NATIONAL SENIOR CERTIFICATE EXAMINATION
EXEMPLAR 2008

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

SECTION A

QUESTION 1

(a) $-\frac{1}{3}\ ;\ -1\ ;\ -3$ (3)

(b) $T_n = ar^{n-1}$

$= (-\frac{1}{81})(3)^{n-1}$

$= -\frac{1}{243}(3)^n$

$= \ -3^{n-5}$

(2)

5 marks

QUESTION 2

(a) $\sum_{r=1}^{5}(r+b) = 1+b+2+b+3+b+4+b+5+b$

$15+5b = 10a$

$5b = 10a-15$

$\therefore b = 2a-3$

(3)

(b) $\log_a 8 - \log_a 5 - \log_a 2 + \log_a 125 = \log_a(\frac{8\ .\ 125}{5\ .\ 2})$

$= \log_a 100$

$= 2\log_a 10$

(3)

(c) $\log_{3x-7}(x^2-2x-3) = 1$

$\therefore 3x-7 = x^2-2x-3$

$x^2-5x+4 = 0$

$(x-1)(x-4) = 0$

$x \neq 1$ or $x = 4$

(5)

(d) $3^{n+1} > 20000$

$\log 3^{n+1} > \log 20000$

$n+1 > \dfrac{\log 20000}{\log 3}$

$n+1 > 9{,}014.....$

$\therefore n > 8{,}014...$

$n = 9$

The ninth term is the first to exceed 20 000.

(4)

15 marks

QUESTION 3

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$10\,000\,000 = \frac{x\left(1+\frac{0{,}15}{12}\right)\left[\left(1+\frac{0{,}15}{12}\right)^{300} - 1\right]}{\frac{0{,}15}{12}}$$

$$125000 = x(1{,}0125)\left[\left(1+\frac{0{,}15}{12}\right)^{300} - 1\right]$$

$123456{,}79.... = x[40{,}544.....]$

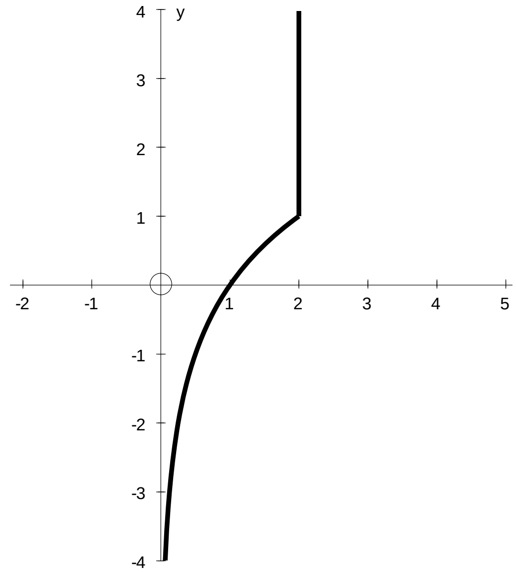
$\therefore x = \text{R}3045.00$

6 marks

QUESTION 4

(a) Not one-to-one – for $y = 2$, there are many x's (2)

(b)



(4)

(c) $x \leq 1$ (2)

(d) $x = 2^y \quad \therefore y = \log_2 x, \quad x \in \left(\frac{1}{4} ; 2\right)$

(2)

(e) $y = -2^x, \quad x \in (-2 ; 1)$ (2)

12 marks

QUESTION 5

(a) $f'(x) = 3x^2 - 12$

$x^2 - 4 = 0$

$\therefore x = \pm 2$

$(2 ; -32)$ or $(-2 ; 0)$

(6)

(b) y-intercept: $y = -16$

x-intercept: $x^3 - 12x - 16 = 0$

$x^2(x+2) - 2x(x+2) - 8(x+2) = 0$

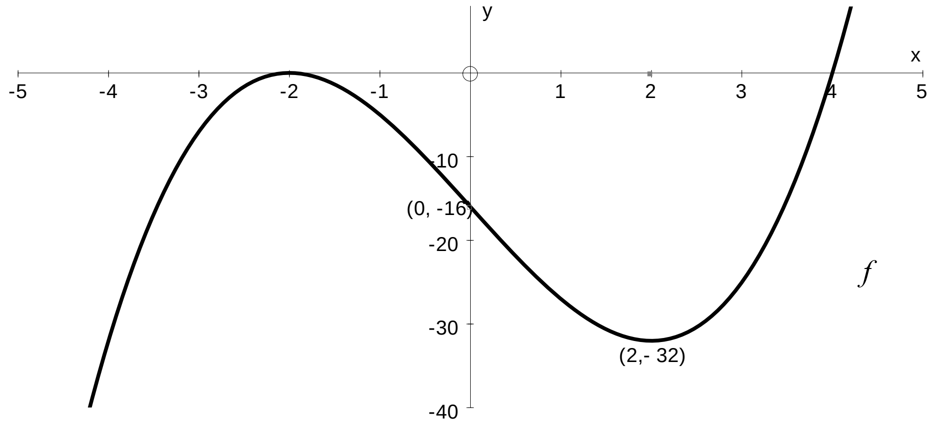
$(x+2)(x^2 - 2x - 8) = 0$

$(x+2)(x-4)(x+2) = 0$

$\therefore x = -2$ or $x = 4$

(5)

(c)



(4)

(d) $y = (x-2)^3 - 12(x-2) - 16$

(2)

17 marks

QUESTION 6

(a) $\dfrac{dL}{dt} = 6 - \dfrac{1}{2}t$

(2)

(b) $\dfrac{dL}{dt} = 6 - \dfrac{1}{2}t$

$6 - \dfrac{1}{2}t = 0$

$\therefore t = 12$ minutes after the start of the race

(3)

(c) $\dfrac{dL}{dt} = 6 - \dfrac{1}{2}t$

$\therefore$ rate at 60 $= 6 - \dfrac{1}{2}(60)$

$= -24m/\text{min}$

The lead is decreasing at a rate of $24m/\text{min}$

(2)

7 marks

QUESTION 7

$f(x) = 3x^2 - 5x + 1$ and $y - 7x + 4 = 0$

$f'(x) = 6x - 5$ and $y = 7x - 4$

$\therefore 6x - 5 = 7$

$6x = 12 \quad \therefore x = 2$

$f(2) = 3(2)^2 - 5(2) + 1$

$= 3$

$y - y_1 = m(x - x_1)$

$y - 3 = 7(x - 2)$

$\underline{y = 7x - 11}$

6 marks

QUESTION 8

(a) $T_n = 5n - 2$

$T_1 = 3$

$T_2 = 8$

$.......5n - 2 \leq 100$

$5n \leq 98$

$n \leq 20,4$

$\therefore n = 20$ ie. Mumsi has to answer 20 questions

(3)

(b) 2, 6, 10, 14, ...

$d = 4$

$T_n = an + b$

$\therefore T_n = 4n - 2$

$4n - 2 \leq 100$

$4n \leq 98$

$n \leq 25,5$

$\therefore n = 25$ ie. David has to answer 25 questions

(4)

7 marks

SECTION B

QUESTION 9

(a) $r = 4k - 3$

$\therefore$ the series converges if $-1 < 4k - 3 < 1$

$2 < 4k < 4$

$\frac{1}{2} < k < 1$

(4)

(b) $4(\frac{4}{5}) - 3 = \frac{1}{5}$

$\sum_{n=1}^{\infty}(\frac{1}{5})^n \rightarrow a = \frac{1}{5}, \; r = \frac{1}{5}$

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{5}}{1-\frac{1}{5}}$

$= \frac{1}{4}$

(5)

9 marks

QUESTION 10

(a) $C = 2\pi r$

$\therefore \frac{1}{2}C = \pi r$ ie. $x = \pi r$

$r = \frac{x}{\pi}$

(2)

(b) $4x = 400$

$\therefore x = 100$

$r = \frac{100}{\pi} = 31{,}83m$

(2)

(c) (i) $2x + 2\pi(r+1) = 406{,}28m$

(ii) $2x + 2\pi(r+9) = 456{,}54m$

(2)

(d) $T_n = 2x + 2\pi(r+n)$

$=200 + 2\pi(\frac{100}{\pi} + n)$

$= 400 + 2\pi n$

$T_1 = 400 = a$

$d = 2\pi$

$S_{10} = \frac{10}{2}[2(400) + (10-1)2\pi]$

$= 4283m$

(4)

12 marks

QUESTION 11

$(1+\frac{0,16}{4})^4 = (1+\frac{i}{2})^2$

$\frac{i}{2} = 0,0816$ (per half-year)

NB. Convert quarterly rate to half-yearly rate so that rate and payments period coincide!

$P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$P_v = \frac{2500[1-(1+0,0816)^{-15}]}{0,0816}$

$P_v = R21\,191,22$

7 marks

QUESTION 12

(a) y-intercept : $2,5m$ (1)

(b) Let $x = 12$

$y = -0,05(12) - 0,005(12)^2 + 2,5$

$y = 1,18$

The ball is higher than the net and passes over the net at a height of 1,18m above the ground. (2)

(c) x-intercept: $-0,005x^2 - 0,05x + 2,5 = 0$

$x^2 + 10x - 500 = 0$

$x = 17,91m$

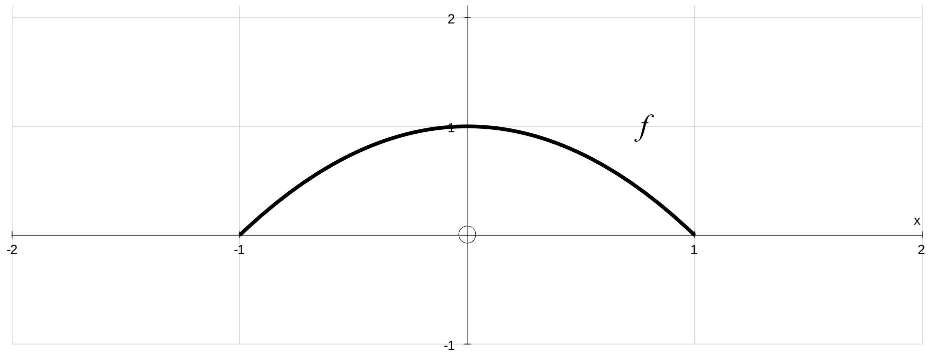
Therefore, Jason's serve is 'in' (within the 18m service line!) (4)

7 marks

QUESTION 13

(a) $p = 3$ (1)

(b) (i) (2)



(ii) Range of $f(x)$: $y \in [0\ ;\ 1]$ (1)

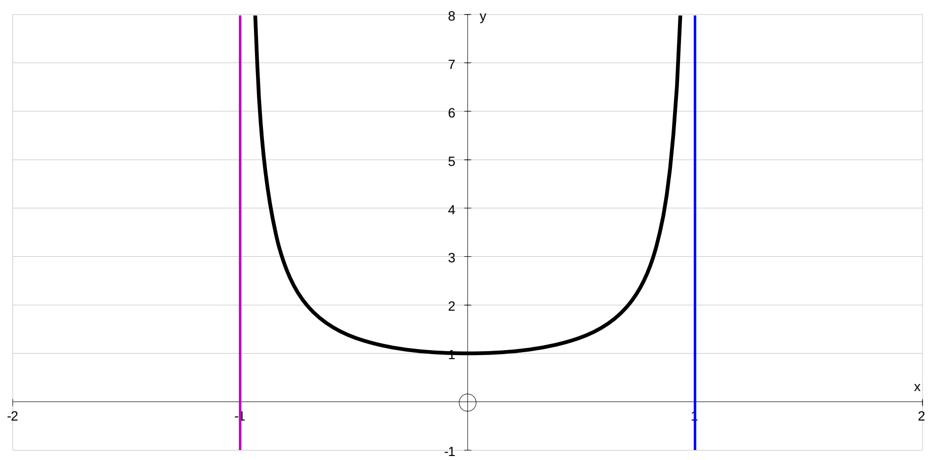
(iii) Range of $g(x)$: $y \in [1\ ;\ \infty)$ (2)

(iv) $g(x) = \dfrac{1}{(1-x)(1+x)}$

$\therefore$ asymptotes are $x = 1$ and $x = -1$

(2)

(v)



[11]

(3)

11 marks

QUESTION 14

(a) $t = \frac{d}{s}$

$\therefore t = \frac{20}{x}\ hrs$

(1)

(b) $C = 245.\frac{20}{x} + (72 + \frac{x}{40})\ .\ 20$

$C = 4900x^{-1} + 1440 + \frac{1}{2}x$

$\frac{dC}{dx} = -4900x^{-2} + \frac{1}{2}$

For min cost, $\frac{dC}{dx} = 0$

$-4900x^{-2} + \frac{1}{2} = 0$

$x^2 = 9800$

$\therefore x = 99 km/hr$

(6)

(c) $C = 245.\frac{20}{x} + (72 + \frac{x}{40})\ .\ 20$

$C = \frac{4900}{99} + 1440 + \frac{99}{2}$

$C = R15,39$

(2)

9 marks

QUESTION 15

(a) $x + y \leq 25$ and $x \geq 14$

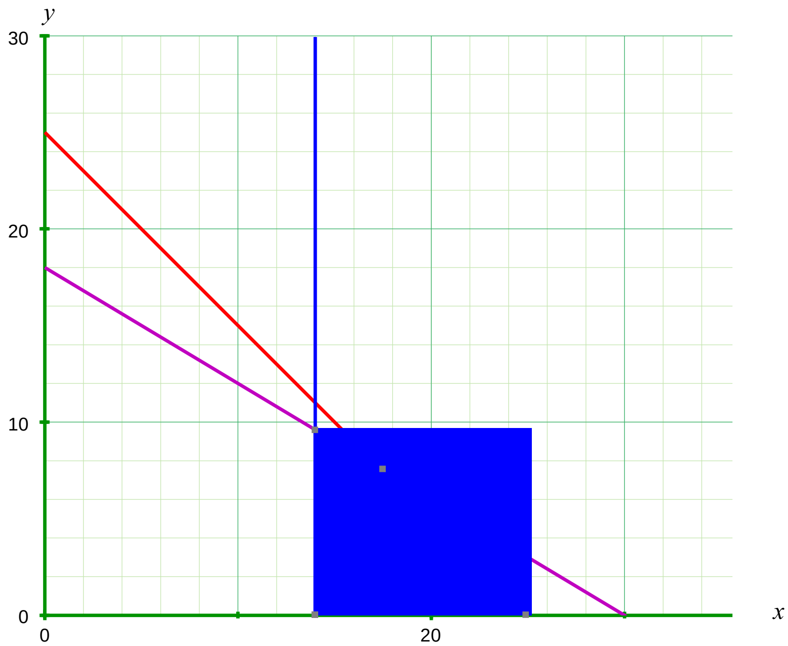
$\frac{x}{5}.1000 + \frac{y}{3}.1000 \leq 6000$

$\frac{x}{5} + \frac{y}{3} \leq 6$

$3x + 5y \leq 90$

(6)

(b)



(5)

(c) $P = 10y + 6x$

(2)

(d) $10y = -6x + P$

$$y = -\frac{6}{10}x + \frac{P}{10}$$

(2)

Therefore, maximum points along $3x + 5y = 90$, $x \in [14\,;17{,}5)$ or the last point within the feasible region such that x and $y \in$ N. 15 men and 9 women.

15 marks

QUESTION 16

THROWS	% HITS	NO. OF HITS
n	p	$n \times \frac{p}{100}$
$n+1$	$p+1$	$n \times \frac{p}{100} + 1$ or $\frac{(n+1)(p+1)}{100}$

$$\frac{np}{100} + 1 = (n+1)(\frac{p+1}{100})$$
$$np + 100 = (n+1)(p+1)$$
$$np + 100 = np + n + p + 1$$
$$\therefore n + p = 99$$

5 marks

TOTAL FOR THIS PAPER: 150 MARKS