PAPER 1 MEMO TRIALS 2017

SECTION A

Q1 a) 1) $3^x = 10$

$x = \log_3 10$

$x = 2,095$

$x = 2,10$ ⟶

2) $(x-3)(x+5) = 9$

$x^2 + 2x - 15 - 9 = 0$

$x^2 + 2x - 24 = 0$

$x_1 = 4$ ⟶ $x_2 = -6$ ⟶

3) $9 \cdot 2^{x-1} = 2 \cdot 3^x$

$\frac{2^x}{3^x} \cdot 2^{-1} = \frac{2}{9}$

$\frac{2^x}{3^x} = \frac{2}{9} \cdot 2$

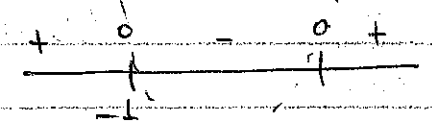
$\frac{2^x}{3^x} = \frac{2^2}{3^2}$

$x = 2$ ⟶

4) $2x^2 - 2 \leq 3x$

$2x^2 - 3x - 2 \leq 0$

C.V. $2 \; / -\frac{1}{2}$



$-\frac{1}{2} \quad 2$

$\therefore x \in [-\frac{1}{2}; 2]$ ⟶

b) 1) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

$= \lim_{x \to 3} (x+3)$

$= 6$ ⟶

2) $\lim_{x \to 3} \frac{x+3}{x(x-3)}$

does not exist ⟶

c) $f(x) = -4x^2$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{-4(x+h)^2 + 4x^2}{h}$

$= \lim_{h \to 0} \frac{-4(x^2 + 2xh + h^2) + 4x^2}{h}$

$= \lim_{h \to 0} \frac{-4x^2 - 8xh - 4h^2 + 4x^2}{h}$

$= \lim_{h \to 0} \frac{h(-8x - 4h)}{h}$

$= \lim_{h \to 0} (-8x - 4h)$

$= -8x$ ⟶

d) 1) $g'(t) = 2t - 3$ ⟶

2) $k(t) = t^{\frac{1}{2}} + 2t^{-\frac{1}{2}}$

$k'(t) = \frac{1}{2} t^{-\frac{1}{2}} - t^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{t}} - \frac{1}{\sqrt{t^3}}$

Q2

a) 1) $A = 150\,000(1-22\%)^5$

$= R43\,307,62$

2) ↑ Price

$A = 150\,000(1+19\%)^5$

$= R357\,953,05$

∴ Sinking fund would be $R357\,953,05 - 43\,307,62$

$= R314\,645,43$ →

b) 1) i monthly $\frac{12\%}{12} = 1\%$

2) $200\,000 = 1200\left[\frac{(1+0,01)^{n+1}-1}{0,01}\right]$

$\frac{8}{3} = (1+0,01)^{n+1}$

$n+1 = \log_{1,01}\frac{8}{3}$

$n+1 = 98,57\ldots$

$n = 94,57\ldots$

∴ 98 months of payments →

THERE ARE ALT METHODS.

Q3

a) 1) $\frac{24}{116}$ 0,21

2) $\frac{65}{116}$ 0,56

3) 0

4) $\frac{84}{116}$ 0,72.

b) POSSIBILITY

11 letters S x 2 "I" x 3

1) $\frac{11!}{2!3!} = 3326400$

2) Begin 3 I's

8 letters S x 2

ways $\frac{8!}{2!} = 20160$

Probability

$\frac{\frac{8!}{2!}}{\frac{11!}{2!3!}} = \frac{20160}{3326400} = \frac{1}{165}$ →

~~0,006...~~

~~0,01~~

SECTION B

Q4 a) 1) $a = 7 \quad d = 2\frac{1}{2}$

$T_{41} = a + 40d$

$= 7 + 40(2\frac{1}{2})$

$= 107 \rightarrow$

2) $S_{101} = \frac{101}{2}\left[2(7) + 100(2\frac{1}{2})\right]$

$= 13332 \rightarrow$

b) $a = 1 \quad r = \frac{3}{4}$

$S_{20} = \frac{a}{1-r}$

$= \frac{1}{1-\frac{3}{4}}$

$= 4 \rightarrow$

c) $\sum_{n=4}^{7} (-1)^{n+1} n^2$

$= (-1)^5(4)^2 + (-1)^6(5)^2$

$+ (-1)^7(6)^2 + (-1)^8(7)^2$

$= 22 \rightarrow$

Q5

a) 1) $S_1 = T_1 = 7 \rightarrow$

2) $T_2 = S_2 - S_1$

$= 11$

$\therefore d = 4 \rightarrow$

3) $T_4 = 7 + 3d$

$= 19 \rightarrow$

b) $a = 5 \quad ar^3 = 40$

$5r^3 = 40$

$r^3 = 8$

$r = 2$

$\therefore T_2 = 10 \rightarrow$

Q6 Sketch

EXAM NUMBER: _____

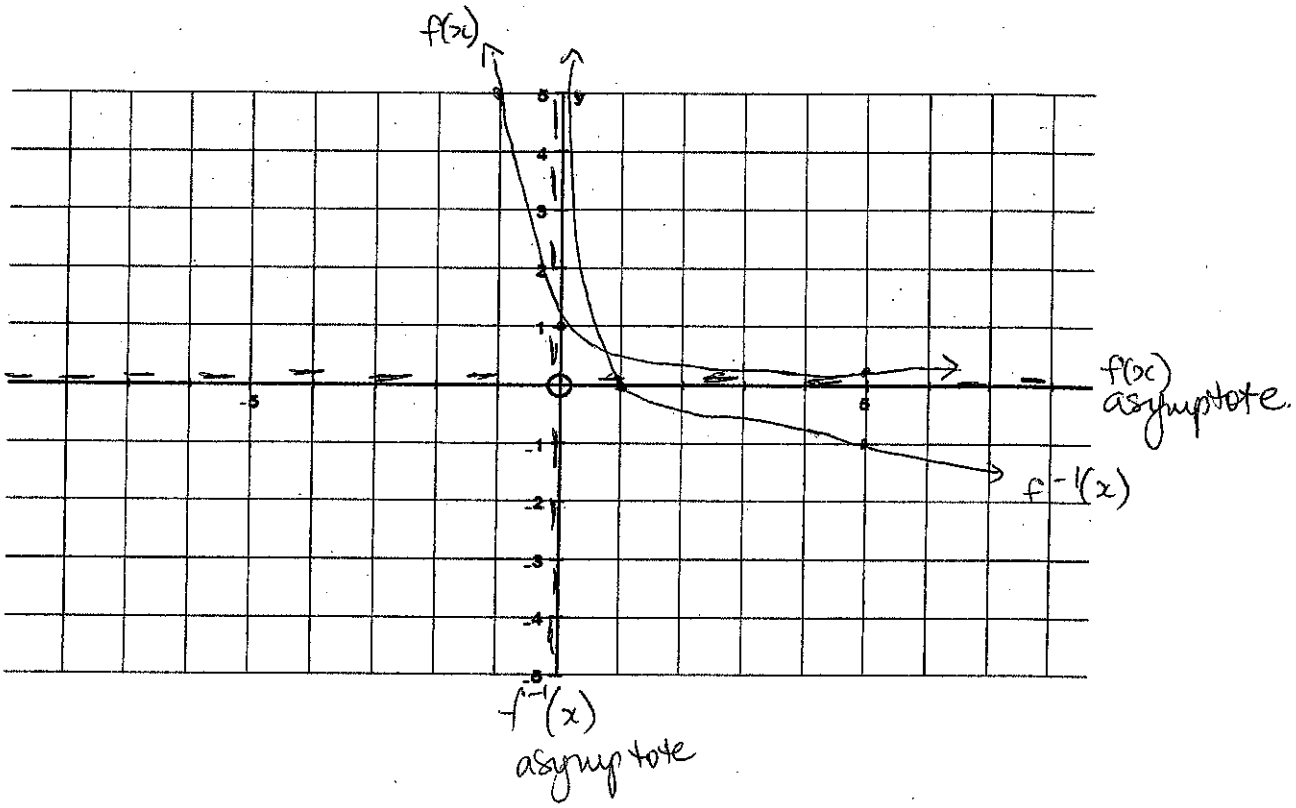
MATHS TEACHER: _____

HAND IN WITH SECTION B

Question 6

Given $f(x) = \left(\frac{1}{5}\right)^x$

Sketch the graphs of f and f^{-1} on the same system of axes. Label both graphs clearly. [4]



Q7

a) $y = 3(x-2)^2 - 3$

T.P. $(2; -3)$

Yint 9

Xint. $y = 3(x^2 - 4x + 4) - 3$

$= 3x^2 - 12x + 9$

$x_1 = 3 \quad x_2 = 1$

$\therefore$ (B)

b) i) ~~(D)~~ (C)

$2x^2 + 5x - 3 = 0$

$x_1 = \frac{1}{2} \quad x_2 = -3$

ii) x value T.P. $x = -\frac{5}{4}$

$\therefore$ Domain to make $f^{-1}(x)$ a function

$x \in \left(-\infty; -\frac{5}{4}\right]$

OR $x \in \left[-\frac{5}{4}; \infty\right)$

iii) $f'(x) > 0$

where $x \in \left(-\frac{5}{4}; \infty\right)$

iv) $f''(x) > 0$

for $x \in R$

c) $s = 5t^2 + vt$

$40 = 5t^2 + 10t$

$0 = 5t^2 + 10t - 40$

$t_1 = 2 \quad t_2 = -4$

$\therefore$ (E)

Q8

$f(x) = 3x^3 + 2x^2 + cx + d$

$f(2) = 0$

$3(8) + 2(4) + 2c + d = 0$

$32 + 2c + d = 0$

$d = -2c - 32$

$f(-3) = 0$

$3(-3)^3 + 2(-3)^2 + c(-3) + d = 0$

$-81 + 18 - 3c + d = 0$

$-63 - 3c + d = 0$

$d = 3c + 63$

$\therefore \quad -2c - 32 = 3c + 63$

$-95 = 5c$

$-19 = c$

$\therefore d = 6$

SECTION C

Q9 $D = 10\log\left(\frac{L}{10^{-16}}\right)$

$107 = 10\log\left(\frac{L}{10^{-16}}\right)$

$10{,}7 = \log\left(\frac{L}{10^{-16}}\right)$

$10^{10{,}7} = \frac{L}{10^{-16}}$

$10^{10{,}7} \times 10^{-16} = L$

$10^{-5{,}3} = L.$

OR 5×10^{-6}

Q10 $y = x^2 - 14x + 53$

1) tangent

$m = 2x - 14$

$m = 2(8) - 14$

$= 2$

$y = 2x + c$

(8, 5) $5 = 16 + c$

$-11 = c$

$y = 2x - 11$

2) $y = 2x - 11$ and $y = -x^2 + 10x - 27$

$-x^2 + 10x - 27 = 2x - 11$

$-x^2 + 8x - 16 = 0$

$(x-4)^2 = 0$

$x = 4$ one pt of contact

Q (4, −3)

OR $-2x + 10 = 2$ } this assumes tangent.

$8 - 2x$ } NOT CORRECT.

$4 = x$

Q11

a) 1) $ST = \sqrt{100 + 100}$ (Pythag)

$= \sqrt{200}$

$= 10\sqrt{2}$

2) $A = l \times b$

$= (10\sqrt{2} - 2x)\, x$

$= 10\sqrt{2}x - 2x^2$

3) $A'(x) = 0$ at max

$10\sqrt{2} - 4x = 0$

$10\sqrt{2} = 4x$

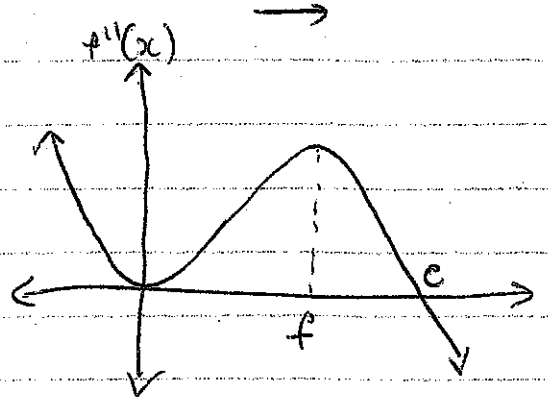
$\frac{10\sqrt{2}}{4} = x$

$x = \frac{5\sqrt{2}}{2}$ m

$\therefore$ length $= 10\sqrt{2} - 5\sqrt{2}$

$= 5\sqrt{2}$ m

b)



Q2

a) 1) 6 cubes high

cubes $= 6 + 4(5+4+3+2+1)$

$= \underline{66} \longrightarrow$

2) 12 high

cubes $= 12 + 4(11+10\ldots\ldots)$

$= 12 + 4(66)$

$= \underline{276} \longrightarrow$

3) x high

cubes $= x + 4(x-1 + x-2 \ldots\ldots)$

$S_n = \frac{x-1}{2}\left[2(x-1) + (x-2)(-1)\right]$

$= \frac{x-1}{2}\left[2x-2-x+2\right]$

$= \left[\frac{x-1}{2}\right](x)$

$\therefore$ cubes $= x + 4\left(\frac{(x-1)(x)}{2}\right)$

$= x + 2x(x-1)$

$= 2x^2 + x - 2x$

$= \underline{2x^2 - x} \longrightarrow$

ALT. a) 1) $\frac{6}{2}\left[1(2) + (6-1)4\right] = 66$

2) $\frac{12}{2}\left[1(2) + (12-1)4\right] = 276$

3) $\frac{x}{2}\left[1(2) + (x-1)4\right]$

$= \frac{x}{2}\left[2+4x-4\right] = \frac{x}{2}\left[-2+4x\right] = \underline{-x+2x^2} \longrightarrow$

(b) $f(x) = x^2 - x + 10$

$g(x) = 5 - x$

$h(x) = \log_2 x$

1) $h(f(x)) = \log_2 (x^2 - x + 10)$

2) $h(g(x)) = \log_2 (5-x)$

3) $\log_2 (x^2-x+10) - \log_2 (5-x) = 3$

$\log_2 \frac{x^2-x+10}{5-x} = 3$

$\frac{x^2-x+10}{5-x} = 2^3$

$x^2 - x + 10 = 8(5-x)$

$x^2 - x + 10 = 40 - 8x$

$x^2 + 7x - 30 = 0$

$x_1 = 3 \longrightarrow \quad x_2 = -10 \longrightarrow$

c) 1) x int

$a(x+1)(x-p) = 0$

$a(x^2 - px + x - p) = 0$

$-ap = p$ (y int)

$\therefore a = -1$

$\therefore -(x^2 - (p-1)x - p) = y$

$-x^2 + (p-1)x + p = y$

2) $y = x + p$ tangent p?

$m_{tangent} = 1$

$m_{curve} = p - 1 - 2x$

$1 = p - 1 - 2x$

$2 + 2x = p$

$\therefore$ tangent

$y = x + 2 + 2x$

When $x = 0$ $y = p$

$p = 0 + 2 + 2(0)$

$p = 2$

$\therefore$ when $p = 2$

$y = x + 2$ will be a tangent to

$y = -x^2 + x + 2$.

ALT. $1 = p - 1 - 2x$

$p = 2 + 2x$

~~$2 + 2x = x + 2$~~

$x + p = p + (p-1)x - x^2$

$0 = px - x - x^2$

$\frac{x^2 + 2x}{x} = p$

$x + 2 = p$

$\therefore 2 + 2x = x + 2$

$x = 0$

$\therefore p = 0 + 2 = 2$