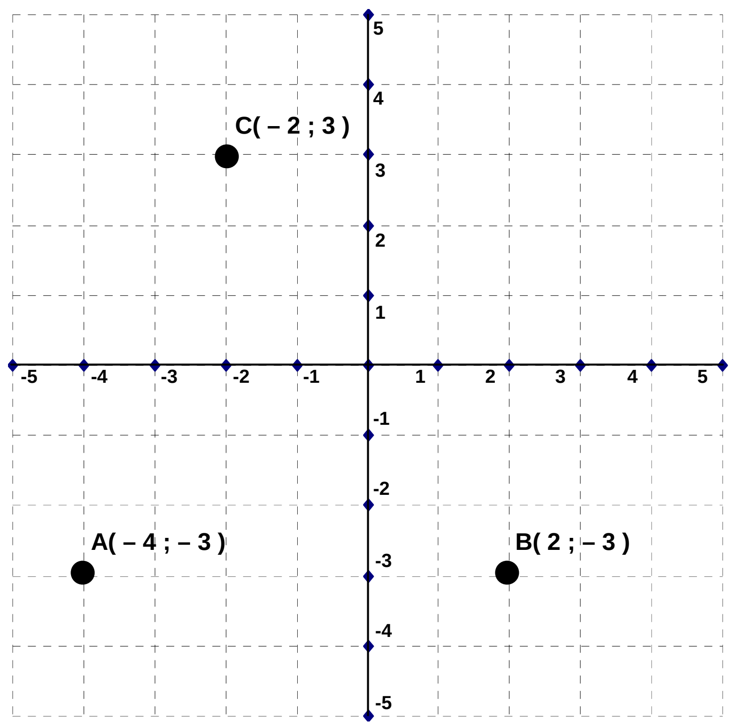
GRADE 11 EXEMPLAR
2007

MATHEMATICS: PAPER II

MEMORANDUM

Time: 3 hours | 150 marks

QUESTION 1



A(– 4 ; – 3), B(2 ; – 3) and C(– 2 ; 3) are the vertices of a triangle.

1.1 Find the gradient of AC. (2)

$$\mathbf{m_{AC} = \frac{3+3}{-2+4} = 3}$$ ✓m ✓a

1.2 Find the inclination of line AC, correct to 1 decimal place. (2)

$\mathbf{\tan\theta = 3}$ ✓m $\quad \mathbf{\theta = \tan^{-1} 3} \quad \mathbf{\theta = 71{,}6°}$ ✓a

1.3 Find the midpoint of AC and hence find the equation of the perpendicular bisector of AC, writing your equation in the form $y = mx + c$. (5)

midpoint AC = (– 3 ; 0) ✓a ✓a

$\mathbf{m_{\perp} = -\frac{1}{3}}$ ✓ca

$\mathbf{y = -\frac{1}{3}(x + 3) = -\frac{1}{3}x - 1}$ ✓ca ✓ca

1.4 Find the area of triangle ABC. (2)

$\mathbf{area = \frac{1}{2} \times base \times height = \frac{1}{2}(6)(6) = 18\ units^2}$ ✓a ✓a

11 marks

QUESTION 2

Consider the following transformations.

2.1 (x ; y) → (x + 2 ; y – 1) Find the image of A(– 3 ; 2) under the transformation. (2)

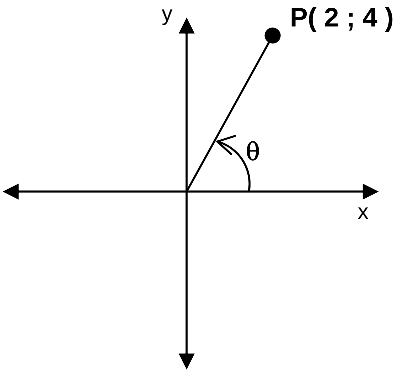
(– 1 ; 1) ✓a ✓a

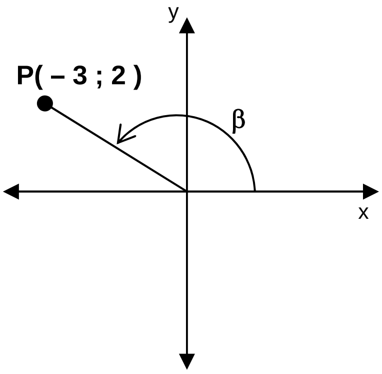
2.2 (x ; y) → (– y ; x) Find the image of B(4 ; – 1) under the transformation. (2)

(1 ; 4) ✓a ✓a

4 marks

QUESTION 3





3.1 With the aid of the diagrams, and **without using a calculator**, find the following (leaving answers in surd form where necessary)

3.1.1 $\tan\theta = \frac{4}{2} = 2$ ✓a (1)

3.1.2 $\cos\beta$ $\quad r = \sqrt{13}$ ✓a $\quad \cos\theta = \frac{-3}{\sqrt{13}}$ ✓ca (2)

3.2 Use your calculator to find the value β (to 1 decimal place). Show all working. (3)

$\tan\beta = \frac{2}{-3}$ ✓a $\quad \beta = -33{,}7°$ ✓ca $\quad \Rightarrow \beta = 146{,}3°$ ✓ca

6 marks

QUESTION 4

A group of 7 shoppers at a local supermarket spent the following amounts (rounded off to the nearest Rand) on a Saturday morning :

37 42 45 51 66 66 141

Find the following (giving your answers, where necessary, to 2 decimal places)

4.1 the median **51** ✓a (1)

4.2 the lower quartile **42** ✓a (1)

4.3 the mode **66** ✓a (1)

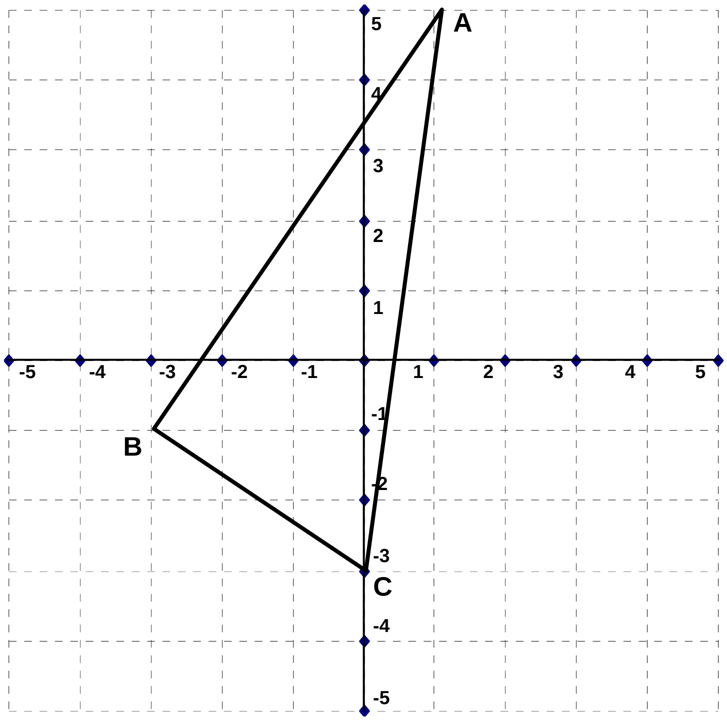
(d) the range **141 – 37 = 104** ✓a (1)

(e) the mean $\frac{448}{7}$ ✓a $= 64$ ✓ca (2)

(f) the standard deviation **(variance)** ✓a **33,12** ✓ca (2)

8 marks

QUESTION 5



A(1 ; 5), B(– 3 ; – 1), and C(0 ; – 3) are the vertices of a triangle.

5.1 Write down the co-ordinates of D if ABCD is a parallelogram. (2)

✓a ✓a

D(4 ; 3)

deduct ✓ if in wrong quadrant

5.2 Show that ABCD is in fact a rectangle. (3)

$m_{AB} = \frac{3}{2}$ ✓a $\quad m_{BC} = \frac{-2}{3}$ ✓a $\quad \therefore$ **sides** $\perp$ ✓a

✓ if works with m's but gets info wrong

5.3 If A, B and E(5 ; y) are three collinear points, find the value of y. (3)

$m_{AB} = \frac{3}{2} \qquad m_{AE} = \frac{y-5}{4}$ ✓a $\quad \frac{y-5}{4} = \frac{3}{2} \Rightarrow y = 11$ ✓a ✓ca

5.4 If the distance between C and F(8 ; p) is 10 units, find the possible values of p. (5)

$(CF)^2 = 10^2 \quad (8-0)^2 + (p-(-3))^2 = 100$ ✓a ✓a

$64 + (p+3)^2 = 100 \qquad (p+3)^2 = 36$ ✓ca

$p + 3 = \pm 6 \qquad p = -9 \text{ or } 3$ ✓ca ✓ca

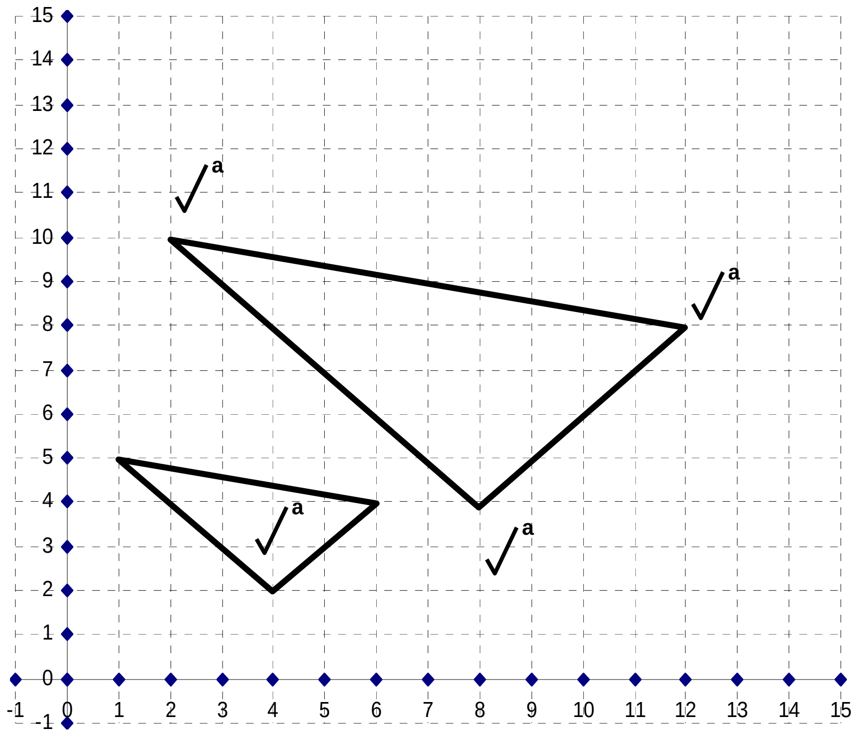
if factorise incorrectly, only lose ✓

13 marks

QUESTION 6

The vertices of ΔDEF are D(4 ; 2), E(6 ; 4) and F(1 ; 5).

ΔD'E'F' is an enlargement of ΔDEF through the origin by a constant factor of k = 2.



6.1 On the plane, draw ΔDEF and ΔD'E'F'. (4)

6.2 Find the length of OD and OD', leaving answers in surd form if necessary. (4)

$OD^2 = 4^2 + 2^2 = 20$ ✓a **OD = $\sqrt{20}$** ✓ca

$OD'^2 = 8^2 + 4^2 = 80$ ✓a **OD' = $\sqrt{80}$** ✓ca

6.3 What is the relationship between the area of ΔDEF and the area of ΔD'E'F'? (1)

area ΔD'E'F' = 4 times area ΔDEF ✓a

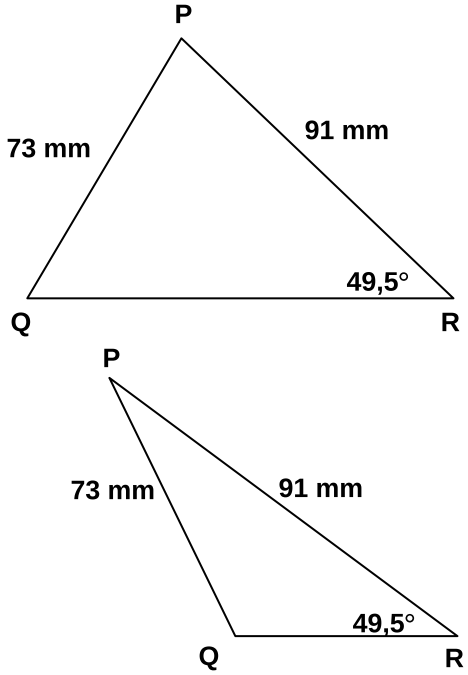
6.4 If ΔD*E*F* was an enlargement of ΔDEF through the origin by a constant factor of k = 3, write down the co-ordinates of F*. (1)

F*(3 ; 15) ✓a

10 marks

QUESTION 7

7.1



7.1.1 Find the two possible sizes of $\hat{Q}$, correct to 1 decimal place. (4)

$$\frac{\sin Q}{91} = \frac{\sin 49{,}5}{73} \checkmark^{a}$$

$$\sin Q = \frac{91(\sin 49{,}5)}{73} = 0{,}9479 \checkmark^{ca}$$

$$\hat{Q} = 71{,}4^\circ \checkmark^{ca} \quad \text{or} \quad 108{,}6^\circ \checkmark^{ca}$$

7.1.2 Hence find the greater length of QR, to 1 decimal place. (3)

$$\frac{QR}{\sin(180 - 49{,}5 - 71{,}4)} \checkmark^{a} = \frac{73}{\sin 49{,}5} \checkmark^{a} \qquad QR = 82{,}4 \text{ mm} \checkmark^{ca}$$

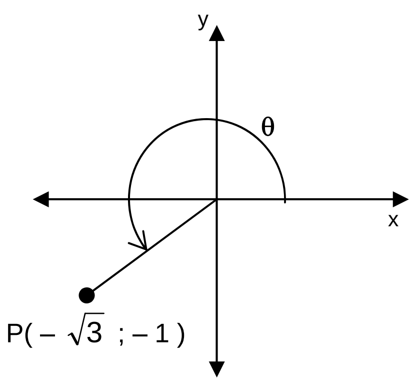
lose ✓ if choose wrong size angle for P

7.2 Simplify $\dfrac{\sin(180^\circ - A)\cdot \tan A \cdot \sin(90^\circ + A)}{\tan(180^\circ + A)\cdot \cos(-A)\cdot \sin(-A)}$ (6)

$$= \frac{(+\sin A)\checkmark^{a}(\tan A)(+\cos A)\checkmark^{a}}{(+\tan A)\checkmark^{a}(+\cos A)\checkmark^{a}(-\sin A)} \checkmark^{a}$$

$$= -1 \checkmark^{ca}$$

7.3



P is the point $(-\sqrt{3}\ ;\ -1)$; $X\hat{O}P = \theta$

7.3.1 **Without using a calculator** find the value of θ (3)

$\tan\theta = \dfrac{-1}{-\sqrt{3}}$ ✓a $\theta = 30°$ ref angle ✓a $\therefore \theta = 210°$ ✓ca

7.3.2 **Without using a calculator**, find the value of sin(2θ). Show all working and leave your answer in surd form. (3)

$2\theta = 420°$ ✓a

$\sin 2\theta = \sin 420° = \sin 60° = \dfrac{\sqrt{3}}{2}$ ✓a ✓ca

19 marks

QUESTION 8

The table represents the percentage of income spent on a certain activity for 50 families.

Percentage (p)	Frequency	Midpoint	Cumulative frequency
$12 < p \le 18$	8	15	8
$18 < p \le 24$	20	**21** ✓m	**28** ✓m
$24 < p \le 30$	12	**27**	**40**
$30 < p \le 36$	8	**33**	**48**
$36 < p \le 42$	2	39	50

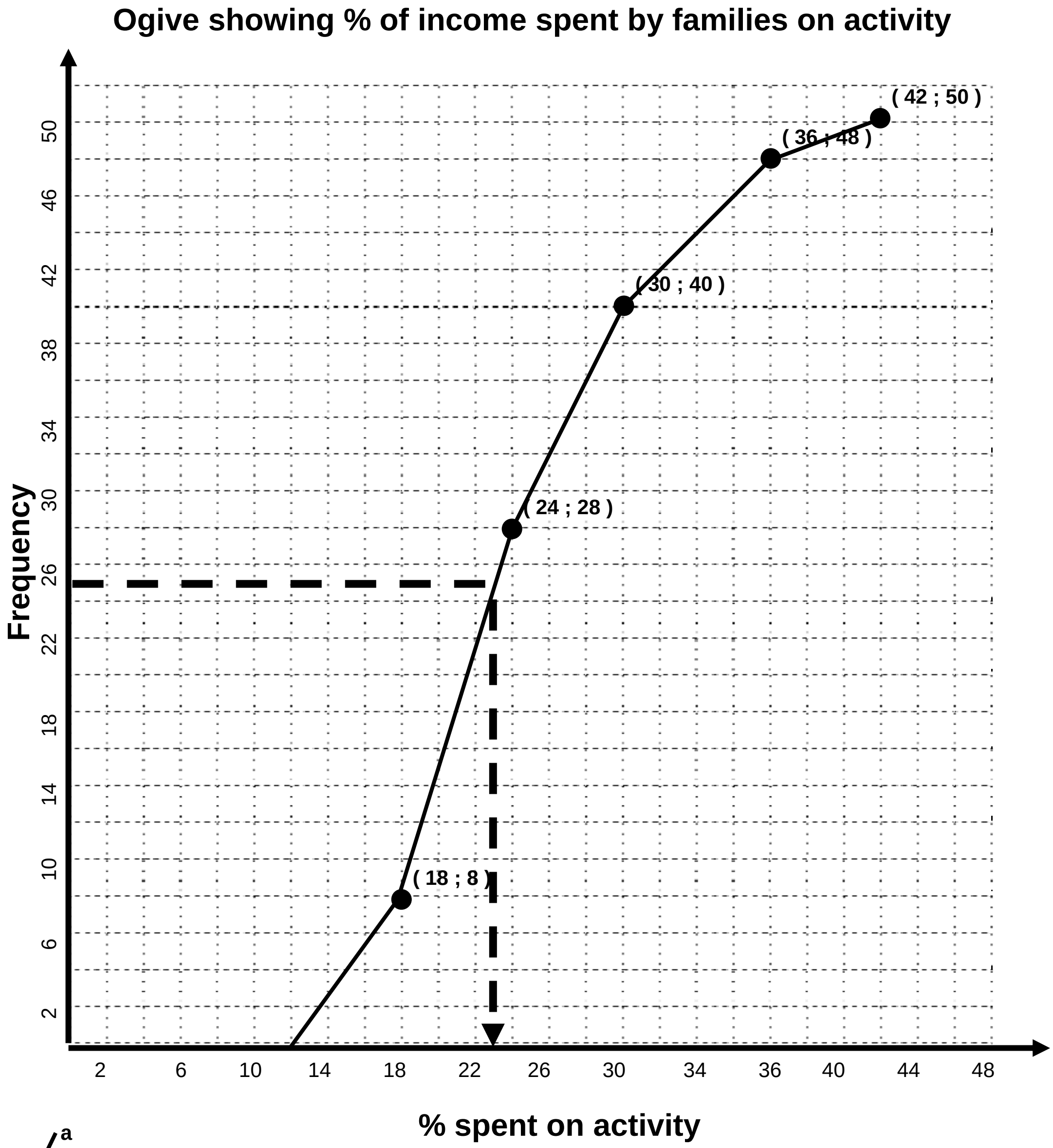
✓all correct

8.1 Complete the table. (3)

8.2 Calculate the mean and standard deviation (to 1 decimal place). (3)

$\bar{x} = \dfrac{1206}{50} = 24{,}1$ ✓a ✓ca $\sigma = 6{,}4$ ✓a

8.3 Draw an ogive (cumulative frequency polygon) of the data.
Use a horizontal scale of 1 block = 2% and a vertical scale of 1 block = 2 families. (4)



✓[a] start at 10
✓[a] appropriate scales and headings ✓[a]
points ✓[a]
ogive

8.4 From your ogive read off the median, clearly showing where you made your reading. (2)

shown on diagram ✓[m] median = about 23% ✓[a]

12 marks

QUESTION 9

The equation of a line is defined by $(3 - 2k)x + (k + 1)y = 12$

9.1 Rewrite the defining equation in the form $y = mx + c$ (2)

$$\mathbf{y = \frac{2k-3}{k+1}x + \frac{12}{k+1}} \checkmark^{a} \checkmark^{a} \quad \textbf{or} \quad \mathbf{y = -\frac{3-2k}{k+1}x + \frac{12}{k+1}}$$

9.2 Find the value of k if

9.2.1 the line is parallel to the line defined by $y = 4x + 7$ (3)

$$\mathbf{\frac{2k-3}{k+1} = 4} \checkmark^{a} \qquad \mathbf{2k - 3 = 4k + 4} \checkmark^{ca} \qquad \Rightarrow \mathbf{k = \frac{-7}{2}} \textbf{ or } \mathbf{-3\frac{1}{2}} \checkmark^{ca}$$

9.2.2 the line passes through the point (– 3 ; 4) (3)

$$\mathbf{(3 - 2k)(-3) + (k + 1)(4) = 12} \checkmark^{a} \checkmark^{ca} \qquad \Rightarrow \mathbf{k = \frac{17}{10}} \checkmark^{ca}$$

9.2.3 the line is parallel to the x-axis (1)

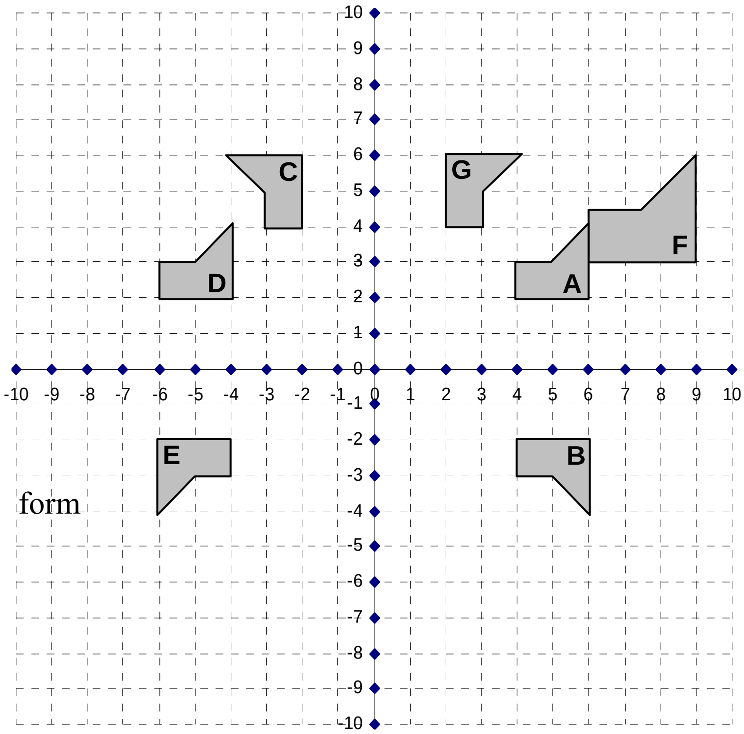
$$\mathbf{\frac{2k-3}{k+1} = 0} \qquad \Rightarrow \mathbf{k = \frac{3}{2}} \checkmark^{a}$$

9.2.4 the line is parallel to the y-axis (1)

$$\mathbf{k = -1} \checkmark^{a}$$

10 marks

QUESTION 10 [12 MARKS]



In the figure on the left, pentagons B, C, D, E, F and G are ALL images of **pentagon A** under different transformations.

Describe EACH transformation in words and then write down the rule for the transformation, giving your answer in the form

$(x ; y) \rightarrow \ldots\ldots$ (12)

A → B **Reflect in x-axis** ✓[a] $\mathbf{(x ; y) \rightarrow (x ; -y)}$ ✓[a]	**A → C** **Rotate anticlockwise 90°** ✓[a] $\mathbf{(x ; y) \rightarrow (-y ; -x)}$ ✓[a]
A → D **Translate 10 units left** ✓[a] $\mathbf{(x ; y) \rightarrow (x - 10 ; y)}$ ✓[a]	**A → E** **Rotate 180°** ✓[a] $\mathbf{(x ; y) \rightarrow (-x ; -y)}$ ✓[a]
A → F **Enlarge by a factor of 1,5** ✓[a] $\mathbf{(x ; y) \rightarrow (1{,}5x ; 1{,}5y)}$ ✓[a]	**A → G** **Reflect in line y = x (or 45° line)** ✓[a] $\mathbf{(x ; y) \rightarrow (y ; x)}$ ✓[a]

12 marks

QUESTION 11

If $\sin 20° = t$ express each of the following in terms of t

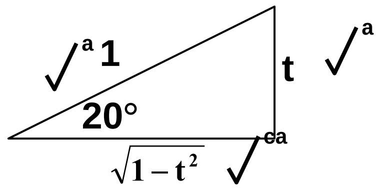
11.1 $\cos 70°$ (1)

$= \sin 20° = t$ ✓a

11.2 $\tan 20°$ (4)

$$= \frac{\sin 20°}{\cos 20°} \checkmark^{a} = \frac{\sin 20°}{\sqrt{1-\sin^2 20°}} \checkmark^{a} = \frac{t}{\sqrt{1-t^2}} \checkmark^{a}\checkmark^{ca}$$

OR using a diagram



✓ca **Hence result**

5 marks

QUESTION 12

12.1 Prove the identity

$$\frac{(\tan^2\theta - \sin^2\theta)\left(\dfrac{\cos^2\theta}{\sin^2\theta} + 1\right)}{\tan^2\theta} = 1 \qquad (5)$$

$$\textbf{LHS} = \frac{\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \checkmark^{a}}{\tan^2\theta}\left(\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}\right) \checkmark^{a}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} \checkmark^{a} \ \frac{1}{\sin^2\theta} \checkmark^{a}$$

$= 1$ **after appropriate cancelling** ✓ca

12.2 If $\sin(2\theta - 40°) = -\frac{1}{2}$ find the values of $\theta \in [\,0° ; 360°]$ **without using a calculator.** (5)

reference angle = 30° ✓a **in 3rd & 4th quadrants**

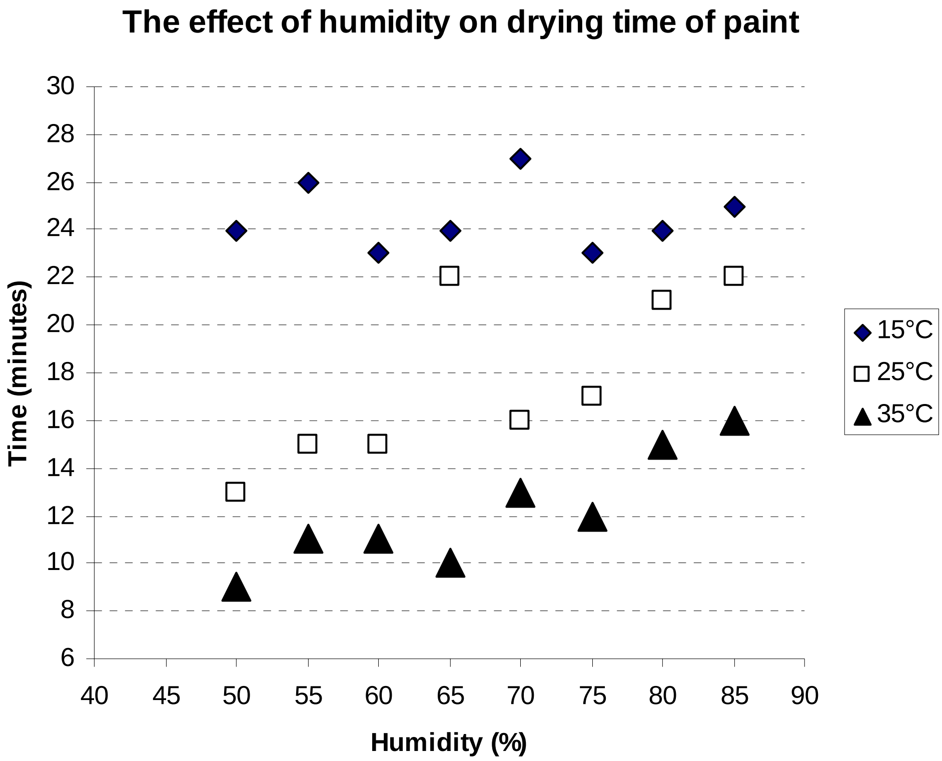
$$2\theta - 40° = \begin{cases} 210° + 360k \checkmark^{ca} \\ 330° + 360k \checkmark^{ca} \end{cases}$$

$$\theta = \begin{cases} 125° + 180k \checkmark^{ca} \\ 185° + 180k \checkmark^{ca} \end{cases}$$

$\theta \in \{5° ; 125° ; 305° ; 185°\}$

10 marks

QUESTION 13



A paint manufacturer wants to establish the effect of humidity on the drying times of it's paints at various temperatures.

The results are shown in the scatterplot, for three different temperatures.

13.1 How long does paint take to dry at 35°C with humidity of 80% ? (1)

15 minutes ✓[a]

13.2

13.2.1 On the scatterplot, roughly draw in a line-of-best-fit for the set of data measured at 25°C.
Explain below what criteria you used to draw the line. ✓[a] (2)

any suitable line with decent explanation – e.g. draw a line so that half the points are above and half below line

✓[a]

13.2.2 Is there an outlier in this data set ? If so, what is the outlying data point ? (1)

yes ; (65% ; 22 min's) ✓[a]

13.2.3 Use your line-of-best-fit to find the drying time of paint at 25°C with humidity of 65%. (1)

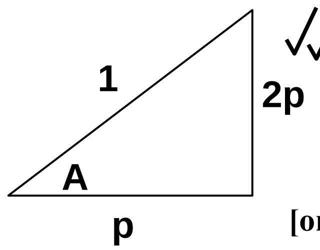
learner's line used correctly ; answer should be in the region of 16 or 17 or 18 minutes ✓[a]

5 marks

QUESTION 14

14.1 $\cos A = p$ and $\sin A = 2p$

14.1.1 find the possible values of p, leaving answers in surd form if necessary (5)



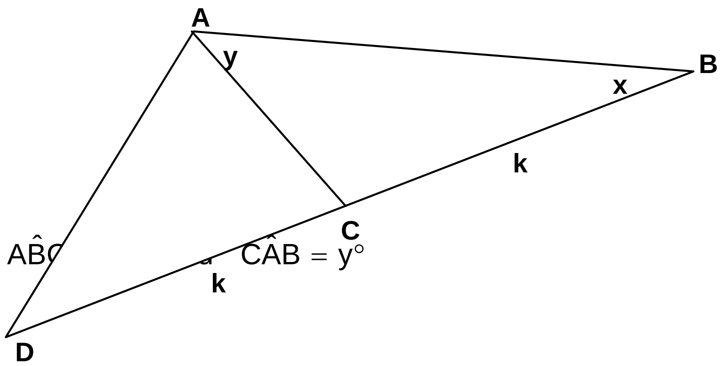
$$p^2 + 4p^2 = 1 \quad \Rightarrow p = \pm\frac{1}{\sqrt{5}}$$

[or use $\sin^2 A + \cos^2 A = 5p^2 = 1$]

14.1.2 and hence, with the aid of your calculator, find the value of A if 90° < A < 360° (to 1 dp) (3)

reference angle = 63,4° $\therefore$ **A = 243,4° (3rd quad)**

14.2



BC = CD = k.

14.2.1 Prove that area of ΔADC $= \dfrac{k^2 \cdot \sin x \cdot \sin(x+y)}{2\sin y}$ (5)

$$A\hat{C}D = x + y$$

$$\frac{AC}{\sin x} = \frac{k}{\sin y} \Rightarrow AC = \frac{k\sin x}{\sin y}$$

correct rule correct subs

$$\text{area } \Delta ADC = \frac{1}{2}(CD)(AC)\sin(A\hat{C}D) = \frac{1}{2}k\frac{k\sin x}{\sin y}\sin(x+y)$$

14.2.2 Find the area of ΔADC to 1 decimal place if k = 14,2, x = 34° and y = 41° (2)

$$\text{area} = \frac{(14{,}2)^2\sin(34°)\sin(34°+41°)}{2\sin(41°)} = 83$$

15 marks

QUESTION 15

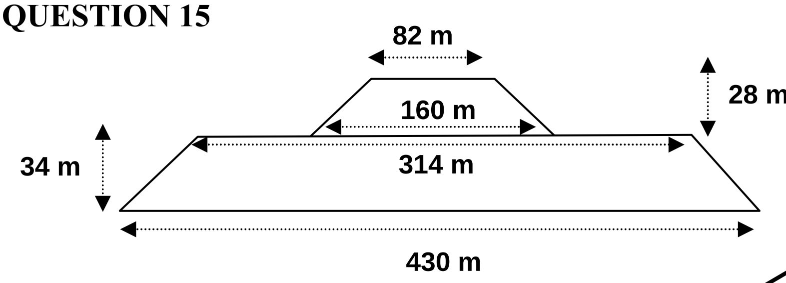


Figure 1 - Side view

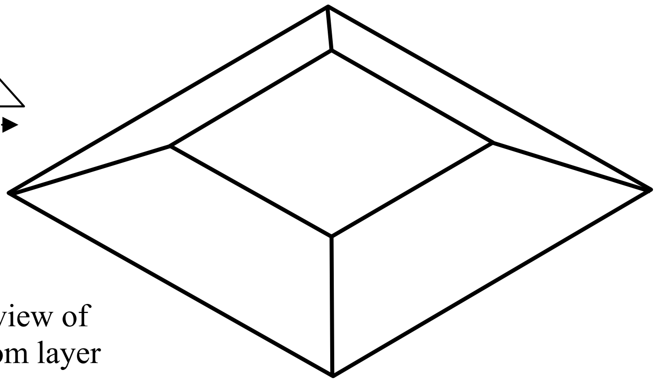
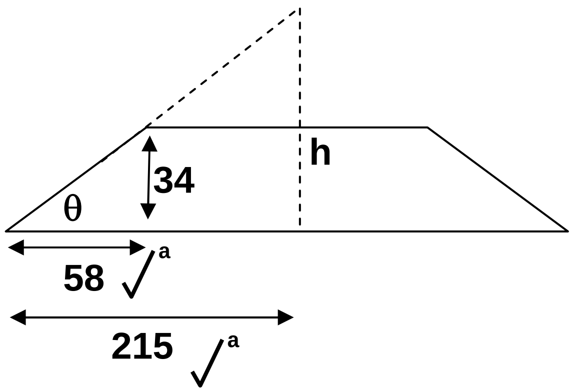


Figure 2 - Oblique view of bottom layer

The ancient Aztec pyramids consist of flat-topped pyramids placed on top of each other. A side view of the Cholula pyramid (which consists of 2 layers) is shown in Figure 1. An oblique view of a single layer is shown in Figure 2. The base and top of each layer is a **square**.

15.1 Show that IF the <u>bottom layer</u> of the Cholula pyramid had been built upwards to a point (like the Egyptian pyramids) then it would have been 126 metres high (to the nearest metre). (6)



$$\tan\theta = \frac{34}{58} = \frac{h}{215}$$ ✓m ✓ca ✓ca

h = 126 m ✓ca

[or use similar triangles]

15.2 Hence calculate the volume of the <u>bottom layer</u> of the Cholula pyramid to the nearest m^3 . (4)

vol of pyramid $= \frac{1}{3}$**(base area)(height)**

vol of FULL pyramid $= \frac{1}{3}(430)^2(126) = 7\,765\,800 \text{ m}^3$ ✓a

vol of upper truncated pyramid $= \frac{1}{3}(314)^2(126-34) = 3\,023\,610\frac{2}{3} \text{ m}^3$ ✓a

vol of bottom layer $= 7765800 - 3\,023\,610\frac{2}{3} = 4\,742\,189$ **(nearest** m^3**)** ✓m ✓ca

10 marks

Total: 150 marks