ST MARY'S DSG, KLOOF

GRADE: 12   12 SEPTEMBER 2017

MATHEMATICS PAPER 2

TIME: 3 HOURS   TOTAL: 150 MARKS

ASSESSOR: S Drew   MODERATORS: J van Rooyen
   E Robertson

EXAMINATION NUMBER: _____TEACHER:_____

INSTRUCTIONS:

1.   This question paper consists of **26 typed pages** and 20 questions. There are also 4
         blank pages.

2.   All answers must be written on the question paper.

3.   There are **4 sections.**

4.   **Write your number and Maths teacher's name** on the top of each section.

5.   A formula sheet has been provided.

6.   Diagrams are not drawn to scale.

7.   Please give all answers correct to **2 decimal places** unless otherwise indicated.

8.   Read all the questions carefully.

9.   An approved non-programmable and non-graphical calculator may be used,
         unless otherwise specified.

10.   Make sure that your calculator is in <u>degree</u> mode.

SECTION A   (24 MARKS)

QUESTION 1

In the diagram below, PQ is a line segment through T with P (–6 ; –8) and Q (8 ; 2) as the end points. M is the midpoint of PQ. A (a ; 3) and B are the end points of the line segment AB which passes through M and O.
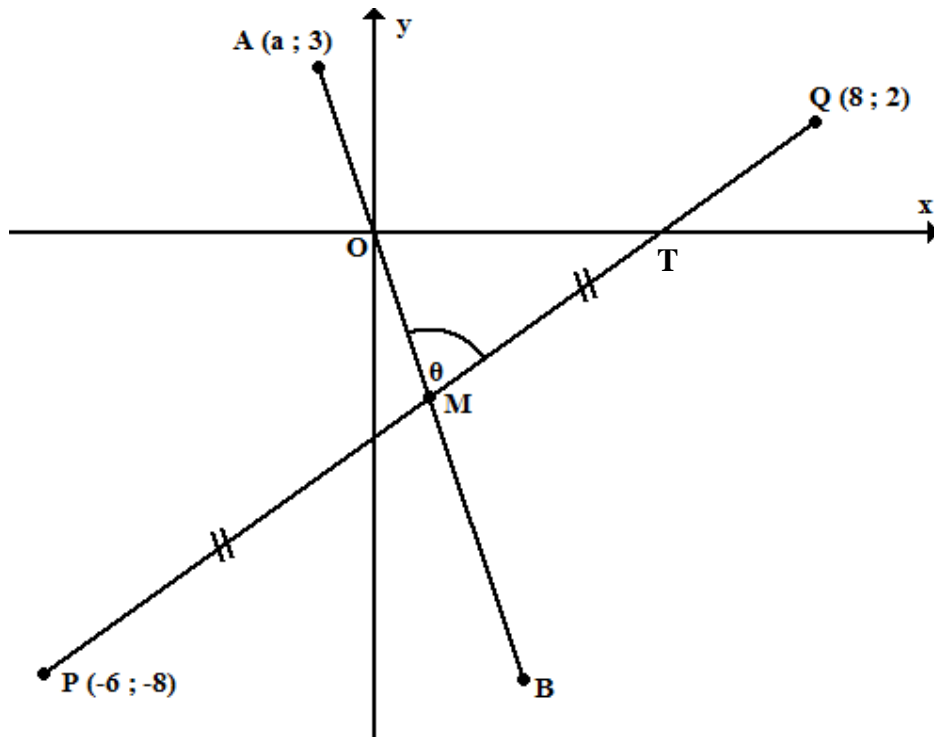
(a)   Determine the co–ordinates of M, the midpoint of PQ.   (2)

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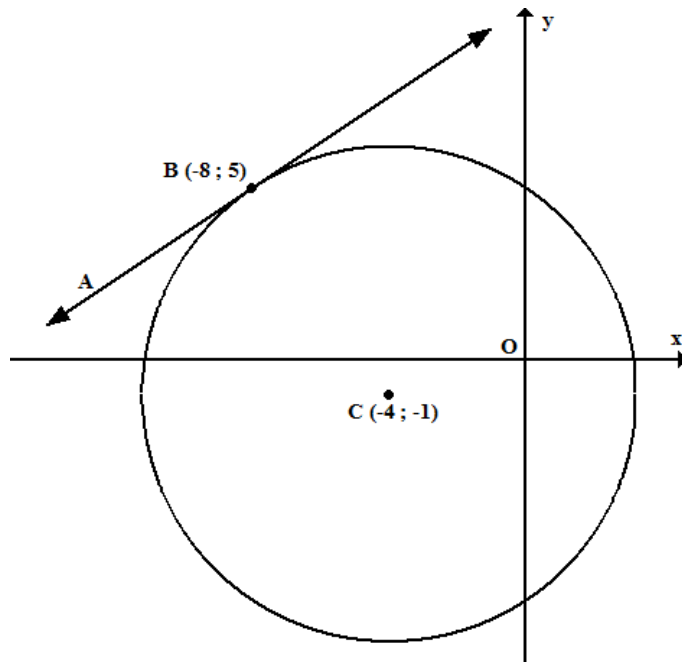(b)   Determine the equation of the line segment AB, that passes through the origin.   (2)

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(c)   Given that A is the point (a ; 3), determine the value of a.   (2)

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(d) (1) Determine the inclination of line PQ.   (2)

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    (2) Hence, calculate the size of θ.   (3)

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QUESTION 2

In the diagram below, C (–4 ; –1) is the centre of the circle. AB is a tangent to the circle with B (–8 ; 5) the point of contact.

(a) Determine the equation of the circle.   (3)

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(b) Determine the equation of AB, the tangent to the circle, in the form y = …..   (4)

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(c)   A second circle centre M, which is not drawn, has the equation:

$$x^2 - 4x + y^2 - 14y = -49$$

Determine whether this circle and the circle in (a) intersect each other.

Show all working.   (6)

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SECTION B   (38 MARKS)

QUESTION 3

A biologist is studying the behaviour of bees in a hive.
Once a bee has located a source of food, it returns to the hive and
performs a dance to indicate to the other bees how far away the source
of the food is. The dance consists of a series of wiggles.
The biologist records the distance, d metres, of the food source from
the hive and the average number of wiggles, w, in the dance.
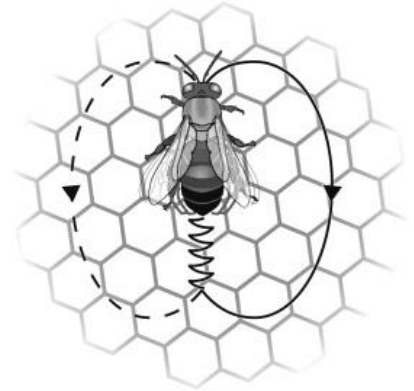
Distance, d	30	50	80	100	150	400	500	650
Average number of wiggles, w	0,725	1,210	1,775	2,250	3,518	6,382	8,185	9,555

Give all answers correct to 3 decimal places.

(a)   Give the equation for the line of best fit in the form $w = a + bd$.   (2)

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(b)   Calculate the correlation coefficient for the above data and then comment on the

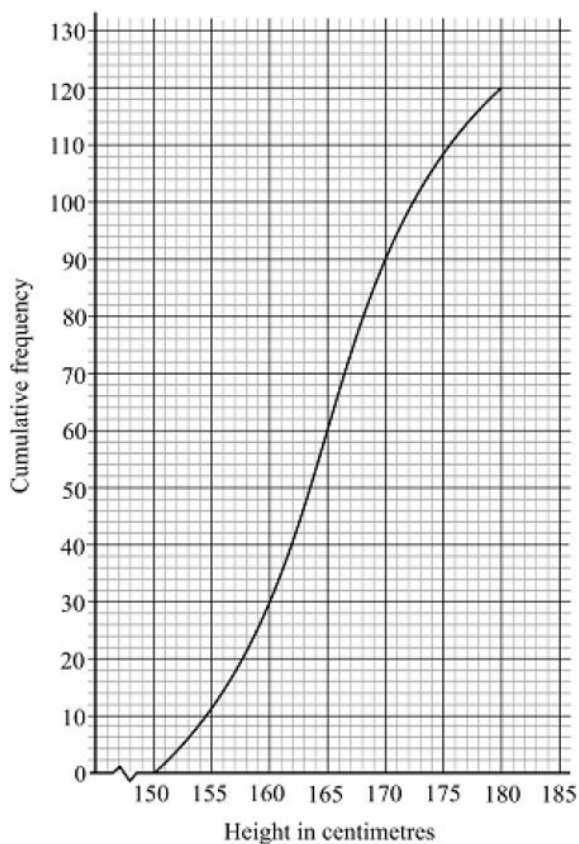strength of the relationship   (2)

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(c)   A new source of food is located 350 m from the hive.

(1)   Use your regression equation to estimate the average number of wiggles

in the corresponding dance.   (1)

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(2)   Comment on the reliability of your estimate.   (1)

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QUESTION 4

The cumulative frequency curve below shows the heights of 120 boys.

Height in centimetres

Use the graph to answer the questions that follow:

(a)   How many boys have heights between 160 and 165cm? _____   (1)

(b)   Determine the median height of the boys. _____   (1)

(c)   Determine the interquartile range.   (2)

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(d)   Given that 10% of the boys are taller than p cm. Find the value of p   .   (2)

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(e) (1) Use the curve to complete the following frequency table.   (3)

h (cm)	$150 \leq h \leq 155$	$155 < h \leq 160$	$160 < h \leq 165$	$165 < h \leq 170$	$170 < h \leq 175$	$175 < h \leq 180$
Midpoint	152,5	157,5	162,5	167,5	172,5	177,5
Frequency	11	19				12

     (2) Hence, calculate the mean height of the boys correct to two decimal places.   (2)

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(f)   10 more boys are measured and their heights are found to be less than 150 cm.
        Without further calculation describe what effect the addition of this new data will
        have on each of the following:

     (1)   the mean   (1)

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     (2)   the standard deviation   (1)

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QUESTION 5

In the diagram, AC is a diameter of the circle with centre O. AC and chord BD intersect at E. AB, BC and AD are also chords of the circle. OD is joined. BE = DE and $\hat{C} = 33°$.
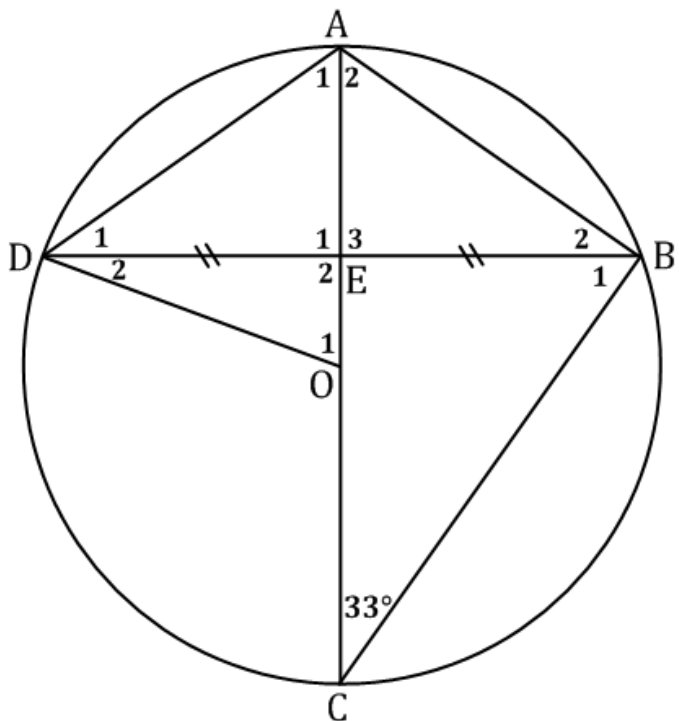
(a)   Calculate, with reasons, the size of:

(1) $\hat{A}_2$ _____
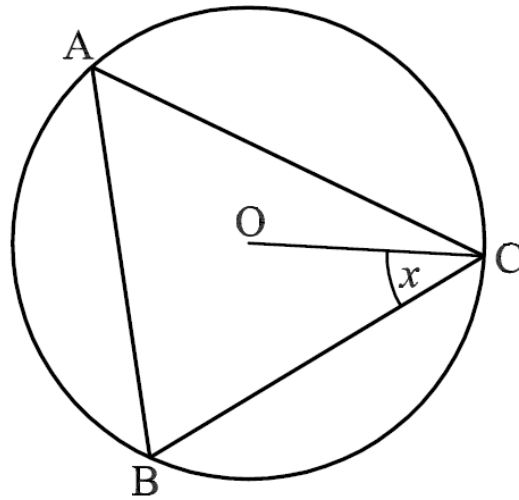
   _____

(2)

(2) $\hat{D}_1$ _____

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(1)

(b)   Show that AC bisects $D\hat{A}B$

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(2)

[5]

QUESTION 6

(a) O is the centre of the circle passing through points A, B and C.
$\hat{OCB} = x$.

Determine with reasons, the size of $\hat{A}$ in terms of x.   (4)

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(b)   In the diagram, ABCD is a cyclic quadrilateral in circle centre O. AB // DC, straight lines.
       BC and AD meet at M.   Let $\hat{A} = x$
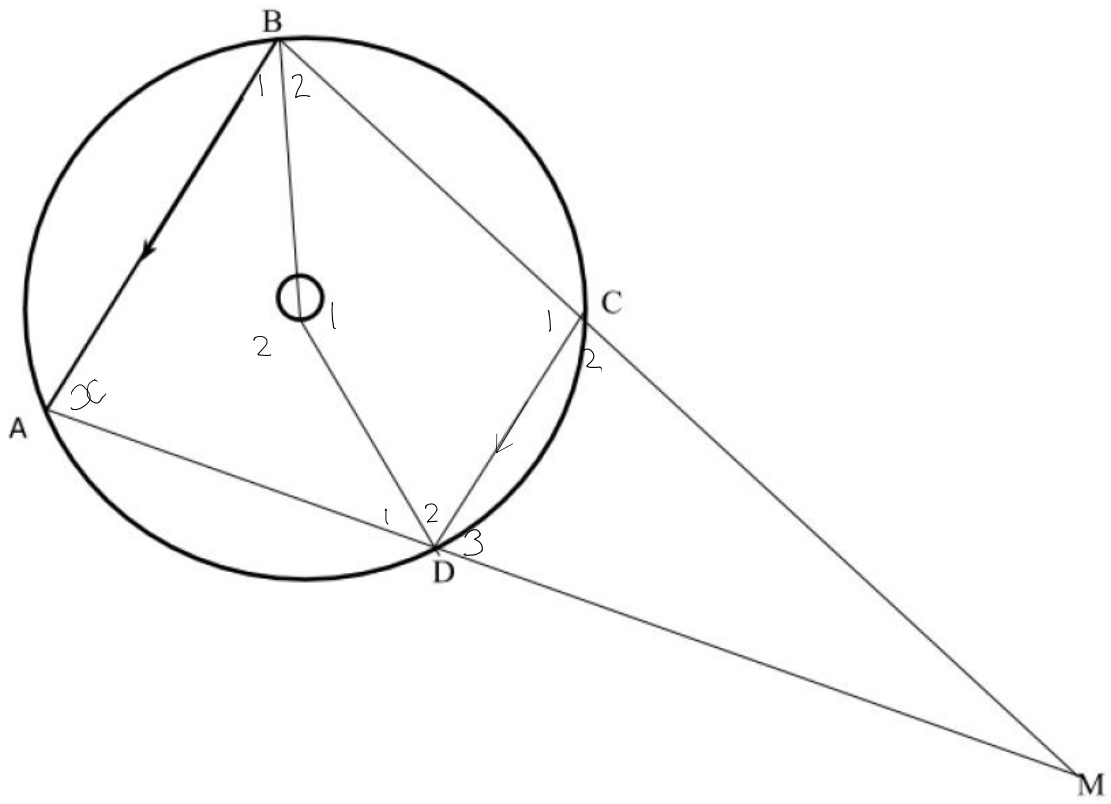
Prove that BODM is a cyclic quadrilateral.   (5)

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QUESTION 7

In the diagram below, two circles intersect at A and B (8;4) as shown. The centres of the circles lie on the line $x = 6$. The tangents at B to the circles are perpendicular to one another and pass through the centres of the circles. N (6 ; 3) is the centre of the smaller circle.

Determine the co-ordinates of M, the centre of the bigger circle.

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SECTION C (48 MARKS)

QUESTION 8

Given: $\sin A = \dfrac{1}{\sqrt{10}}$ where $\hat{A}$ lies in the first quadrant.

$\cos B = \dfrac{-1}{2}$ where $\hat{B}$ lies in the second quadrant.

Determine, without a calculator the value of $\sin (A - B)$

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[6]

QUESTION 9

In each of the following, select the correct answer and ring the letter corresponding to the correct answer.

(a) What is the amplitude of the graph of the equation $y = 2\sin\dfrac{1}{2}\theta$?

   (A) $\dfrac{1}{2}$ (B) 2   (C) $180°$ (D) $360°$ (1)

(b) A sound wave is modelled by the curve $y = \sin(4x - 15°)$. What is the period of the sound wave?

   (A) $105°$ (B) $75°$ (C) $90°$ (D) $4°$ (1)

[2]

QUESTION 10

This sketch shows the graphs of $f(x) = \sin(x + 45^o)$ and $g(x)$ both for $x \in [-90^o; 135^o]$:
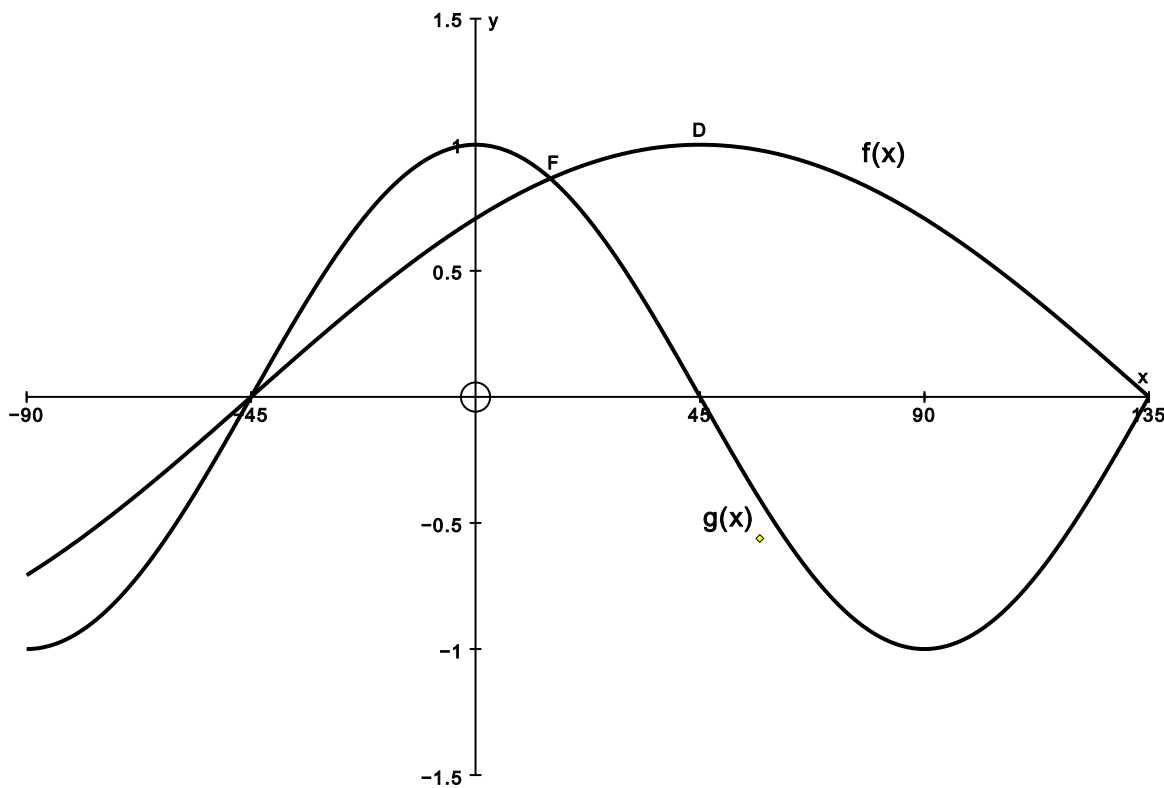
(a)   Write down the equation of *g(x)* .   (1)

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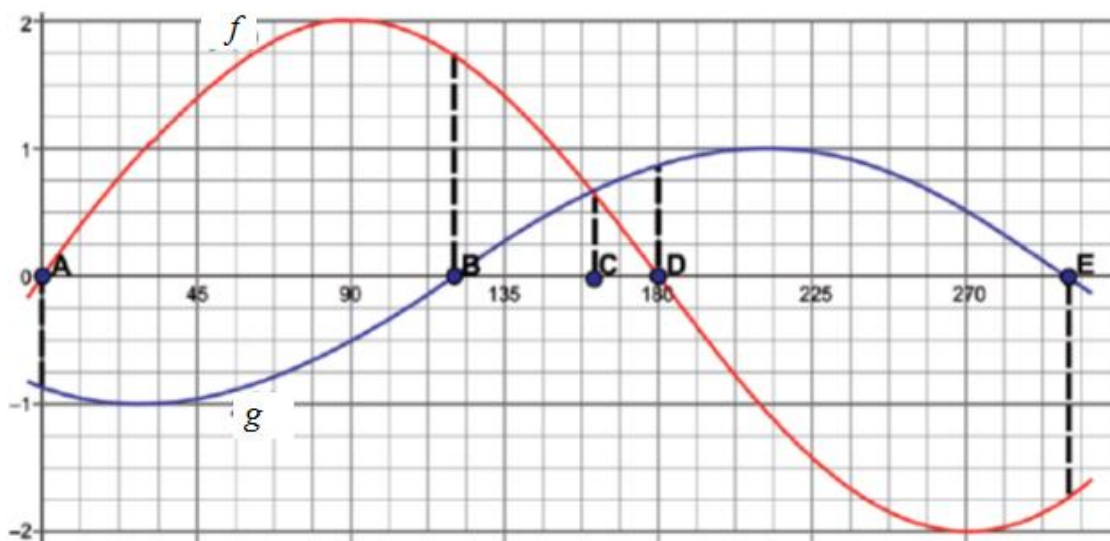(b)   Determine h if $h(x) = f(x - 90^\circ)$ and moved down 2 units.   (2)

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(c)   For which values of *x* is $f'(x).g'(x) > 0$ for $x \in [0^o; 135^o]$?   (3)

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QUESTION 11

Study graphs f and g below and then answer the questions that follow:

In each of the following, select the correct answer and ring the letter corresponding to the correct answer.

(a) Which one of the options below describes the solution/s for $g(x) = 0,75$ for the interval $x \in (0° ; 315°)$

    (A) $x = 67,5°$ (B) $x = 168,75°$ and $x = 253,13°$

    (C) $x = 22,5°$ and $x = 157,5°$ (D) $x = 202,5°$ (2)

(b) Which one of the options below describes the region for the values of *x* where
     $f(x) \le g(x)$ *and* $f(x).g(x) \le 0$?

    (A) $A \le x \le B$ (B) $B \le x \le C$

    (C) $C \le x \le D$ (D) $D \le x \le E$

                                                                              (2)

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QUESTION 12

(a) Prove that: $\dfrac{\cos^4 x + \sin^2 x \cos^2 x}{1 - \sin x} = 1 + \sin x$ (4)

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(b) If $\cos^2 12° - \sin^2 12° = m$, express each of the following in terms of m.

(1) $\cos 24°$ (1)

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(2) $\dfrac{\sqrt{3}}{2} . \cos 6° + \dfrac{1}{2} . \sin 6°$ (4)

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(c) Determine the minimum value of : $3 - 4\sin x \cos x$ (4)

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(d)   Simplify without the use of a calculator.

$$\frac{\cos(180°+x)\,.\,\sin 74°}{\sin 37°\,\sin 53°.\sin(x-90°)}$$ (4)

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[17]

QUESTION 13

(a)   Study the following true statements and then answer the questions that follow.

$$\frac{1 - \cos 10°}{1 + \cos 10°} = \tan 5° \times \tan 185°$$

$$\frac{1 - \cos 20°}{1 + \cos 20°} = \tan 10° \times \tan 190°$$

$$\frac{1 - \cos 30°}{1 + \cos 30°} = \tan 15° \times \tan 195°$$

$$\frac{1 - \cos 80°}{1 + \cos 80°} = \tan 40° \times \tan 220°$$

$$\frac{1 - \cos 100°}{1 + \cos 100°} = \tan 50° \times \tan 230°$$

(1)   Write down a possible value for P and a possible value for Q if

$$\frac{1 - \cos 60°}{1 + \cos 60°} = \tan P \times \tan Q$$

P =_____   Q = _____   (4)

(2)   Hence, complete the following statement in terms of A only and then write
       your answer as a single trig ratio.:
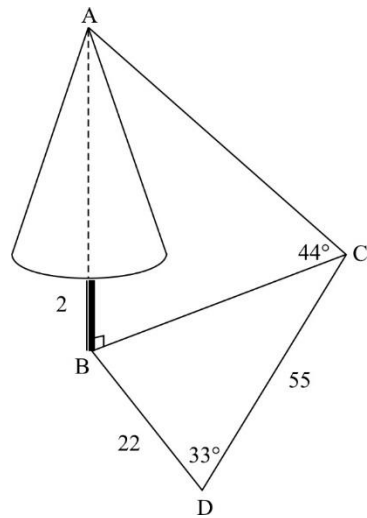
$$\frac{1 - \cos 2A}{1 + \cos 2A} = \tan (...............) \ \times \tan (................)$$

$$= _____$$ (2)

[6]

QUESTION 14

In the diagram below, a tall tree is roughly illustrated by a right cone. The centre of the circular base of the cone is 2 metres above a point, B, on the horizontal ground.

D and C are points in the same horizontal plane as B so that BD = 22 m, DC = 55 m and $B\hat{D}C = 33°$. The angle of elevation of the top of the tree, A, from C is 44°. Note that $C\hat{B}D$ is **not** a right angle!

(a)   Find the height AB of the tree.   (5)

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(b)   If the radius of the base of the cone is 2 m, find the **volume of the cone** depicting the foliage of the tree in the diagram above.   (2)

The volume of a cone is given by: $\dfrac{\text{base area} \times \text{height}}{3}$
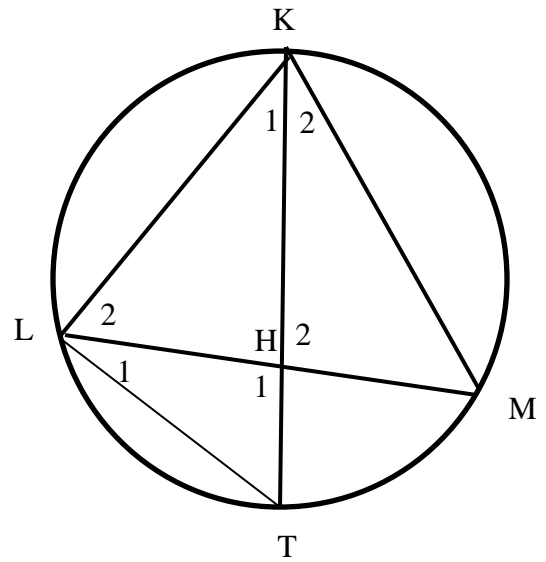
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SECTION D (40 MARKS)

QUESTION 15

In the diagram, KT bisects $L\hat{K}M$ so that $\hat{K}_1 = \hat{K}_2$.
 Note that H is **not the centre** of the circle.

(a)   Prove that TL is a tangent to the circle through L, H and K.   (3)

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(b)   Show that $TL^2 = TK.TH$ (4)

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QUESTION 16

In the figure below, $\triangle ABC$ has D and E on BC. BD = 10cm and DC = 15cm.
AT : TC = 2 : 1 and AD // TE
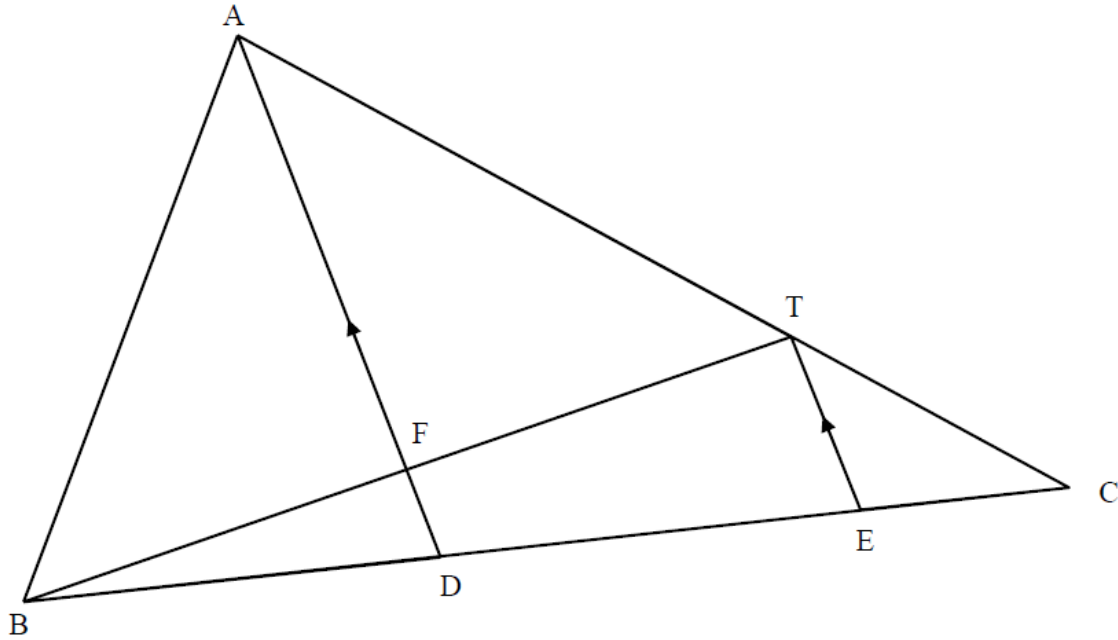
(a)   Write down the numerical value of $\dfrac{CE}{ED}$ (1)

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(b)   If D is the midpoint of BE and FD = 2,5 cm calculate TE.   (3)

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(c)   Calculate the value of $\dfrac{\text{Area } \triangle ADC}{\text{Area } \triangle ABD}$ (2)

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(d)   Calculate the value of $\dfrac{\text{Area } \triangle TEC}{\text{Area } \triangle ABC}$ (3)

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[9]

QUESTION 17

In the triangle below, $PQ \parallel BC$.

$AP = \sin x$, $PB = 2$, $AQ = 2\cos x$, $QC = 3$

Determine the values of x for which the diagram is valid and where $x \; \varepsilon \; [0° ; 360°]$

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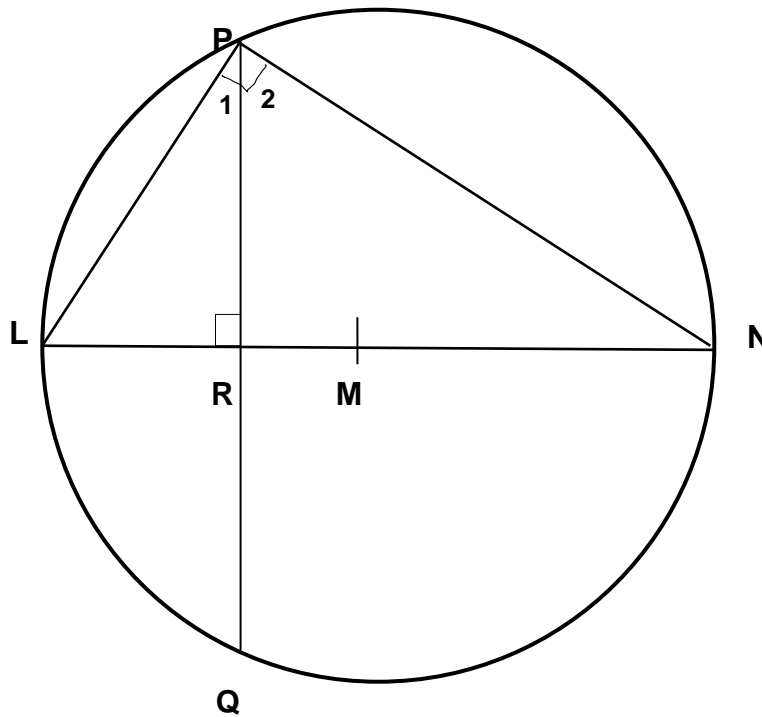_____

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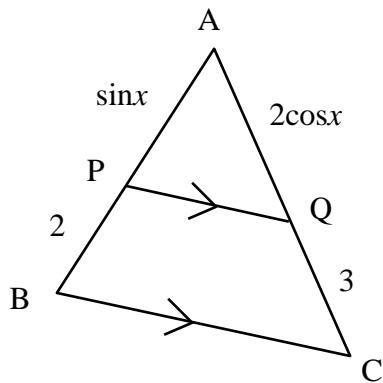
[6]

QUESTION 18

(a) <u>**Complete the theorem which states**</u>:

"The _____drawn from the vertex of the right angle of a

right angled triangle to the _____, divides the triangle into two

triangles that are _____

and to the_____" (2)

(b) Refer to the diagram. M is the centre of the circle.
Chord PQ intersects LN perpendicularly at R.

If LP = $\sqrt{6}$, RM = 2 units and LR = x, calculate the radius of the circle, centre M. (5)

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QUESTION 19

The diagram shows an equilateral triangle ABC with sides of length 6 cm.
P is the midpoint of AB and Q is the midpoint of AC.
APQ is a sector of a circle, centre A.
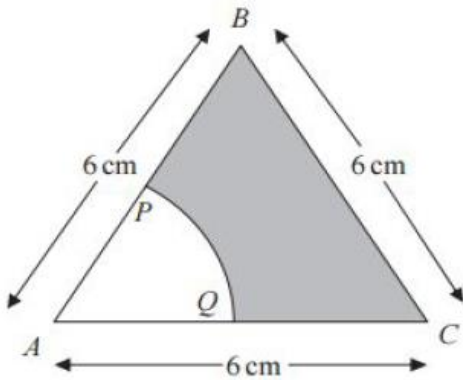Calculate the area of the shaded region.

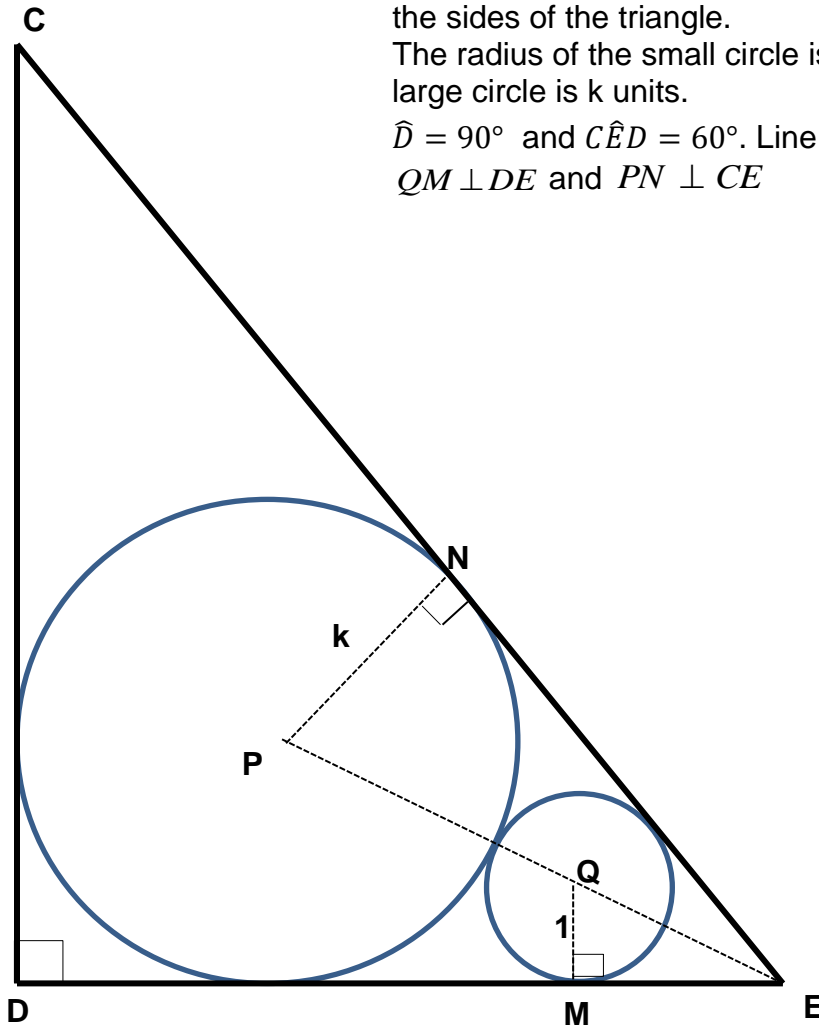Diagram **NOT**
accurately drawn

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[6]

In △ CDE, two circles, centres P and Q touch each other and the sides of the triangle.

The radius of the small circle is 1 unit and the radius of the large circle is k units.

$\hat{D} = 90°$ and $C\hat{E}D = 60°$. Line PQE bisects angle $C\hat{E}D$

$QM \perp DE$ and $PN \perp CE$

Determine the value of k, showing your method clearly.

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[5]