distance of 3m. Unfortunately, the ball does not go into the basket but hits the front rim which is 3m above the floor.

Determine how far Braydon is from the rim, that is the horizontal distance between Braydon's hand and the front of the rim. (6) P.S

$y = a(x-p)^2 + q$ ✓

$y = a(x-3)^2 + 3,1625$

$1,7 = a(0-3)^2 + 3,1625$ ✓

$1,7 = 9a + 3,1625$

$= 9a$

$-1,4625 = 9a$

$a = -0,1625$ ✓

$y = -0,1625(x-3)^2 + 3,1625$

$\therefore 3 = -0{,}1625(x-3)^2 + 3{,}1625$ ✓

$-0{,}1625 = -0{,}1625(x-3)^2$

$1 = (x-3)^2$ ✓

$1 = x^2 - 6x + 9$

$0 = x^2 - 6x + 8$

$= (x-4)(x-2)$

$\therefore x = 4$ or $x \neq 2$

$\therefore 4\text{ m}$ ✓

**[25]**

**QUESTION 8**

A factory needs to make a closed cylinder container (with radius r and height h) to hold 10 cm³ of substance.

a) Show that the total surface area of metal needed for the container is:

$S = 2\pi r^2 + \frac{20}{r}$ (3)

C

$10 = \pi r^2 h$

$h = \frac{10}{\pi r^2}$ ✓

$S = 2\pi r^2 + 2\pi r h$ ✓

$= 2\pi r^2 + 2\pi r \left(\frac{10}{\pi r^2}\right)$ ✓

$= 2\pi r^2 + \frac{20}{r}$

b) As the special metal used is expensive, the manufacturers wish to minimise the surface area. Calculate the required radius and height in order to achieve this. (round off to 2 decimal places) (5)

C

$S = 2\pi r^2 + 20r^{-1}$

$S' = 4\pi r - \frac{20}{r^2}$ ✓ ✓

$0 = 4\pi r - \frac{20}{r^2}$ ✓

$\frac{20}{r^2} = 4\pi r$

$20 = 4\pi r^3$

$r^3 = \frac{5}{\pi}$

$r = 1{,}17$ ✓

$h = 2{,}34$ ✓

c) Calculate the minimum surface area. (round off to 2 decimal places.) (2)

R

$$SA = 2\pi(1{,}17)^2 + \frac{20}{1{,}17}$$

$$= 25{,}70\,cm^2$$

[10]

**QUESTION 9**

a) The sum of the first $n$ terms of a geometric sequence $9+6+4+\ldots$ is greater than 25. Calculate the smallest value of $n$. (6)

R

C

$$25 \le \frac{9\left(\left(\frac{2}{3}\right)^n - 1\right)}{\frac{2}{3} - 1}$$

~~$\frac{74}{3} \le 9\left(\frac{2}{3}^n - 1\right)$~~

~~$\frac{101}{27} < \frac{2^n}{3}$~~

$$-\frac{25}{3} \ge 9\left(\left(\frac{2}{3}\right)^n - 1\right)$$

$$\frac{2}{27} \ge \left(\frac{2}{3}\right)^n$$

$$\log_{\frac{2}{3}} \frac{2}{27} = n$$

$$n = 6{,}419 \quad \therefore n = 7$$

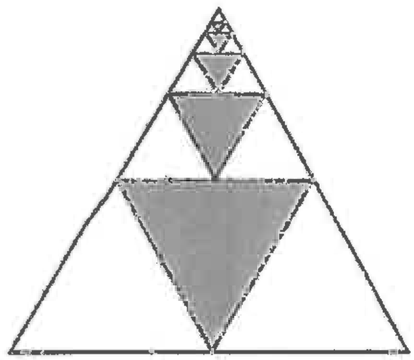
b) In the figure below, the largest triangle has an area of one square unit. The biggest grey triangle has an area of $\frac{1}{4}$ sq. units and each subsequent triangle's area is $\frac{1}{4}$ the size of the triangle bigger than it. These triangles continue indefinitely. Determine the area of the unshaded part of the triangle. (5)

C

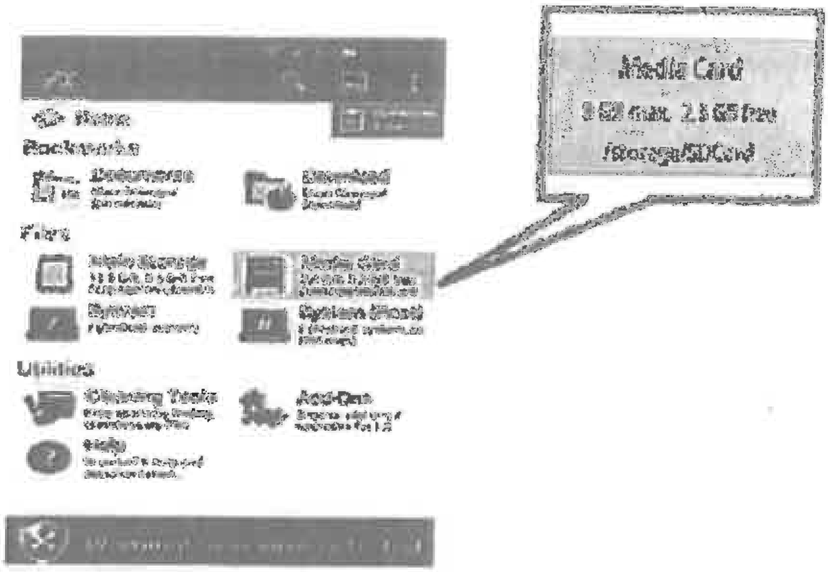
$$\frac{1}{4} \,;\, \left(\frac{1}{4} \times \frac{1}{4}\right) \,;\, \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)$$

$$1 - \frac{\frac{1}{4}}{1 - \frac{1}{4}}$$

$$= \frac{2}{3}$$



c) Nathi checks his phone at 07:00 am and sees the following on his screen.





He ignores the warning and leaves his phone unprotected.
The virus duplicates a game (25.02 MB in size) that he had downloaded from the internet onto his Media Card.

At 08:00 am it had pasted one copy of the game onto his media card.

At 09:00 am it had pasted two additional copies of the game onto his media card.

At 10:00 am it had pasted three additional copies and so on....

At what time, on the hour will the Media Card be full if this trend continues?

(note: 1GB = 1000MB) (6) P/S

Free = 2300MB

25,02 , 50,04 ; 75,06 ...

$$2300 = \frac{n}{2}\left[2(25{,}02) + (n-1)(25{,}02)\right]$$

$$4600 = n\left[50{,}04 + 25{,}02n - 25{,}02\right]$$

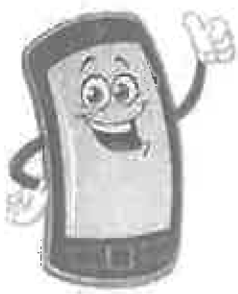
$$4600 = n\left[25{,}02n + 25{,}02\right]$$

$$0 = 25{,}02n^2 + 25{,}02n - 4600$$

$$n = \frac{-25{,}02 \pm \sqrt{(25{,}02)^2 - 4(25{,}02)(-4600)}}{2(25{,}02)}$$

$$n = 13{,}07 \quad \text{or} \quad n = -14{,}07$$

$\therefore$ 9pm



[17]

## QUESTION 10

For each of the 3 years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.

At the end of 2012 the populations of these two towns were equal.

Determine the ratio of the population of town X (call it $P_x$) to the population of town Y (Call it $P_y$) at the beginning of 2010. (4) P/S

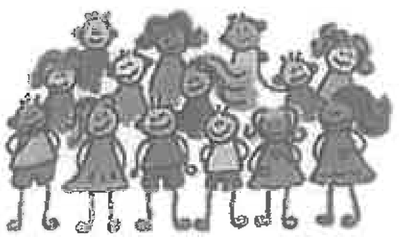
$X : x(1-0{,}08)^3$ ✓

$Y : y(1+0{,}12)^3$ ✓

$x(1-0{,}08)^3 = y(1+0{,}12)^3$ ✓

$\frac{x}{y} = \frac{(1+0{,}12)^3}{(1-0{,}08)^3}$ ✓

$= 1{,}8$



[4]