NATIONAL SENIOR CERTIFICATE EXAMINATION
EXEMPLAR 2008

MATHEMATICS: PAPER I
(LO 1 AND LO 2)

Time: 3 hours | 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 12 pages, including a Formula Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Questions 15 and 16 must be answered on the answer sheet provided and this sheet must be handed in with your answer booklet.
5. Number your answers exactly as the questions are numbered.
6. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.
7. Round off your answers to one decimal digit where necessary.
8. All the necessary working details must be clearly shown.
9. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

Given the geometric sequence $-\frac{1}{81}\ ;\ -\frac{1}{27}\ ;\ -\frac{1}{9}\ ;\ ....$:

(a) Write down the next three terms of the sequence. (3)

(b) Find an expression for the n^{th} term. (2)

5 marks

QUESTION 2

(a) If $\sum_{r=1}^{5}(r+b)=10a$, determine b in terms of a. (3)

(b) Simplify: $\log_a 8-\log_a 5-\log_a 2+\log_a 125$ (3)

(c) Solve for x: $\log_{3x-7}(x^2-2x-3)=1$ (5)

(d) Given $T_n=3^{n+1}$. Which term is the first to exceed 20 000? (4)

15 marks

QUESTION 3

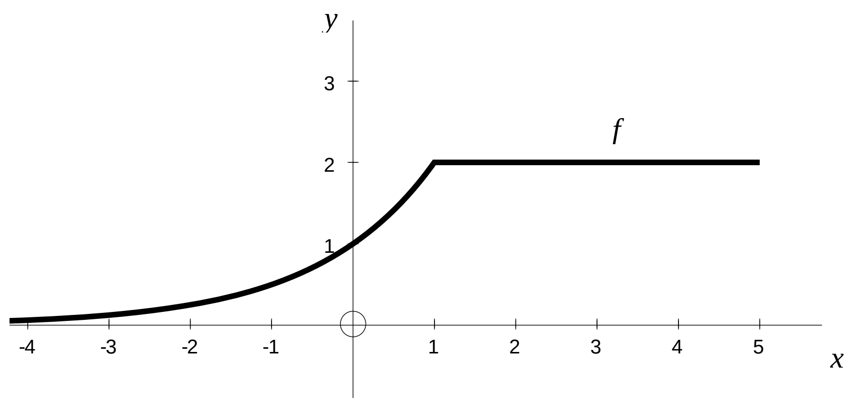
Sarah is 25 years old and wishes to accumulate R10 000 000 by her 50^{th} birthday. She will deposit equal monthly payments into an account that pays 15% p.a. compounded monthly. Payments start on her 25^{th} birthday and end on the month just before her 50th birthday (so she makes 300 payments).

Find the monthly deposit that Sarah will make.

6 marks

QUESTION 4

Use the sketch of f in the diagram to answer the following questions:



(a) Is f a one-to-one function? Explain. (2)

(b) Sketch the graph of the inverse of f. (4)

(c) Maintaining the range, give a restriction on the domain of f that would ensure that f^{-1} is a function. (2)

(d) If f is an exponential function when $x \in (-2\ ;\ 1)$, find the equation of $f^{-1}(x)$. Give your answer in the form y = …….. (2)

(e) Write down the equation of $h(x)$ which is the reflection of f about the x-axis when $x \in (-2\ ;\ 1)$. (2)

12 marks

QUESTION 5

Given $f(x) = x^3 - 12x - 16$

(a) Determine the co-ordinates of the local maximum/ minimum. (6)

(b) Determine the x- and y-intercepts. (5)

(c) Sketch $f(x)$, clearly labelling all relevant points. (4)

(d) If $f(x)$ is shifted so that its local maximum coincides with the origin, what is the equation of this new graph? It is not necessary to simplify the equation. (2)

17 marks

QUESTION 6

The lead, L, in metres, of a runner in the last 75 minutes of a marathon is given by:
$L = 1000 + 6t - \frac{t^2}{4}$, where t is the time in minutes.

(a) Determine $\frac{dL}{dt}$. (2)

(b) Calculate the time at which the runner has the greatest lead. (3)

(c) At what rate is the runner's lead decreasing when $t = 60$ minutes? (2)

7 marks

QUESTION 7

Given $f(x) = 3x^2 - 5x + 1$.
Find the equation of the tangent to $f(x)$ which is parallel to $y - 7x + 4 = 0$.

6 marks

QUESTION 8

An exercise has 100 questions.

(a) A maths teacher asks Mumsi to do each question numbered $5n - 2$, where $n \in \mathrm{N}$.
How many questions is this? (3)

(b) David is told to do questions 2, 6, 10, 14, ...
Find a formula to generate this sequence and hence determine how many questions David will answer. (4)

7 marks

SECTION B

QUESTION 9

The series $(4k-3)+(4k-3)^2+(4k-3)^3+.........$ is given.

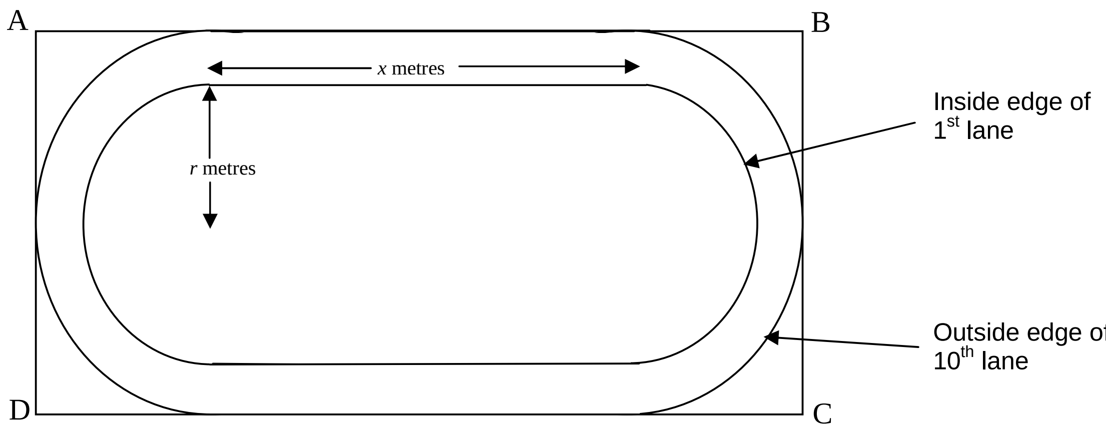
(a) For which values of k will the series converge? (4)

(b) Determine $\sum_{n=1}^{\infty}(4k-3)^n$ if $k=\frac{4}{5}$. (5)

9 marks

QUESTION 10

Part of a community service project for a matric class was to build a sports field. The athletics track was built within a rectangular field ABCD. The inside lane consisted of four **equal** parts: 2 straights, each of length x metres and 2 semi-circular sections, also each of length x metres.



Answer the following questions, giving all answers correct to 2 decimal digits, where necessary:

(a) Express r in terms of x and π. (2)

(b) If the distance of one lap on the inside edge of the 1st lane is set at 400m, determine r. (2)

(c) Each lane is 1 metre wide.
Find the distance that a runner would cover, if they ran one complete lap on the inside edge of:

(i) the 2nd lane (2)

(ii) the 10th lane (2)

(d) Jack decided to train on the track by running the first lap in lane 1, the second lap in lane 2 and so on. By using a suitable formula, or otherwise, calculate how far he would run if he used all 10 lanes. Give your answer to the nearest metre. (4)

12 marks

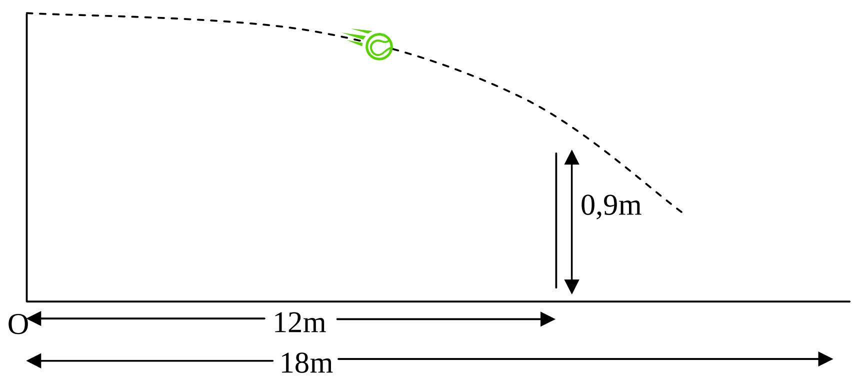
QUESTION 11

Ted needs to take out a loan to buy a new bicycle. He is able to repay the loan by means of 15 semi-annual payments of R2 500, starting in 6 months from now, when interest is 16% p.a. compounded quarterly.
Calculate the amount of the loan.

7 marks

QUESTION 12

Jason is playing tennis. The height of the ball above the ground is given by
$y = -0,05x - 0,005x^2 + 2,5$.
The origin is the point where Jason is standing and x is the horizontal distance (in metres) that the ball covers.



(a) What is the height of the ball as Jason serves? (1)

(b) The net is 0,9m high and 12m away. Show that the ball passes over the net. (2)

(c) For the serve to be 'in', it must land between the net and the service line, which is 18m away. Showing all working, determine whether Jason's serve is 'in'. (4)

7 marks

QUESTION 13

(a) If $p \in \{2\ ;\ \frac{3}{4}\ ;\ \frac{1}{2}\ ;\ 1\ ;\ 3\}$, for which values of p will $\frac{1}{p}$ have the smallest value? (1)

(b) Consider $f(x) = 1 - x^2$ where $x \in [-1\ ;\ 1]$ and $g(x) = \frac{1}{f(x)}$

(i) Sketch $f(x)$. (2)

(ii) What is the range of $f(x)$? (1)

(iii) Hence, write down the range of $g(x)$. (2)

(iv) Write down the equations of the vertical asymptotes of $g(x)$. (2)

(v) Sketch $g(x)$. (3)

11 marks

QUESTION 14

Twenty kilometres from home, Alex remembered that he had left the heater on, which will cost him R2,45 per hour. He decides to return home to turn off the heater and save electricity!

Driving at x km/h will cost him $(72 + \frac{x}{40})$ cents per km for petrol.

(a) Write down an expression for time, t in hours, that Alex takes to return home. (1)

(b) At what speed should he drive to minimise the total additional cost of petrol and electricity? (6)

(c) Calculate this additional cost. (2)

9 marks

QUESTION 15

A sports meeting starts at 09h00 on Friday and ends on Sunday afternoon.
The manager of a team comprising *x* men and *y* women has the following restrictions to take into account:

The maximum number in a team is 25, of which at least 14 must be men.

The team has only R6000 to spend on accommodation. Rooms are made available for the duration of the meeting (two nights accommodation is required). 5 men are allocated to a room, whilst only 3 women are allocated to a room. Each room costs R500 per night.

(a) Give all inequalities, showing that one of the restrictions simplifies to the inequality $3x+5y \leq 90$.

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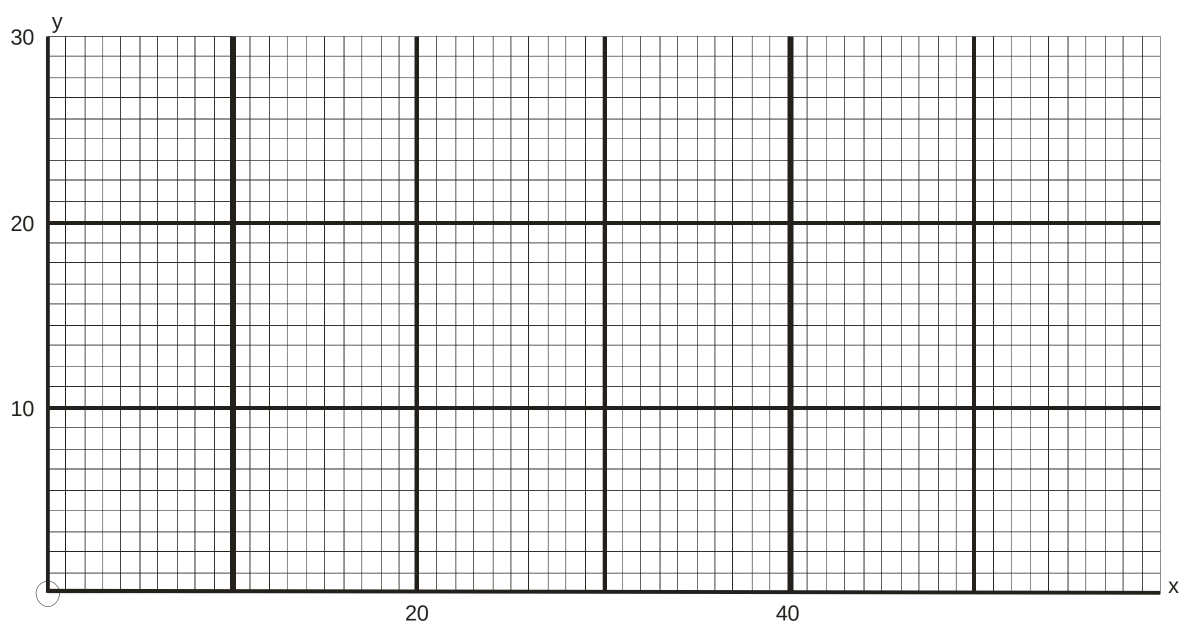
____ (6)

(b) Represent the inequalities graphically, on the axes below, and indicate the feasible region.



(5)

(c) If individual women obtain 10 points for a win and individual men obtain 6 points for a win, write down an expression for the number of points it is possible for the team to obtain (assuming only 1 event per person).

(2)

(d) Determine the number of men and women that should be selected, in order to give the maximum possible points at the meeting.

(2)

15 marks

QUESTION 16

After throwing a dart n times at a dart board, my percentage of hits is p %.
With my next throw, I make a hit and my percentage of hits is now $(p+1)\%$.
Prove that $n + p = 99$.

5 marks

TOTAL FOR THIS PAPER: 150 MARKS

MATHEMATICS
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} \left[a + (i-1)d \right] = \frac{n}{2}\left[2a + (n-1)d \right]$$

$$\sum_{i=1}^{n} a r^{i-1} = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad \sum_{i=1}^{\infty} a r^{i-1} = \frac{a}{1-r} ; \; -1 < r < 1, \; r \neq 0$$

$$T_n = an^2 + bn + c$$

$$T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)}{2} s$$

where f is the first term of the first difference and s is the second difference

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i} \right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2} \right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\ ;\ y) = \left((x_A\cos\alpha - y_A\sin\alpha)\ ;\ (y_A\cos\alpha + x_A\sin\alpha)\right)$$

$$\bar{x} = \frac{\sum x}{n} \qquad \bar{x} = \frac{\sum fx}{n}$$

$$\text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \qquad \text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$s.d = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$