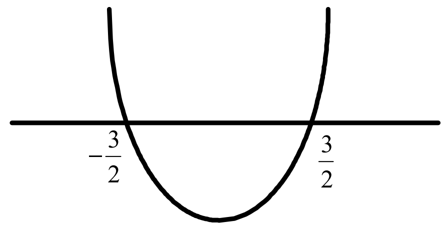
MATHEMATICS EXEMPLAR EXAMINATION
GRADE 12
PAPER 1

MEMORANDUM

QUESTION 1

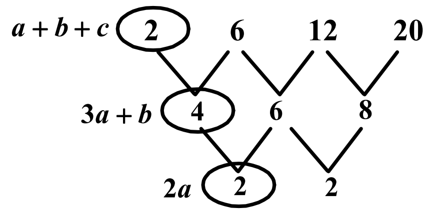
1.1.1	$\frac{x^2-1}{x+1}=2$ $\therefore x^2-1=2(x+1)$ $\therefore x^2-1=2x+2$ $\therefore x^2-2x-3=0$ $\therefore (x-3)(x+1)=0$ $\therefore x=3$ or $x=-1$ But $x\neq -1$ $\therefore x=3$ is the solution OR $\frac{x^2-1}{x+1}=2$ $\therefore \frac{(x+1)(x-1)}{(x+1)}=2$ $\therefore x-1=2$ provided $x\neq -1$ $\therefore x=3$	✓ standard form = 0 ✓ factorisation ✓ both answers ✓ excluding $x=-1$ (4) ✓ factorisation ✓ simplification ✓ correct answer ✓ excluding $x=-1$
1.1.2	$8-(x-2)(x-3)=0$ $\therefore 8-(x^2-5x+6)=0$ $\therefore 8-x^2+5x-6=0$ $\therefore -x^2+5x+2=0$ $\therefore x^2-5x-2=0$ $\therefore x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-2)}}{2(1)}$ $\therefore x=\frac{5\pm\sqrt{33}}{2}$ $\therefore x=5,37$ or $x=-0,37$ – 1 for inaccurate rounding off for both answers.	✓ simplification ✓ standard form ✓ substitution into formula ✓ correct answers (4)

1.2.1	$4x^2 < 9$ $\therefore 4x^2 - 9 < 0$ $\therefore (2x+3)(2x-3) < 0$ $\therefore -\frac{3}{2} < x < \frac{3}{2}$  	✓ factorisation ✓ endpoints ✓ inequality notation (3)
1.2.2	$x \in \{-1;0;1\}$	✓ correct answer (1)
1.3	$3x - y = 2$ $\therefore 3x - 2 = y$ $\therefore 3(3x-2) + 9x^2 = 4$ $\therefore 9x - 6 + 9x^2 = 4$ $\therefore 9x^2 + 9x - 10 = 0$ $\therefore (3x+5)(3x-2) = 0$ $\therefore x = -\frac{5}{3}$ or $x = \frac{2}{3}$ $\therefore y = -7$ or $y = 0$	✓ $3x - 2 = y$ ✓ substitution ✓ standard form ✓ factorisation ✓ both x-values ✓ $y = -7$ ✓ $y = 0$ (7)

QUESTION 2

2.1	Area of triangle 1: $\frac{1}{2}(2cm)(2cm) = (1)(2)cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm) = (2)(3)cm^2$ a Area of triangle 3: $\frac{1}{2}(6cm)(4cm) = (3)(4)cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm) = (4)(5)cm^2$ The areas form the following pattern: $(1)(2);(2)(3);(3)(4);(4)(5);$........... Area of triangle *n*: $(n)(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2 = 10100cm^2$	✓ determining areas ✓ establishing pattern ✓ obtaining general term ✓ area of 100th triangle (4)

	OR Area of triangle 1: $\frac{1}{2}(2cm)(2cm)$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm)$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm)$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm)$ The bases are in arithmetic sequence: 2; 4; 6; 8; 10; .... General term is $2+(n-1)2=2n$ The heights are in arithmetic sequence: 2; 3; 4; 5; .... General term is $2+(n-1)(1)=n+1$ Therefore, the general term for areas is: Area of triangle n: $\frac{1}{2}(2n)(n+1)cm^2$ $=n(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2=10100cm^2$	✓ determining areas ✓ general terms of arithmetic sequences ✓ obtaining area of nth triangle ✓ area of 100th triangle
	OR Area of triangle 1: $\frac{1}{2}(2cm)(2cm)=2cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm)=6cm^2$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm)=12cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm)=20cm^2$ The areas form a quadratic number pattern: 2; 6; 12; 20;	✓ determining areas ✓ quadratic pattern ✓ obtaining general term ✓ area of 100th triangle

	 $2a = 2$ $\quad 3a + b = 4$ $\quad a + b + c = 2$ $a = 1$ $\quad \therefore 3(1) + b = 4$ $\quad \therefore 1 + 1 + c = 2$ $\therefore b = 1$ $\quad \therefore c = 0$ Area of triangle *n*: $(n^2 + n)\,cm^2$ Area of triangle 100: $\left[(100)^2 + 100\right]cm^2 = 10100cm^2$	
2.2	$n(n+1) = 240$ $\therefore n^2 + n - 240 = 0$ $\therefore (n+16)(n-15) = 0$ $\therefore n = -16$ or $n = 15$ But $n \neq -16$ $\therefore n = 15$ The 15th triangle will have an area of $240\,\text{cm}^2$	✓ equating general term to 240 ✓ factorising ✓ obtaining 15 triangles (3)

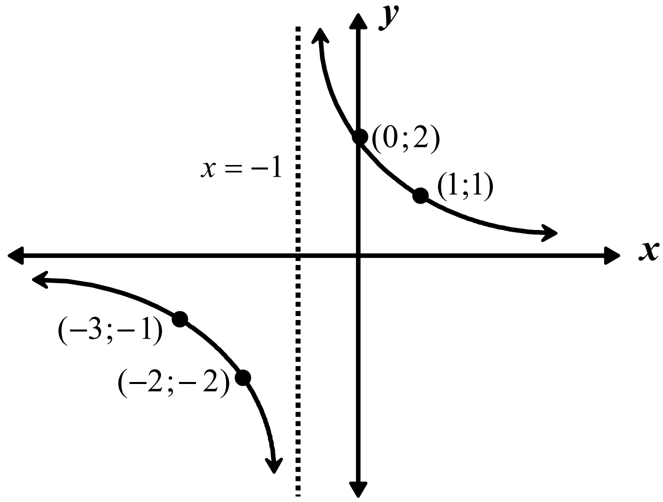
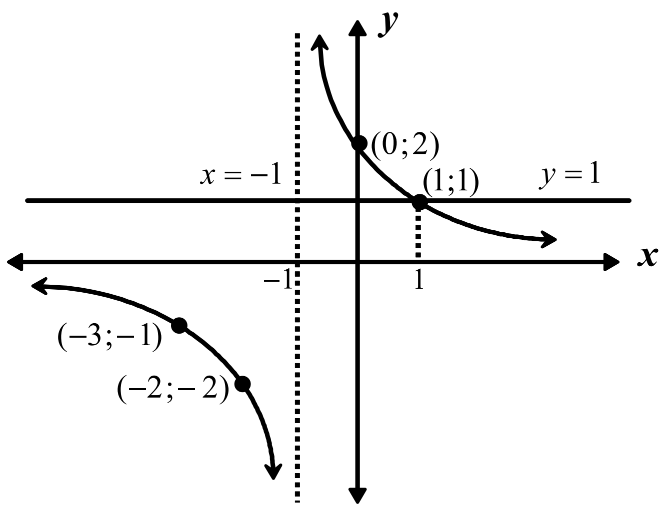
QUESTION 3

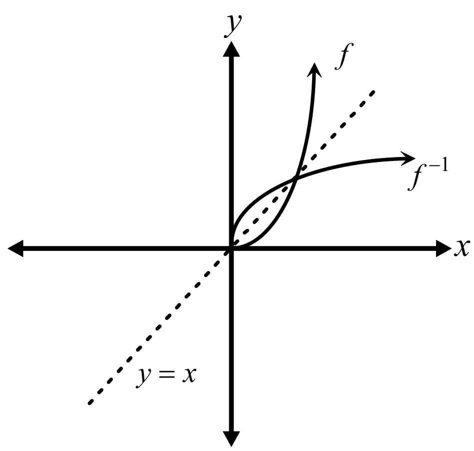
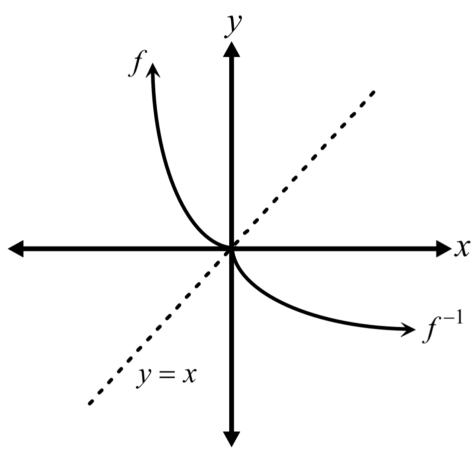
3.1.1	$\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + \ldots\ldots\ldots\ldots + \frac{180}{181}$ $a = \frac{1}{181}$ $\quad d = \frac{1}{181}$ $\quad n = 180$ $\therefore S_{180} = \frac{180}{2}\left[\frac{1}{181} + \frac{180}{181}\right] = 90[1] = 90$ OR $S_{180} = \frac{180}{2}\left[2\left(\frac{1}{181}\right) + (179)\frac{1}{181}\right] = 90[1] = 90$	✓ correct *a* and *d* ✓ correct *n* ✓ S_n formula ✓ correct answer (4)

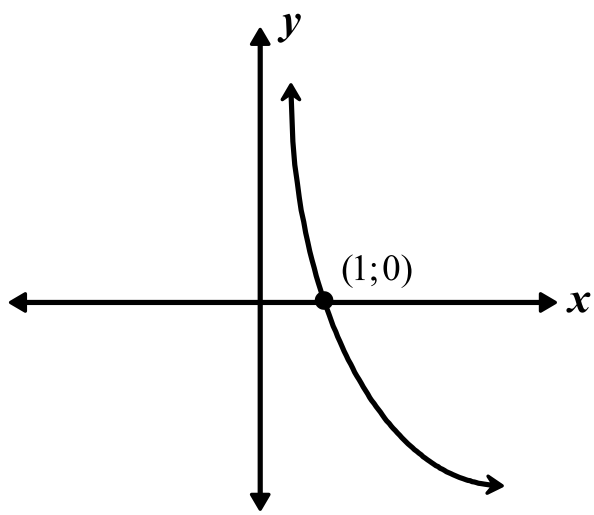
3.1.2	$\left(\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+....+\left(\frac{1}{181}+\frac{2}{181}+....+\frac{180}{181}\right)$ $=\frac{1}{2}+1+1\frac{1}{2}+2+..........+90$ $a=\frac{1}{2} \quad d=\frac{1}{2} \quad T_n=90$ $\therefore \frac{1}{2}+(n-1)\frac{1}{2}=90$ $\therefore 1+n-1=180$ $\therefore n=180$ $\therefore S_{180}=\frac{180}{2}\left[\frac{1}{2}+90\right]=90\left[90\frac{1}{2}\right]=8145$ OR $S_{180}=\frac{180}{2}\left[2\left(\frac{1}{2}\right)+(179)\left(\frac{1}{2}\right)\right]=90\left[90\frac{1}{2}\right]=8145$	✓ simplifying fractions to get series ✓ $\frac{1}{2}+(n-1)\frac{1}{2}=90$ ✓ $n=180$ ✓ substitution into S_n formula to get 8145 (4)
3.2	$ar^5=\sqrt{3}$ $ar^7=\sqrt{27}$ $\therefore \frac{ar^7}{ar^5}=\frac{\sqrt{27}}{\sqrt{3}}$ $\therefore r^2=\sqrt{\frac{27}{3}}$ $\therefore r^2=\sqrt{9}$ $\therefore r^2=3$ $\therefore r=\sqrt{3}$ (terms are positive) $\therefore a(\sqrt{3})^5=\sqrt{3}$ $\therefore a=\frac{\sqrt{3}}{(\sqrt{3})^5}$ $\therefore a=\frac{1}{(\sqrt{3})^4}$ $\therefore a=\frac{1}{(3^{\frac{1}{2}})^4}$ $\therefore a=\frac{1}{9}$	✓ $ar^5=\sqrt{3}$; $ar^7=\sqrt{27}$ ✓ dividing ✓ $r=\sqrt{3}$ ✓ correct working with surds ✓ $a=\frac{1}{9}$ (5)

3.3.1	$\sum_{n=1}^{\infty} 2(\frac{1}{2}x)^n$ $= 2(\frac{1}{2}x)^1 + 2(\frac{1}{2}x)^2 + 2(\frac{1}{2}x)^3 + 2(\frac{1}{2}x)^4 + ...$ $= x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + ...$ The series converges for: $-1 < \frac{1}{2}x < 1$ $\therefore -2 < x < 2$	✓ $r = \frac{1}{2}x$ ✓ $-1 < \frac{1}{2}x < 1$ ✓ $-2 < x < 2$ (3)
3.3.2	$a = \frac{1}{2} \quad r = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ $\therefore S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$	✓ a and r ✓ S_∞ formula ✓ $\frac{2}{3}$ (3)
3.4	$4 + 6 + 9 + 13,5 + ...$ $a = 4 \quad r = \frac{3}{2} \qquad S_n = 2000\ 000$ $\therefore 2000\ 000 = \frac{(4)\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2} - 1}$ $\therefore 2000\ 000 = 8\left[\left(\frac{3}{2}\right)^n - 1\right]$ $\therefore 250\ 000 = \left(\frac{3}{2}\right)^n - 1$ $\therefore 250\ 001 = \left(\frac{3}{2}\right)^n$ $\therefore \log_{\frac{3}{2}}(250\ 001) = n$ $\therefore n = 30,65422881$ Malibongwe will be able to pay off the R2000 000 on the last day of March (31 days)	✓ constant ratio ✓ correct substitution into the S_n formula ✓ use of logs ✓ $n = 30,65422881$ ✓ 31 days (5)

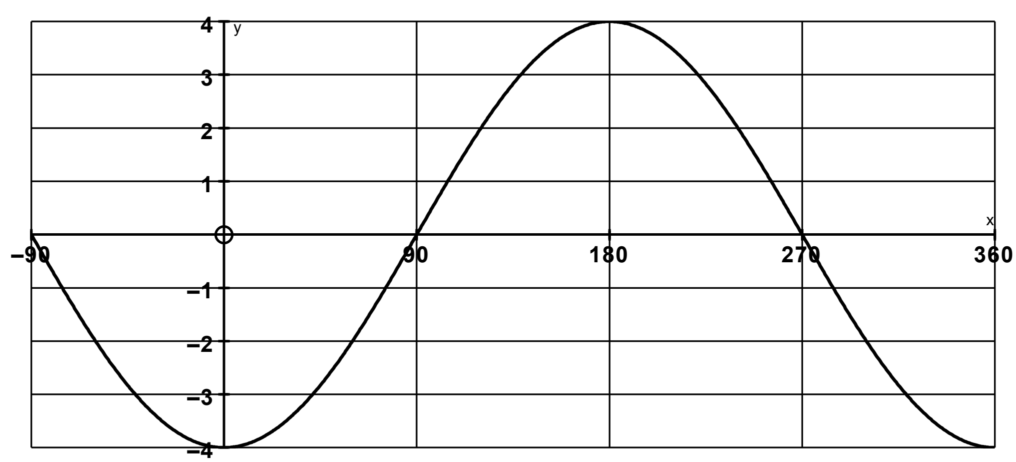
QUESTION 4

4.1.1	vertical: $x=-1$ horizontal: $y=0$	✓ vertical asymptote ✓ horizontal asymptote (2)
4.1.2		✓ $x=-1$ ✓ $y=0$ ✓ left branch ✓ coordinates on left branch ✓ right branch ✓ coordinates on right branch (6)
4.1.3	$y=\frac{2}{x+1-3}+2$ $\therefore y=\frac{2}{x-2}+2$	✓ denominator: $x-2$ ✓ +2 (2)
4.1.4	 Therefore $\frac{2}{x+1}\geq 1$ for $-1<x\leq 1$	✓ $-1<x$ ✓ $x\leq 1$ (2)

4.2.1	$f(x)=2x^2$ where $x \geq 0$ OR $f(x)=2x^2$ where $x \leq 0$	✓ $x \geq 0$ OR $x \leq 0$ (1)
4.2.2	 OR  	✓ f ✓ f^{-1} (2)
4.2.3	$y=\left(\frac{1}{2}\right)^x$ $x=\left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x)=\log_{\frac{1}{2}} x$	✓ $x=\left(\frac{1}{2}\right)^y$ ✓ $g^{-1}(x)=\log_{\frac{1}{2}} x$ (2)

4.2.4		✓ shape ✓ $(1;0)$ (2)
4.2.5	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✓ $x > 1$ (1)

QUESTION 5

5.1	$y = -2f(x)$ $\therefore y = -2(2\cos x)$ $\therefore y = -4\cos x$	
		✓ amplitude ✓ domain (2)

5.2	Amplitude is 4	✓ amplitude (1)
5.3	$y = f\left(\frac{x}{2}\right)$ $\therefore y = 2\cos\left(\frac{1}{2}x\right)$ period is $\frac{360°}{\frac{1}{2}} = 720°$	✓ 720° (1)
5.4	$g(x) = f(x) - 2$ $g(x) = 2\cos x - 2$ maximum is 0	✓ max (1)

QUESTION 6

6.1	$y = a(x+p)^2 + q$ $\therefore y = a(x+1)^2 + 6$ Substitute $(0;2)$: $\therefore 2 = a(0+1)^2 + 6$ $\therefore 2 = a + 6$ $\therefore 2 - 6 = a$ $\therefore a = -4$ $\therefore f(x) = -4(x+1)^2 + 6$	✓ $y = a(x+1)^2 + 6$ ✓ Substitute $(0;2)$ ✓ $a = -4$ ✓ $f(x) = -4(x+1)^2 + 6$ (4)
6.2	Range: $y \in (-\infty; 6]$	✓ $y \in (-\infty; 6]$ (1)
6.3	$y = -4(x+1)^2 + 6 \quad (f)$ $\therefore -y = -4(x+1)^2 + 6 \quad (g)$ $\therefore g(x) = 4(x+1)^2 - 6$	✓ $g(x) = 4(x+1)^2 - 6$ (1)

QUESTION 7

7.1.1	$i_{eff} = \left(1+\frac{0,08}{2}\right)^2 - 1$ $\therefore i_{eff} = 0,0816$	✓ formula ✓ 0,0816 (2)
7.1.2	$P = 100\ 000(1,0816)^{-4}$ $\therefore P = R73\ 069,02$ OR $P = 100\ 000\left(1+\frac{0,08}{2}\right)^{-8}$ $\therefore P = R73\ 069,02$	✓ formula ✓ answer (2)
7.2	$90\ 000 = 200\ 000(1-0,08)^n$ $\therefore \frac{9}{20} = 0,92^n$ $\therefore \log_{0,92}\left(\frac{9}{20}\right) = n$ $\therefore n = 9,576544593$ 9 years and 7 months	✓ correct substitution into formula ✓ use of logs ✓ answer (3)

QUESTION 8

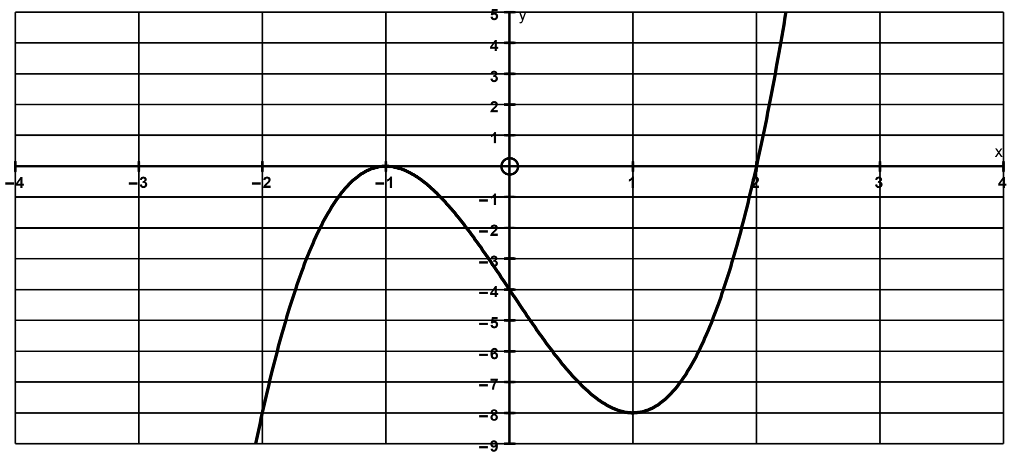
8.1	$2\ 500\ 000 = \frac{x\left[(1,0075)^{361}-1)\right]}{0,0075}$ $\therefore \frac{2\ 500\ 000 \times 0,0075}{\left[(1,0075)^{361}-1)\right]} = x$ $\therefore x = R1354,67$	✓ correct formula ✓ $n = 361$ ✓ $\frac{0,09}{12} = 0,0075$ ✓ $F = 2\ 500\ 000$ ✓ answer (5)
8.2	$2\ 500\ 000 = \frac{x\left[1-\left(1+\frac{0,07}{12}\right)^{-240}\right]}{\left(\frac{0,07}{12}\right)}$ $\therefore \frac{2\ 500\ 000 \times \left(\frac{0,07}{12}\right)}{\left[1-\left(1+\frac{0,07}{12}\right)^{-240}\right]} = x$ $\therefore x = R19\ 382,47$	✓ correct formula ✓ $n = 240$ ✓ $\frac{0,07}{12}$ ✓ $P = 2\ 500\ 000$ ✓ answer (5)

QUESTION 9

9.1	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{-2(x+h)^2+1-(-2x^2+1)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{-2(x^2+2xh+h^2)+1+2x^2-1}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{-2x^2-4xh-2h^2+1+2x^2-1}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{h(-4x-2h)}{h}$ $\therefore f'(x)=\lim_{h\to 0}(-4x-2h)$ $\therefore f'(x)=-4x-2(0)$ $\therefore f'(x)=-4x$ – 1 for inaccurate notation	✓ $-2(x+h)^2+1$ ✓ $-(-2x^2+1)$ ✓ $-2x^2-4xh-2h^2$ ✓ $\dfrac{h(-4x-2h)}{h}$ ✓ $-4x$ (5)
9.2	$y=\left(2\sqrt{x}-\dfrac{1}{3x}\right)^2$ $\therefore y=4x-\dfrac{4\sqrt{x}}{3x}+\dfrac{1}{9x^2}$ $\therefore y=4x-\dfrac{4x^{\frac{1}{2}}}{3x}+\dfrac{1}{9}x^{-2}$ $\therefore y=4x-\dfrac{4}{3}x^{-\frac{1}{2}}+\dfrac{1}{9}x^{-2}$ $\therefore \dfrac{dy}{dx}=4-\dfrac{4}{3}\times-\dfrac{1}{2}x^{-\frac{3}{2}}+\dfrac{1}{9}\times-2x^{-3}$ $\therefore \dfrac{dy}{dx}=4+\dfrac{2}{3}x^{-\frac{3}{2}}-\dfrac{2}{9}x^{-3}$ $\therefore \dfrac{dy}{dx}=4+\dfrac{2}{3x^{\frac{3}{2}}}-\dfrac{2}{9x^3}$ – 1 for inaccurate notation	✓ $4x-\dfrac{4}{3}x^{-\frac{1}{2}}+\dfrac{1}{9}x^{-2}$ ✓✓✓ $4+\dfrac{2}{3x^{\frac{3}{2}}}-\dfrac{2}{9x^3}$ (4)

QUESTION 10

10.1	$f(x)=ax^3+bx$ $\therefore f'(x)=3ax^2+b$ $\therefore f'(-1)=3a(-1)^2+b$ $\therefore f'(-1)=3a+b$ Now $y=x+4$ $m_t=1$ $1=3a+b$ Now at $x=-1$ $y=-1+4=3$ $\therefore$ Substitute $(-1;3)$ into the equation of f: $f(-1)=a(-1)^3+b(-1)$ $\therefore 3=-a-b$ $\therefore a+b=-3$ Solving simultaneously: $a=2$ and $b=-5$	✓ $f'(x)=3ax^2+b$ ✓ $m_t=1$ ✓ $1=3a+b$ ✓ $a+b=-3$ ✓ $a=2$ ✓ $b=-5$ (6)
10.2.1	y-intercept: $(0;-4)$ x-intercepts: $0=2x^3-6x-4$ $\therefore 0=x^3-3x-2$ $\therefore 0=(x+1)(x^2-x-2)$ $\therefore 0=(x+1)(x-2)(x+1)$ $\therefore x=-1$ or $x=2$ $(-1;0)$ $(2;0)$	✓ y-intercept ✓ $0=2x^3-6x-4$ ✓ $(x+1)(x^2-x-2)=0$ ✓ $(x+1)(x-2)(x+1)$ ✓ x-intercepts (5)
10.2.2	$f(x)=2x^3-6x-4$ $\therefore f'(x)=6x^2-6$ $\therefore 0=6x^2-6$ $\therefore 0=x^2-1$ $\therefore x=\pm 1$ $f(1)=-8$ $f(-1)=0$ Turning points are $(1;-8)$ and $(-1;0)$	✓ $f'(x)=6x^2-6$ ✓ $0=6x^2-6$ ✓ $x=\pm 1$ ✓ $(1;-8)$ and $(-1;0)$ (4)

10.2.3		✓ intercepts with the axes ✓ turning points ✓ shape (3)
10.2.4	$f'(x)=6x^2-6$ $\therefore f''(x)=12x$ $\therefore 0=12x$ $\therefore x=0$ $f(0)=-4$ Point of inflection at $(0;-4)$	✓ $f''(x)=12x$ ✓ $x=0$ ✓ $(0;-4)$ (3)
10.2.4	$p>0$ or $p<-8$	✓ $p>0$ ✓ $p<-8$ (2)

QUESTION 11

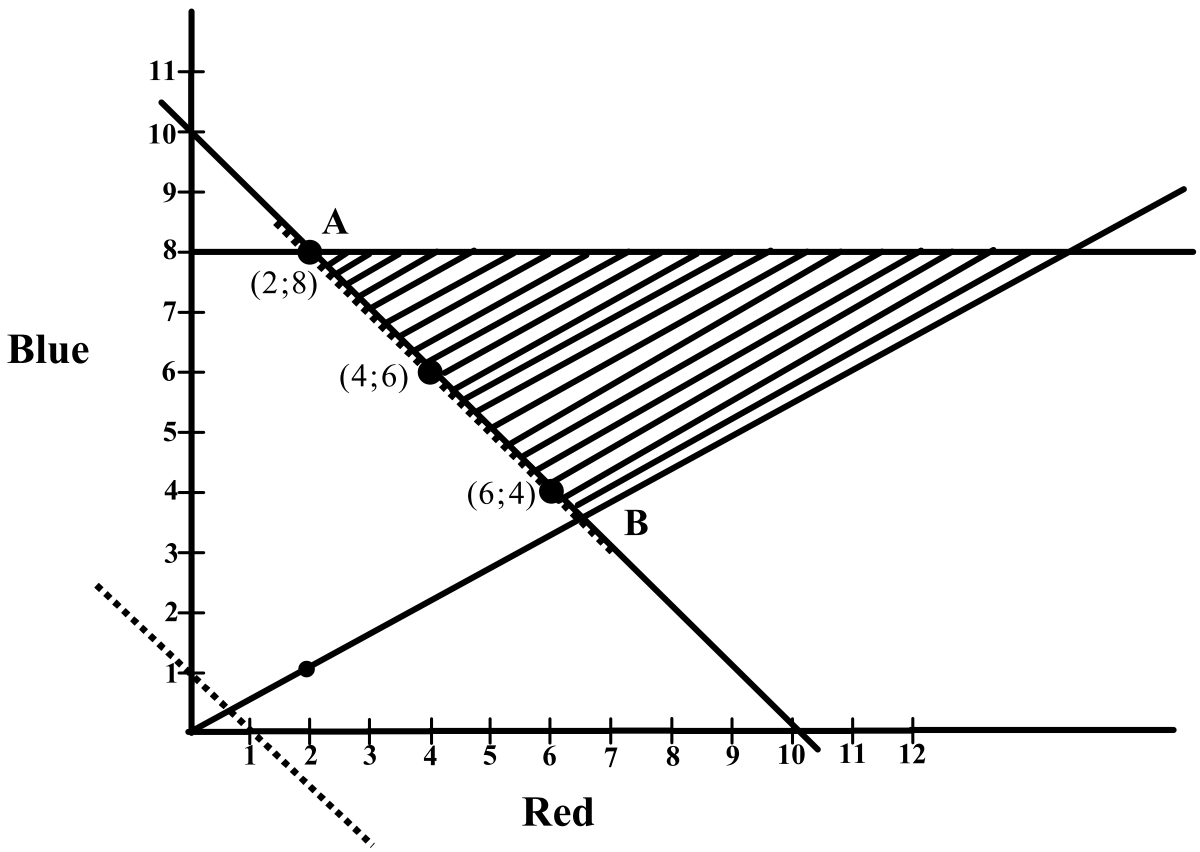
11.1	$\text{A}=(2x)(3x)+2(y)(3x)+2(y)(2x)$ $\therefore 200=6x^2+6xy+4xy$ $\therefore 200=6x^2+10xy$ $\therefore 100=3x^2+5xy$ $\therefore 100-3x^2=5xy$ $\therefore \frac{100}{5x}-\frac{3x^2}{5x}=y$ $\therefore y=\frac{20}{x}-\frac{3x}{5}$	✓ $6x^2+6xy+4xy$ ✓ $200=$ ✓ arriving at answer (3)

11.2	$V=(2x)(3x)(y)$ $\therefore V=(2x)(3x)\left(\frac{20}{x}-\frac{3x}{5}\right)$ $\therefore V=(6x^2)\left(\frac{20}{x}-\frac{3x}{5}\right)$ $\therefore V=120x-\frac{18x^3}{5}$	✓ $V=(2x)(3x)(y)$ ✓ $V=120x-\frac{18x^3}{5}$ (2)
11.3	$V(x)=120x-\frac{18}{5}x^3$ $\therefore V'(x)=120-\frac{18}{5}\times 3x^2$ $\therefore 0=120-\frac{54}{5}x^2$ $\therefore 0=600-54x^2$ $\therefore 54x^2=600$ $\therefore x^2=\frac{600}{54}$ $\therefore x^2=\frac{100}{9}$ $\therefore x=\frac{10}{3}$	✓ $V'(x)$ ✓ $V'(x)=0$ ✓ $x=\frac{10}{3}$ (3)

QUESTION 12

12.1	$x+y\geq 10$ $y\geq\frac{1}{2}x$ $y\leq 8$	✓ $x+y\geq 10$ ✓ $y\geq\frac{1}{2}x$ ✓ $y\leq 8$ (3)
12.2	see next page	
12.3	$C=40x+40y$ $\therefore 40x+40y=C$ $\therefore 40y=-40x+C$ $\therefore y=-1x+\frac{C}{40}$	✓ $C=40x+40y$ ✓ search line on diagram ✓ $(2;8)$ ✓ $(4;6)$ ✓ $(6;4)$ (5)

12.2



✓ $x + y \geq 10$

✓ $y \geq \frac{1}{2}x$

✓ $y \leq 8$

✓ feasible region

(4)