

**MATHEMATICS EXEMPLAR EXAMINATION
GRADE 12
PAPER 1**

TIME: 3 HOURS

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.
 2. Clearly show ALL calculations, diagrams, graphs, etcetera that you have used in determining your answers.
 3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
 4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
 5. Diagrams are NOT necessarily drawn to scale.
 6. Number the answers correctly according to the numbering system used in this question paper.
 7. It is in your own interest to write legibly and to present the work neatly.
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QUESTION 1

1.1 Solve for x rounded off to two decimal places where necessary:

$$1.1.1 \quad \frac{x^2 - 1}{x + 1} = 2 \quad (4)$$

$$1.1.2 \quad 8 - (x - 2)(x - 3) = 0 \quad (4)$$

1.2 Consider the inequality: $4x^2 < 9$

Determine the solution to this inequality if:

$$1.2.1 \quad x \in \{\text{real numbers}\} \quad (3)$$

$$1.2.2 \quad x \in \{\text{integers}\} \quad (1)$$

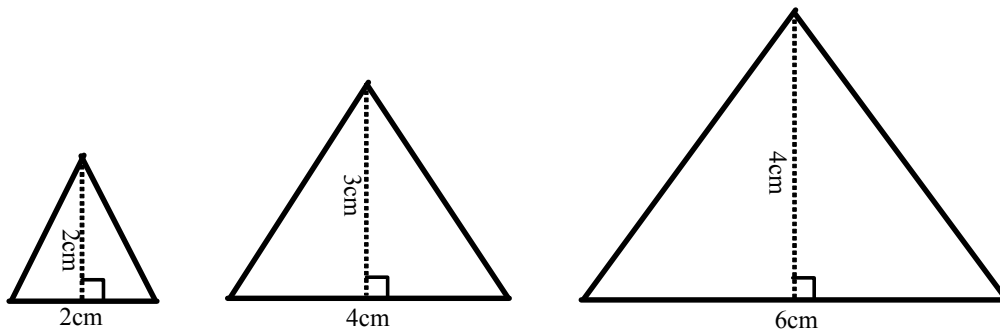
1.3 Solve for x and y :

$$3x - y = 2 \quad \text{and} \quad 3y + 9x^2 = 4 \quad (7)$$

[19]

QUESTION 2

A sequence of isosceles triangles is drawn. The first triangle has a base of 2cm and height of 2cm. The second triangle has a base that is 2cm longer than the base of the first triangle. The height of the second triangle is 1cm longer than the height of the first triangle. This pattern of enlargement will continue with each triangle that follows.



2.1 Determine the area of the 100th triangle. (4)

2.2 Which triangle will have an area of 240cm²? (3)

[7]

QUESTION 3

3.1 Given: $\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + \dots + \frac{180}{181}$

3.1.1 Calculate the sum of the given series. (4)

3.1.2 Hence calculate the sum of the following series:

$$\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots + \left(\frac{1}{181} + \frac{2}{181} + \dots + \frac{180}{181}\right) \quad (4)$$

3.2 In a geometric sequence in which all terms are positive, the sixth term is $\sqrt{3}$ and the eighth term is $\sqrt{27}$. Determine the first term and constant ratio. (5)

3.3 Consider the series $\sum_{n=1}^{\infty} 2\left(\frac{1}{2}x\right)^n$

3.3.1 For which values of x will the series converge? (3)

3.3.2 If $x = \frac{1}{2}$, calculate the sum to infinity of this series. (3)

3.4 Malibongwe bought a franchise business on the 28th February 2009. He took out a loan to pay for the franchise which cost R2 000 000. On the 1st March 2009, he made a profit of R4. On the 2nd March, the profit made was R6. On the 3rd March, he made a profit of R9 and on the 4th March, a profit of R13,50. The profits made per day were deposited into a bank account. After how many days will he be able to pay off the R2 000 000 loan, using the amount saved in the bank account? (5)

[24]

QUESTION 4

- 4.1 Given: $f(x) = \frac{2}{x+1}$
- 4.1.1 Write down the equations of the asymptotes. (2)
- 4.1.2 Sketch the graph of f indicating the coordinates of the y -intercept as well as the asymptotes. (6)
- 4.1.3 Write down the equation of the graph formed if the graph of f is shifted 3 units right and 2 units upwards. (2)
- 4.1.4 Determine graphically the values of x for which $\frac{2}{x+1} \geq 1$ (2)
- 4.2 Consider the functions: $f(x) = 2x^2$ and $g(x) = \left(\frac{1}{2}\right)^x$
- 4.2.1 Restrict the domain of f in one specific way so that the inverse of f will also be a function. (1)
- 4.2.2 Hence draw the graph of your new function f and its inverse function f^{-1} on the same set of axes. (2)
- 4.2.3 Write the inverse of g in the form $g^{-1}(x) = \dots\dots\dots$ (2)
- 4.2.4 Sketch the graph of g^{-1} . (2)
- 4.2.5 Determine graphically the values of x for which $\log_{\frac{1}{2}} x < 0$ (1)
- [20]**

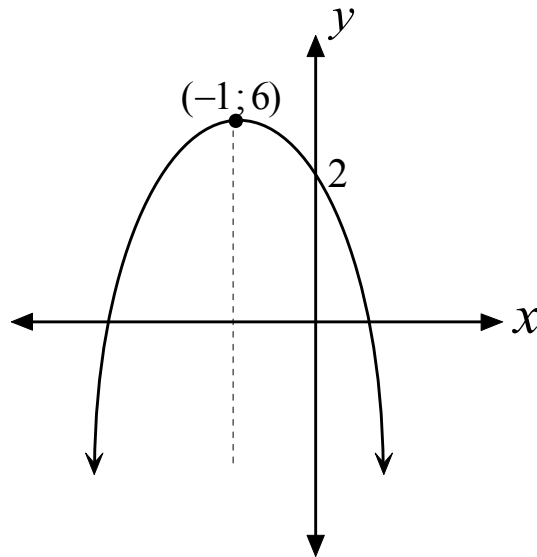
QUESTION 5

Consider: $f(x) = 2 \cos x$

- 5.1 Draw a sketch graph of $y = -2f(x)$ for $x \in [-90^\circ; 360^\circ]$ (2)
- 5.2 Write down the amplitude of $y = -2f(x)$. (1)
- 5.3 Write down the period of $y = f\left(\frac{x}{2}\right)$. (1)
- 5.4 Write down the maximum value of the graph of $g(x) = f(x) - 2$ (1)
- [5]**

QUESTION 6

Sketched below is the graph of $f(x) = a(x + p)^2 + q$.



- 6.1 Determine the equation of f in the form $f(x) = a(x + p)^2 + q$. (4)
- 6.2 Write down the range of f . (1)
- 6.3 If the graph of f is reflected about the x -axis to form the graph of g , write the equation of g in the form $g(x) = a(x + p)^2 + q$. (1)
- [6]**

QUESTION 7

- 7.1 Patricia deposited a certain amount of money into a bank account paying 8% per annum compounded half-yearly. After four years, the money has a value of R100 000.
- 7.1.1 Convert the nominal interest rate into the equivalent annual effective rate. (2)
- 7.1.2 Hence, or otherwise, calculate the amount of money originally deposited into the bank account by Patricia. (2)
- 7.2 A motor car which cost R200 000 depreciates at a rate of 8% per annum on the reducing balance method. Calculate how long it will take for the car to depreciate to a value of R90 000 under these conditions. (3)
- [7]**

QUESTION 8

Mpho takes out a retirement annuity that will supplement his pension when he retires in thirty years' time. He estimates that he will need R2 500 000 in this retirement fund at that stage. The interest rate he earns is 9% per annum compounded monthly.

8.1 Calculate his monthly payment into this fund if he starts paying immediately and makes his final payment in 30 years' time. (5)

8.2 The retirement fund does not pay out the R2 500 000 million when Mpho retires. Instead he will be paid monthly amounts, for a period of twenty years, starting one month after his retirement. If the interest that he earns over this period is calculated at 7% per annum compounded monthly, determine the monthly payments he will receive. (5)

[10]**QUESTION 9**

9.1 If $f(x) = -2x^2 + 1$, determine $f'(x)$ from first principles. (5)

9.2 Determine the following and leave your answer with positive exponents:

$$\frac{dy}{dx} \text{ if } y = \left(2\sqrt{x} - \frac{1}{3x}\right)^2 \quad (4)$$

[9]**QUESTION 10**

10.1 The equation of the tangent to the curve of $f(x) = ax^3 + bx$ at $x = -1$ is $y - x - 4 = 0$. Calculate the value of a and b . (6)

10.2 Given: $f(x) = 2x^3 - 6x - 4$

10.2.1 Determine the coordinates of the points of intersection of f with the axes. (5)

10.2.2 Determine the coordinates of the turning points of f . (4)

10.2.3 Sketch the graph of f showing the intercepts with the axes and the turning points. (3)

10.2.4 Determine the coordinates of the point of inflection on the graph of f . (3)

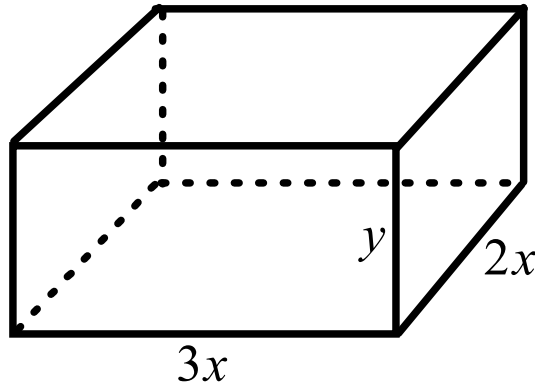
10.2.5 Determine graphically the values of p for which the following equation will have one real solution:

$$2x^3 - 6x - 4 = p \quad (2)$$

[23]

QUESTION 11

The sides of the base of a rectangular cardboard box are $3x$ and $2x$ cm respectively. The height is y cm. The box is open on the top (without a lid).



- 11.1 If the total exterior surface area of the box is 200cm^2 , prove that:

$$y = \frac{20}{x} - \frac{3x}{5} \quad (3)$$

- 11.2 Express the volume of the box in terms of x . (2)

- 11.3 Determine the value of x if its volume is to be a maximum. (3)

[8]

QUESTION 12

In order to paint the walls of his home, Joseph will require at least 10 litres of purple paint. Purple paint is obtained by mixing quantities of red and blue paint. To obtain a suitable shade of purple paint, the volume of blue paint used must be at least half the volume of red paint used. The hardware store where Joseph intends buying the paint has only 8 litres of blue paint in stock. Let the number of litres of red paint be x and the number of litres of blue paint be y .

- 12.1 Write down the inequalities in terms of x and y which represent the constraints of this situation. (3)

- 12.2 On the attached diagram provided, represent the constraints graphically and clearly indicate the feasible region. (4)

- 12.3 The cost of both red and blue paint is R40 per litre, but the paint is only sold in 2-litre tins. Determine the number of litres of red and blue paint which can be bought maintaining a minimum cost. Show all possible combinations. (5)

[12]