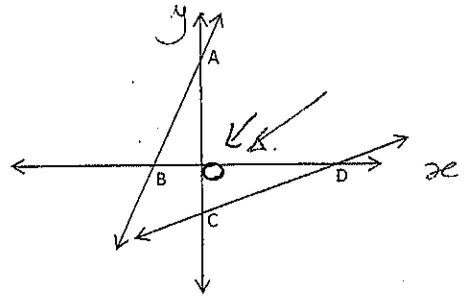
TREV. TRIALS (2017) P2.

SECTION A

QUESTION 1

AB is the line $2x - y + 6 = 0$ and CD is the line $x - 2y = 4$.



a) Write down the co-ordinates of A, B, C and D, the intercepts of the lines with the axes

$y = 2x + 6$ $\qquad$ $2y = x - 4$

$y = \frac{1}{2}x - 2$

$A(0; 6)$ $\qquad$ $C(0; -2)$

$x = 0$

$0 = 2x + 6$ $\qquad$ $y = 0$ $\qquad$ $x - 2(0) = 4$

$-6 = 2x$ $\qquad$ $x = 4$

$-3 = x$ $\qquad$ $D\ (4; 0)$

$B(-3; 0)$

6

(6) K

b) Write down the equation of the line that is perpendicular to AB and passes through A

$AB = \ y = 2x + 6$

New Eqn : $y = -\frac{1}{2}x + 6$

2

(2) K

c) Prove ΔOAB similar to ΔODC

In ΔOAB & ΔODC

$B\hat{O}A = D\hat{O}C$ (vert opp ∠s)

$A\hat{B}O = O\hat{C}D$ (both 63,4°)

$\therefore \Delta OAB \ ||| \ \Delta ODC$ (A.A.A)

$\tan A\hat{B}D = 2$

$A\hat{B}D = 63,4°$

$\tan O\hat{D}C = 0,5$

$O\hat{D}C = 26,6°$

$O\hat{C}D = 63,4°$

(∠s of Δ)

OR $\frac{OC}{OB} = \frac{2}{3}$ $\qquad$ $\frac{OD}{OA} = \frac{2}{3}$

$\therefore \Delta OAB \ ||| \ \Delta OCD$, prop sides.

(7) R.

d) Prove that ABCD is a cyclic quadrilateral

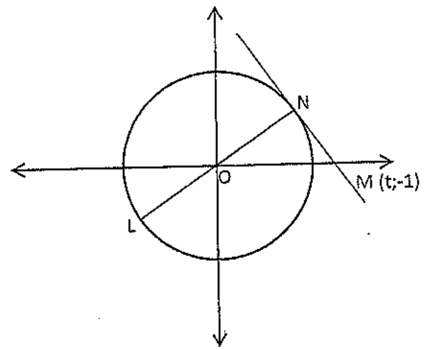
$A\hat{B}O = D\hat{C}O$ (proved)

$\therefore ABCD$ is cyclic (equal ∠s in same seg)

(2) R.

[17]

QUESTION 2



In the given diagram O (0 ; 0) is the centre of the circle. L (x ; y) and N (12 ; 5) are two points on the circle. LON is a straight line. The point M (t ; -1) lies on the tangent to the circle at N.

a) Write down the equation of the circle (2) K.

$(12)^2 + (5^2) = r^2$ $\qquad$ $x^2 + y^2 = 169$

$169 = r^2$

b) Determine the value of t

$m_{ON} = 5/12$

$M_{NM} = -12/5$ (rad ⊥ tang)

$\therefore \frac{5-(-1)}{12-t} = \frac{-12}{5}$

$30 = -12(12-t)$

$30 = -144 + 12t$

$174 = 12t$

$14{,}5 = t$

(5) R.

c) Another circle is defined by the equation $x^2+6x+y^2+4y=4$.

Rewrite this equation in the form $(x-a)^2+(y-b)^2=r^2$ and determine the value of $a+b+r$

$x^2+6x+9+y^2+4y+4=4+13$

$(x+3)^2+(y+2)^2=17$

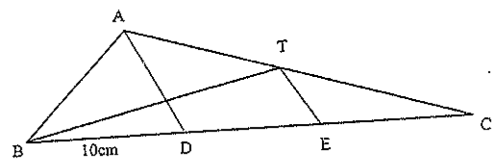
$a=-3 \quad b=-2 \quad r=\sqrt{17}$

$\therefore a+b+r = -5+\sqrt{17}$ OR $-0{,}88$

(5) C.

[12]

QUESTION 3



In the above diagram ΔABC has D and E on line BC so that AD ∥ TE

BD is 10 cm and DC = 15 cm and AT:TC = 2:1

a) Show that D is the midpoint of BE.

$\frac{DE}{EC} = \frac{2}{1}$ (prop int th)

$DE = 2/3\,(15)$

$= 10$ cm

$= BD$

(3) R

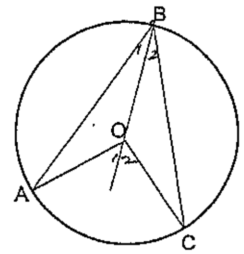
(b) Determine the value of $\frac{\text{Area } \Delta BTE}{\text{Area } \Delta TEC}$

$\frac{\text{Area } \Delta BTE}{\text{Area } \Delta TEC} = \frac{BE}{EC} = \frac{20}{5} = 4$ (common vertex, common ht)

(2)

[5]

QUESTION 4



Using the given diagram of circle ABC with O as the centre, **prove the theorem** which states $A\hat{O}C = 2\hat{B}$

R.T.P. : $A\hat{O}C = 2\hat{B}$

Proof: Construct BO produced

$AO = OB = OC$ (radii)

Let $A = x$ then $B_1 = x$ (isos Δ)

Let $C = y$ then $B_2 = y$ (")

$\hat{O}_1 = 2x$ & $\hat{O}_2 = 2y$ (ext ∠ of Δ)

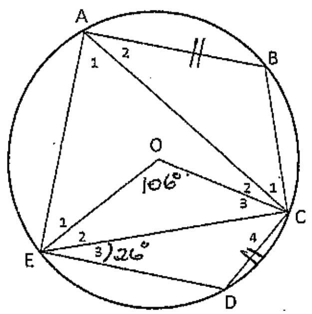
$\hat{O}_1 + \hat{O}_2 = 2x + 2y$

$= 2(x+y)$

$A\hat{O}C = 2\hat{B}$

[4]

QUESTION 5



O is the centre of circle ABCDE with $D\hat{E}C = 26°$, AB = DC and $E\hat{O}C = 106°$.

Calculate the size of:

a) $B\hat{C}A$

$B\hat{C}A = 26°$ (equal chords sub = ∠s)

(2) K

b) $\hat{A}$

$\hat{A} = \cancel{53}\ 53°$ (∠ at ⊙ = 2∠ at circum)

(2) K

c) $O\hat{C}D$

$O\hat{C}E = O\hat{E}C$ (radii ⇒ isos △)

$O\hat{C}E = 37°$ (int ∠s of isos △)

$\hat{D} = \cancel{128}\ 127°$ (opp ∠s of cyclic quad)

$D\hat{C}E = \cancel{26}\ 27°$ (int ∠s of △)

$O\hat{C}D = \cancel{63}°$ 64

(6) R

[10]

QUESTION 6

a) Without the use of a calculator determine:

1) $2\sin15°\sin75°$

$= 2\sin 15 \cos 15°$

$= \sin 2(15°)$

$= \sin 30°$

$= \frac{1}{2}$

~~(4)~~ (3) R

2) $\cos 2A$ if $\cos A = -\frac{12}{13}$ and $A \in [0°;180°]$

$\cos 2A = 2\cos^2 A - 1$

$= 2\cdot(-\frac{12}{13})^2 - 1$

$= 2\left(\frac{144}{169}\right) - 1 = \frac{119}{169}$

~~(4)~~ (3) R

b) Simplify without the use of a calculator:

$\frac{1-\cos 2x}{\sin 2x}$

$= \frac{1-(2\cos^2 x - 1)}{2\sin x \cos x}$

$= \frac{1-2\cos^2 x + 1}{2\sin x \cos x}$

$\frac{2(1-\cos^2 x)}{2\sin x \cos x}$

$= \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\sin x}{\cos x} = \tan x.$

~~(5)~~ (6) C

c) Give the general solution for x if: $\sin^2 x = \cos x + 2$ 2cosx

$\sin^2 x = 2\cos x + 2$

$1 - \cos^2 x - 2\cos x - 2 = 0.$

$\cos^2 x + 2\cos x + 1 = 0.$

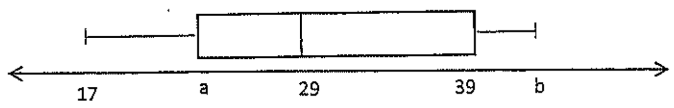
$(\cos x + 1)^2 = 0.$

$\cos x = -1$ $\therefore x = \pm 180° + 360° k,$

(5) (5) C

KεZ ~~[18]~~ [17]

QUESTION 7



In the given diagram, the box and whisker plot depicts the number of push ups done by 36 students in one minute. The interquartile range and the range are 15 and 25 respectively.

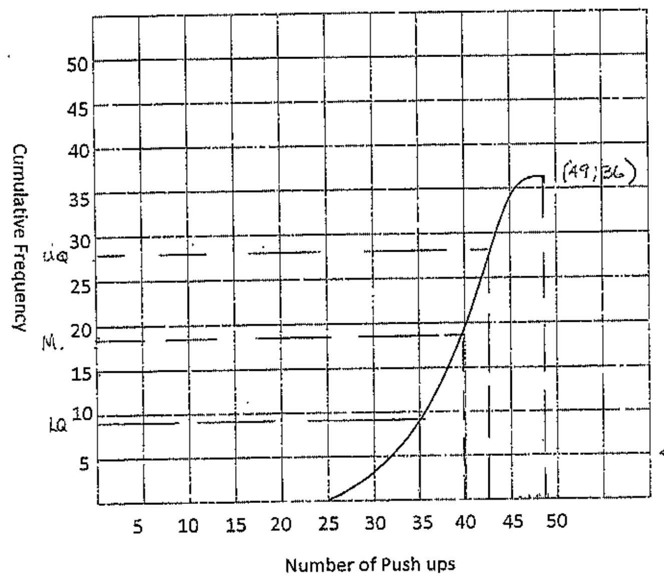
a) Determine a and b

$a = 39 - 15$ ✓ $b = 25 + 17$ ✓

$= 24$ ✓ $= 42$ ✓

(2) R.

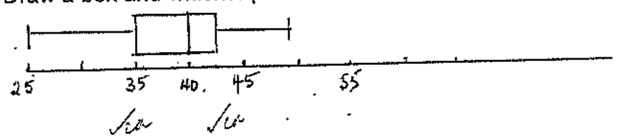
b) The sports coach puts the pupils through a training program and then re-tests them and plots the new data on a cumulative frequency curve to see if anyone had improved.



$4\frac{37}{9\frac{1}{4}}$

✓m.

1) Draw a box and whisker plot of the new data



(3) C.

2) Would you say this data is skewed ? Why?

Yes ✓it is skewed left. (negatively) since lower 50% data is more spread out. ✓

(2) 1R 1P/S

3) Which part of the distribution indicates the best improvement?

Lower 50%. shows best improvement. ✓

(1) PS

4) Comment on the effectiveness of the training program based on the data, giving reasons for your conclusions

Program was especially effective ✓ for the unfit which showed best improvement. ✓ However the upper 50% improved too which is difficult to achieve with those already fit. ∴ good program.

(3) PS.

[11]

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SECTION B

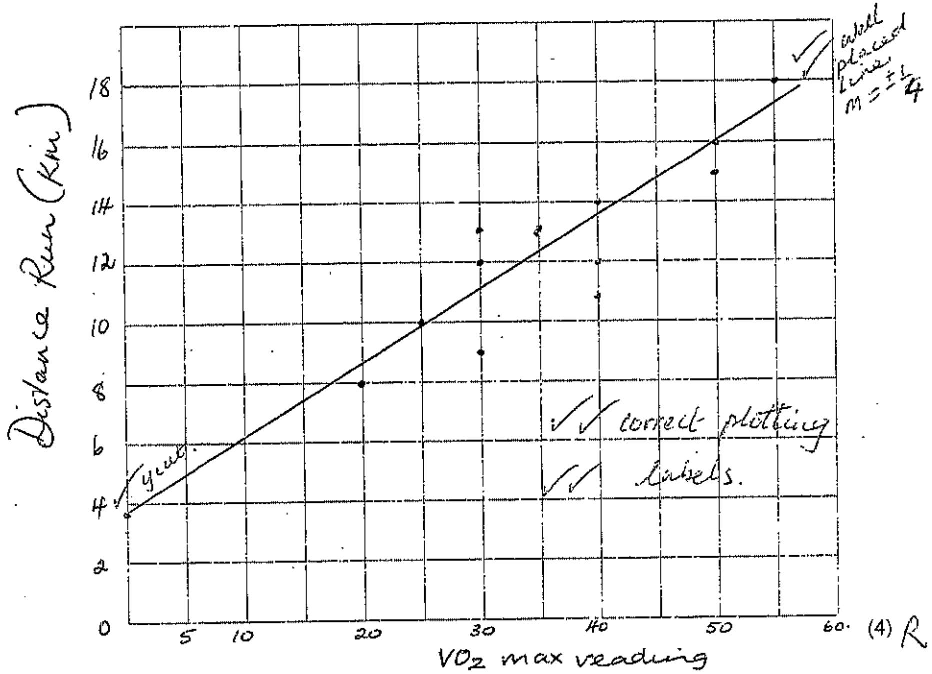
QUESTION 8

An athlete's ability to take and use oxygen effectively is called his / her VO_2-max.

Twelve athletes with pre-recorded VO_2-max readings ran for one hour. The distances (in kilometres) that they each covered are represented in the table:

VO_2-max reading	20	55	30	25	40	30	50	40	35	30	50	40
Distance run (*km*)	8	18	13	10	11	12	16	14	13	9	15	12

a) Draw a scatter plot of the data. (Place VO_2-max on the horizontal axis)



(4) R

b) Use the correlation coefficient to describe the correlation between the two sets of data.

$r = 0,89$ ∴ strong positive correlation

(2) R.

c) Determine the equation of the least squares (line of best fit) and draw it on the graph (Values given correct to 2 dec places.) terminology

$a = 3,69$ $b = 0,24$

$y = 3,69 + 0,24x$. + 3 graph

(5) R.

d) Use the method of interpolation to predict the distance run if an athlete has a VO_2-max value of 26.

$y = 3,69 + 0,24(26)$

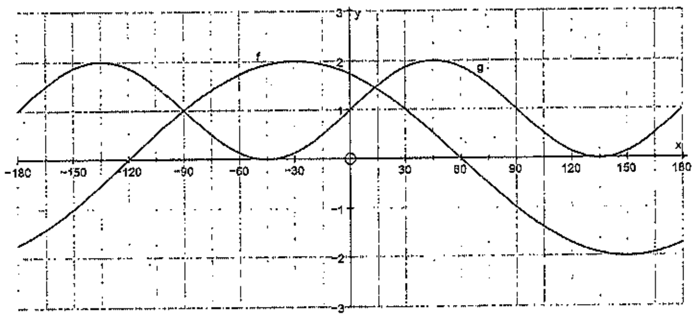
$y = 9,93$ km.

(2) R.

[13]

QUESTION 9

In the figure, the graphs of $f(x) = 2\cos(x + a)$ and $g(x) = 1 + \sin bx$ are given for $x \in [-180°;180°]$



a) Determine the values of a and b using the graphs.

$a = 30°$ $b = 2$

(2) R.

b) Determine the values of x for which $\frac{g(x)}{f(x)} \leq 0$

$[-180°; -120°] \cup [45°] \cup [60°; 180°]$

(3) PS.

c) The y axis is translated 30° to the right. Determine the new equation of f in the form $y = c\sin(x + d)$ with reference to the new set of axes.

$y = 2\cos(x \mp 60°)$

(2) R

[7]

QUESTION ~~9~~ 10

a) Prove that: $\sin 3x - \sin x = 2\cos 2x \sin x$

LHS $\sin 3x - \sin x$

$= \sin(2x + x) - \sin x$

$= \sin 2x \cos x + \sin x \cos 2x - \sin x$

$= 2\sin x \cos x \cos x + \sin x \cos 2x - \sin x$ more lines

$= \sin x[2\cos^2 x + \cos 2x - 1]$

$= \sin x[2\cos^2 x + 2\cos^2 x - 1 - 1]$

$= \sin x[4\cos^2 x - 2] = 2\sin x[2\cos^2 x - 1]$

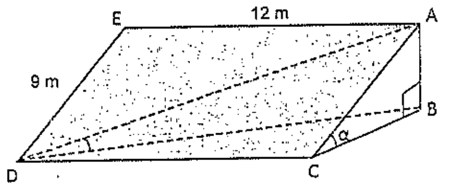
$= 2\sin x \cos 2x$

$= RHS.$

(6) C

b) A rectangular roof which is 12m by 9m slopes at an angle of α. ~~Determine~~ the slope of its diagonal ($A\hat{D}B$) in terms of α.

Write an expression for..



In $\triangle DCA$ In $\triangle ABC$

$DA = 15\text{m}$ (Pythag) $\sin\alpha = \frac{AB}{9}$

$9\sin\alpha = AB$

In $\triangle ADB$

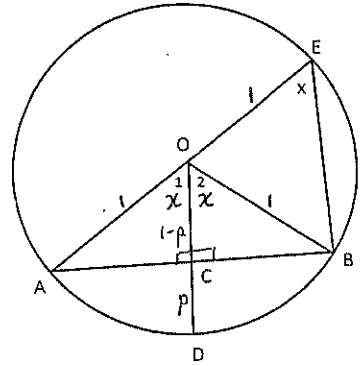
$\sin A\hat{D}B = \frac{AB}{AD}$

$\sin A\hat{D}B = \frac{9\sin\alpha}{15}$

~~(8)~~ (5) PS

~~[12]~~ [11]

QUESTION ~~9~~ 11



Line AE is a diameter with O as the centre

In the given diagram O is the centre of the circle with a radius of 1 unit. OD ⊥ AB at C.

$DC = p$, $\hat{O}_1 = \hat{O}_2 = x$. Determine an expression for:

a) p in terms of x

$\cos x = 1 - p$ $\therefore p = 1 - \cos x$

(2) PS

b) AB in terms of x

$\sin x = \frac{CB}{1}$

$AB = 2CB$ (⊥ line from ⊙ to chord)

$AB = 2\sin x$

(3) PS

c) Hence determine BE in terms of p

$\hat{B} = 90°$ (∡ in semi ⊙)

$AE^2 - AB^2 = BE^2$ (Pythag) $BE^2 = 2^2 - (2\sin x)^2$

$= 4 - 4\sin^2 x$

$= 4(1 - \sin^2 x)$

$= 4\cos^2 x$

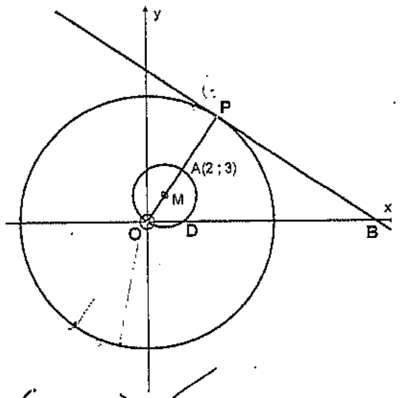
$BE^2 = 4(1-p)^2$

$BE = 2(1-p)$

PS (3) [8]

QUESTION 12

A(2 ; 3), the midpoint of radius OP, lies on the circumference of the smaller circle with diameter OA. Write down:



a) The equation of the larger circle.

$P(4;6)$ ✓

$\therefore$ Eqn x^2+y^2

$= 16+36$ ✓

$\therefore x^2+y^2 = 52$. ✓

(3) R

b) The equation of the smaller circle.

$M(1;\frac{3}{2})$ ✓

$r^2 = 1 + \frac{9}{4}$

$= \frac{13}{4}$ ✓

$\therefore$ Eqn $(x-1)^2+(y-\frac{3}{2})^2 = \frac{13}{4}$ ✓

(3) C

c) Determine the equation of the tangent to the large circle at P.

$m_{OP} = \frac{6}{4} = \frac{3}{2}$ ✓ $\therefore m_{tang} = -\frac{2}{3}$ ✓

$y = mx + c$

$6 = -\frac{2}{3} \cdot 4 + c$ ✓ m(sub).

$y = -\frac{2}{3}x + 8\frac{2}{3}$ ✓

(4) C.

d) Hence, determine the equation of a tangent to the large circle which is parallel to the tangent in c)

Pt $(-4;-6)$ ✓ $y = mx + c$

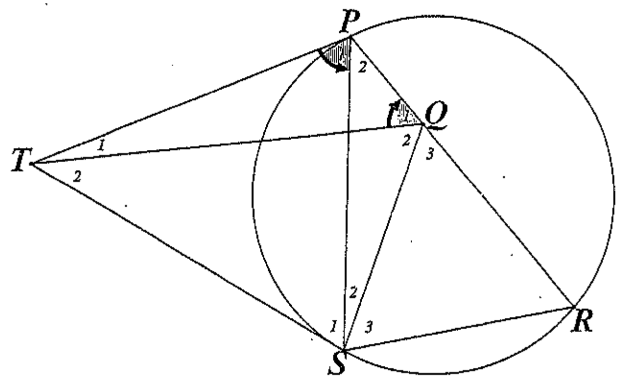
$-6 = -\frac{2}{3}(-4) + c$

$y = -\frac{2}{3}x - 8\frac{2}{3}$ ✓

(2) C.

[12]

QUESTION 13



TP and TS are tangents to the circle. Q is a point on PR such that $\hat{Q}_1 = \hat{P}_1$.

Prove the following, **stating all necessary reasons:**

a) TQ // SR.

$\hat{P}_1 = R$ ✓ (tan chord Th.) ✓

$R = \hat{Q}_1$ ✓ ($\hat{P}_1 = \hat{Q}_1$)

$\therefore$ TQ // SR (= Corr ∠s)

(4) R

b) TQ bisects $S\hat{Q}P$.

TP = TS (tang. from same pt.) ✓

$\hat{P}_1 = \hat{S}_1$ (isos Δ) ✓

$\hat{Q}_1 = \hat{S}_1$ ✓

$\therefore$ PQST is cyclic (conv. of ∠s same seg) ✓

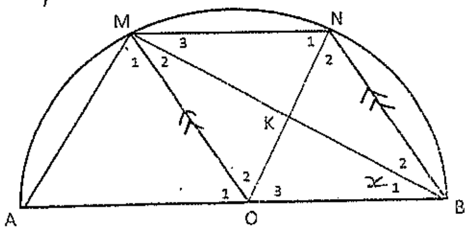
$\therefore \hat{P}_1 = \hat{Q}_2$ (∠s in same seg) ✓

$\hat{Q}_1 = \hat{Q}_2$

~~(4)~~ (5) PS.

~~[8]~~ [9]

QUESTION 14



In the above diagram AOB is the diameter of the semi-circle, centre O, MO || NB, ON and MB intersect at K and $\hat{B}_1 = x$.

a) Prove that MB bisects $N\hat{B}O$

$ON = AO = OM = OB$ (radii)

$\hat{M}_2 = x$ (isos Δ)

$O_1 = 2x$ (ext $\angle$s of Δ) / $\angle$ @ centre

$\hat{B}_2 = x$ ($\hat{B} = \hat{O}_1$ corr $\angle$s MO // NB)

(or. alt $\angle$s)

(3) PS

b) Express $M\hat{K}N$ in terms of x

$\hat{N}_2 = 2x$ (isos Δ)

$M\hat{K}N = 3x$ (ext $\angle$ of Δ)

(3) PS

c) ~~Calculate the value of~~ Express $\hat{M}_3$ in terms of x.

$\hat{O}_2 = 2x$ (alt $\angle$s MO // NB)

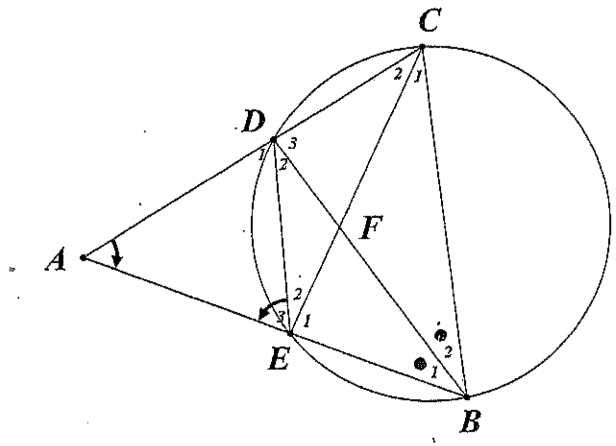
$\hat{O}_3 = 180° - 4x$ ($\angle$s on str. line)

$M_3 = 90° - 2x$ ($\angle$ at O = 2$\angle$ at circum)

(3) PS

[9]

QUESTION 15



In the diagram, BC is the diameter of the circle BCDE. BD bisects $A\hat{B}C$, and $\hat{A} = \hat{E}_3$.

Prove $AD^2 = DF.DB$

In Δ DEF + Δ DEB

$\hat{E}_2 = \hat{B}_2$ ($\angle$s in same seg)

$\therefore E_2 = \hat{B}_1$

$\hat{D}_2$ is common

$\therefore \Delta DFE /// \Delta DEB$ (A.A.A.)

(5) PS

$\therefore \frac{DF}{DE} = \frac{DE}{DB}$ (/// Δs th.)

$DE^2 = DF.DB$

$\therefore AD^2 = DF.DB$ ($AD = DE$ base $\angle$s =)

[5]