education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2009

MEMORANDUM

This memorandum consists of 13 pages.

QUESTION 1

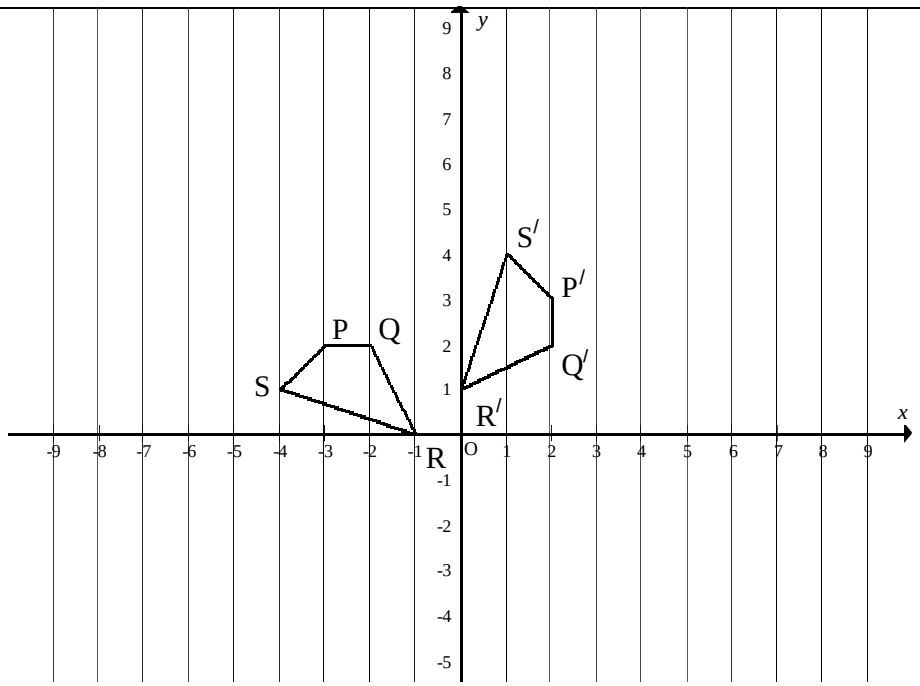
1.1	$m_{BC} = \frac{1-0}{6-3}$ $m_{BC} = \frac{1}{3}$	ü substitution into gradient formula ü answer (2)
1.2	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3}$ ---------- AB//BC ∴ Equation of AD is: $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$ **OR** $y - 6 = \frac{1}{3}(x-1)$ $y - 6 = \frac{1}{3}x - \frac{1}{3}$ $y = \frac{1}{3}x + \frac{17}{3}$	ü $m_{AC} = \frac{1}{3}$ ü substitution of (1 ; 6) into a straight line equation ü equation (3) ü $m_{AC} = \frac{1}{3}$ ü substitution of (1 ; 6) into a straight line equation ü equation (3)
1.3	$y = \frac{1}{3}x + \frac{17}{3}$ $t = \frac{1}{3}(7) + \frac{17}{3}$ $t = 8$ **OR** $\frac{t-6}{7-1} = \frac{1}{3}$ $t - 6 = 2$ $\therefore t = 8$	ü ü substitution of x value into a straight line equation. (2)

1.4	$AD = \sqrt{(8-6)^2 + (-1-3)^2}$ $AD = \sqrt{40}$ $AD = 2\sqrt{10}$ $BC = \sqrt{(6-3)^2 + (1-0)^2}$ $BC = \sqrt{10}$ $AB = \sqrt{(6-0)^2 + (1-3)^2}$ $AB = \sqrt{40}$ $AB = 2\sqrt{10}$	ü using distance formula ü answer for AD ü answer for BC ü answer for AB (4)
1.5	$m_{AB} = \frac{6-0}{1-3}$ $m_{AB} = -3$ $m_{BC} = \frac{1-0}{6-3} = \frac{1}{3}$ $m_{AB}.m_{BC} = \frac{1}{3} \times -3$ $= -1$ $\therefore AB \perp BC$	ü $m_{AB} = -3$ ü $m_{AB} \times m_{BC} = -1$ ü conclusion (3)
1.6	Area of Quad ABCD = area of ?A DC + area of ABC $= \frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ = 20 + 10 =30 square units Or Area of ABCD = $\frac{1}{2}$(sum of parallel sides)$\times h$ $= \frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ $= \sqrt{10}(3\sqrt{10})$ = 30 square units	ü formula for area of ? ü ü $\frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ ü answer (4) ü formula for area of trapezium ü ü $\frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ ü Answer (4)
1.7	From 1.1 $m_{BC} = \frac{1}{3}$ $\tan\theta = \frac{1}{3}$ $\therefore \theta = 18{,}43^o$	ü ü $\tan\theta = \frac{1}{3}$ ü $\theta = 18{,}43^o$ (3) **[21]**

QUESTION 2

2.1	Midpoint AB $\left(\frac{-8+0}{2};\frac{1+5}{2}\right)$ $= (-4; 3)$	ü substitution into midpoint formula (1)
2.2	$M_{AD} = \frac{5+1}{0-3} = \frac{-2}{1}$ $y - y_1 = m(x - x_1)$ $y - 5 = -2(x - 0)$ $y = -2x + 5$	ü substitution into gradient formula ü $M_{AD} = \frac{-2}{1}$ ü substitution of $(0; 5)$ in a straight line equation ü Answer (4)
2.3	$AM^2 = (5-3)^2 + (0+4)^2$ $AM^2 = 2^2 + 4^2$ $AM = \sqrt{20}$	ü substitution into distance formula ü simplification ü answer (3)
2.4	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $(x+4)^2 + (y-3)^2 = 20$ $x^2 + y^2 + 8x - 6y + 5 = 0$	P $(x+4)^2$ P $(y-3)^2$ P 20 ü answer (4)
2.5	$AT = TK = 6$ $CD \perp AK$ Therefore, ACKD is a kite since diagonal CD bisects diagonal AK at right angles. **OR** $C\hat{A}D = 90°$ $M_{KC}.M_{KD} = \frac{6}{-12}.\frac{6}{3} = -1$ $\therefore C\hat{K}D = 90°$? CAD & ? CKD are right angles & congruent ACKD is a kite	ü AT = TK ü CD perpendicular to AK ü Kite ü reason (4) ü ü $M_{KC}.M_{KD} = \frac{6}{-12}.\frac{6}{3} = -1$ ü ? CAD & ? CKD are right angles & congruent ü ACKD is a kite (4) **[16]**

QUESTION 3

3.1.1	$P'\left(-\sqrt{3};-2\right)$	ü Pcoordinates P' (2)
3.1.2	$P'(-\sqrt{3};2)$	ü Pcoordinates P' (2)
3.2.1	$Q'(2;2)$	ü ü coordinates Q' (2)
3.2.2		ü coordinates P' ü coordinates Q' ü coordinates R' ü coordinates S' (4)
3.2.3	$P''(4;6)$	ü ü answer (2)
3.2.4	Not rigid. The shape remains the same, whilst the size changes.	ü Not rigid ü explanation (2)
3.2.5	$(x;y)\rightarrow(y;-x)$ $(y;-x)\rightarrow(2y;-2x)$ $\therefore(x;y)\rightarrow(2y;-2x)$	ü $(y;-x)$ P ü $(2y;-2x)$ (3)
3.2.6	Area of PQRS : area $P''Q''R''S''$ $= 1^2 : 2^2$ $= 1 : 4$	ü squaring ü answer (2) **[19]**

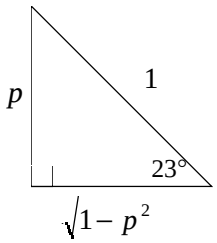
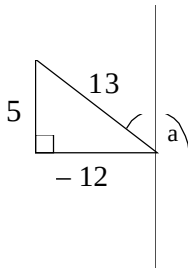
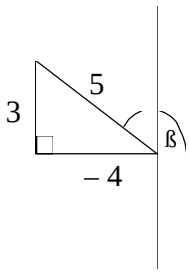
QUESTION 4

4. 1	$x' = x\cos(135°) - y\sin(135°)$ $x' = -x\cos 45° - y\sin 45°$ $x' = x\left(\frac{-\sqrt{2}}{2}\right) - y\left(\frac{\sqrt{2}}{2}\right)$ $x' = -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y$ and $y' = y\cos(135°) + x\sin(135°)$ $y' = -y\cos 45° + x\sin 45°$ $y' = y\left(-\frac{\sqrt{2}}{2}\right) + x\left(\frac{\sqrt{2}}{2}\right)$ $y' = -\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x$	ü 135° ü substitution ü answer for x ü answer for y (4)
4.2	$x' = -\frac{\sqrt{2}}{2}(2) - \frac{\sqrt{2}}{2}(4)$ $x' = -\sqrt{2} - 2\sqrt{2}$ $x' = -3\sqrt{2}$ $y' = -\frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(2)$ $y' = -\sqrt{2}$ $\therefore M(-3\sqrt{2};-\sqrt{2})$	ü x coordinates ü y coordinates (2) **[6]**

QUESTION 5

5.1	$\dfrac{\tan(180°+x)\cos(360°-x)}{\sin(180°-x)\cos(90°+x)+\cos(540°+x)\cos(-x)}$ $=\dfrac{\tan x.(\cos x)}{(\sin x).(-\sin x)-\cos x.\cos x}$ $=\dfrac{\dfrac{\sin x}{\cos x}\cos x}{-\sin^2 x-\cos^2 x}$ $=\dfrac{\sin x}{-(\sin^2 x+\cos^2 x)}$ $=-\sin x$	P $\tan x$ P $\cos x$ P $-\sin x$ P $-\sin x$ P $-\cos x$ ü $\cos x$ P simplification P answer (8)
5.2	$\dfrac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\dfrac{1-(1-2\sin^2 x)-\sin x}{2\sin x.\cos x-\cos x}$ $=\dfrac{2\sin^2 x-\sin x}{2\sin x.\cos x-\cos x}$ $=\dfrac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\dfrac{\sin x}{\cos x}$ $=\tan x$	P $1-2\sin^2 x$ P $2\sin x.\cos x$ PP factorisation P answer (5) **[13]**

QUESTION 6

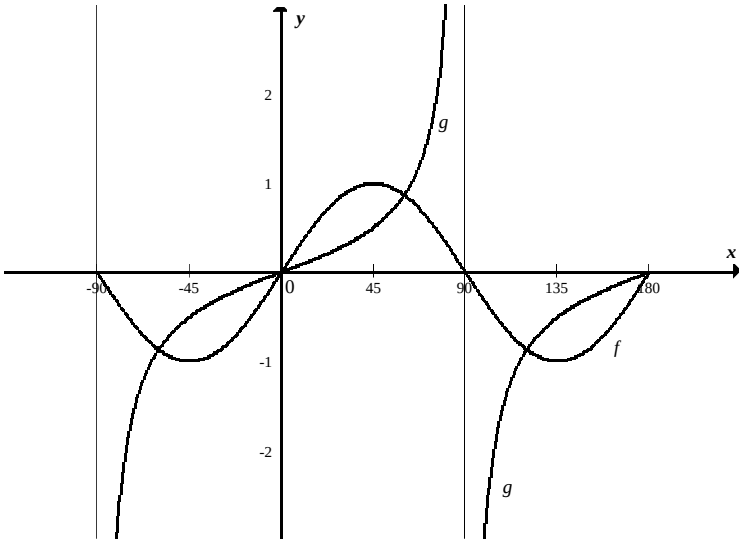
6.1.1	$\cos 113°$ $= \cos(90° + 23°)$ $= -\sin 23°$ $= -p$	P reduction P answer (2)
6.1.2	$\cos 23°$ $= \sqrt{1-p^2}$  **OR** $\cos^2 23° + \sin^2 23° = 1$ $\cos^2 23° = 1 - p^2$ $\cos 23° = \sqrt{1-p^2}$	P diagram P answer (2) **OR** P identity P answer (2)
6.1.3	$\sin 46°$ $= 2\sin 23°.\cos 23°$ $= 2p\sqrt{1-p^2}$	P expansion P answer (2)
6.2.1	$\sin\alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos\alpha = -\frac{12}{13}$  	P simplification P diagram P answer (3)
6.2.2	$\tan\beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$  $\cos(\alpha + \beta)$ $= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48-15}{65}$ $= \frac{33}{65}$	P diagram P expansion P $-\frac{4}{5}$ P $\frac{3}{5}$ P answer (5)

6.3	$\frac{1}{2}\cos x = 0,435$ $\cos x = 0,87$ $x = 29,54°$ or $x = 330,46°$	P simplification PP answers (3) **[17]**

QUESTION 7

7.1	$\frac{7}{PB} = \sin 18°$ $PB = \frac{7}{\sin 18°}$ $PB = 22,65$ m (22,65247584...)	P ratio P answer (2)
7.2	$\frac{18}{PA} = \cos 23°$ $PA = \frac{18}{\cos 23°}$ $PA = 19,55$ m (19,55448679..)	P ratio P answer (2)
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55).\cos 42°$ $= 237,0847954...$ $AB = 15,40$ m (15,3975581...)	P use of cosine rule P substitution P 237,0847… P answer (4) **[8]**

QUESTION 8

8.1		tan graph P shape P asymptotes P intercepts Sine graph P shape P intercepts P period (6)
8.2	$\sin 2x = \frac{1}{2}\tan x$ $2\sin x.\cos x = \frac{\sin x}{2\cos x}$ $4\sin x.\cos^2 x - \sin x = 0$ $\sin x(4\cos^2 x - 1) = 0$ $\sin x = 0$ or $\cos^2 x = \frac{1}{4}$ $x = 0°$ or $180°$ or $\cos x = \pm\frac{1}{2}$ $x = 60°$; $-60°$ or $120°$	P equating P $2.\sin x.\cos x$ P $\frac{\sin x}{2\cos x}$ P simplification P factorisation P $\sin x = 0$ P $x = 0°$ or $180°$ P $\cos^2 x = \frac{1}{4}$ P $\cos x = \pm\frac{1}{2}$ P answers (10)
8.3	$\{x \mid -60° < x < 0°\} \cup \{x \mid 60° < x < 90°\} \cup \{x \mid 120° < x < 180°\}$ **OR** $x \in (-60° ; 0°) \cup (60° ; 90°) \cup (120° ; 180°)$ **OR** $-60° < x < 0°$ or $60° < x < 90°$ or $120° < x < 180°$	PPP answers (3) **[19]**

QUESTION 9

	$= \frac{1}{3\sin^2 x + 3\cos^2 x + \cos^2 x}$ $= \frac{1}{3(\sin^2 x + \cos^2 x) + \cos^2 x}$ $= \frac{1}{3 + \cos^2 x}$	ü expansion ü identity ü simplification üanswer(s) (4) **[4]**

QUESTION 10

10.1 Mean $= \frac{5500}{10} = 550$ kilocalories — ü ümean (2)

10.2 $\sigma = 69{,}03$ kilocalories (done by calculator) — ü ü ü ü standard deviation (4)

OR

x	$(x - \bar{x})$	$(x_i - \bar{x})^2$
440	-1 10	12100
520	-3 0	900
480	-7 0	4900
560	10	100
615	65	4225
550	0	0
620	70	4900
680	130	16900
545	-5	25
490	-6 0	3600
Sum		47650

ü ü $(x_i - \bar{x})^2$

ü sum = 47650

$\sigma^2 = \frac{47650}{10}$

$= 4765$

$\sigma = 69{,}03$

üanswer

10.3 Snack foods have a greater variation. The standard deviation for snack foods is 69,03 kilocalories whilst the standard deviation for breakfast cereals is 28 kilocalories. i.e energy levels of breakfast cereals is spread closer to the mean than in those of the snack food.

ü snack foods
ü explanation (2)
[8]

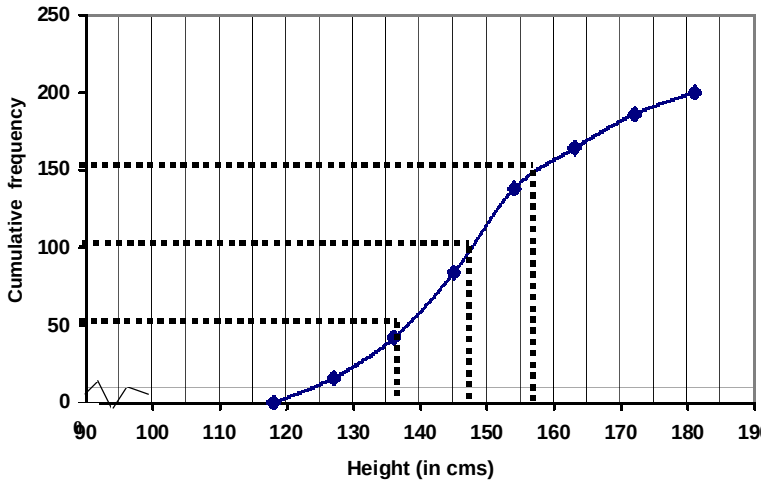
QUESTION 11

11.1

Height (in cms)	Frequency	Cumulative Frequency
$118 \leq h < 127$	16	16
$127 \leq h < 136$	26	42
$136 \leq h < 145$	42	84
$145 \leq h < 154$	54	138
$154 \leq h < 163$	26	164
$163 \leq h < 172$	22	186
$172 \leq h < 181$	14	200

ü ü answers in cumulative frequency column (2)

11.2

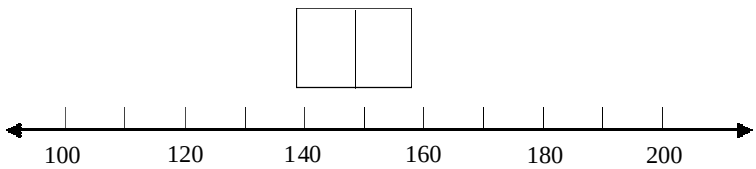


ü cumulative totals
ü points at upper limits of intervals
ü curve (3)

11.3 Lower quartile ≈ 138 cms
Median ≈ 148 cms
Upper quartile ≈ 158 cms

ü ü ü correctly read off ogive (3)

11.4



ü minimum and maximum values
ü quartiles and median
ü whiskers (3)

11.5 The heights of players are spread fairly evenly.

ü spread evenly (1)

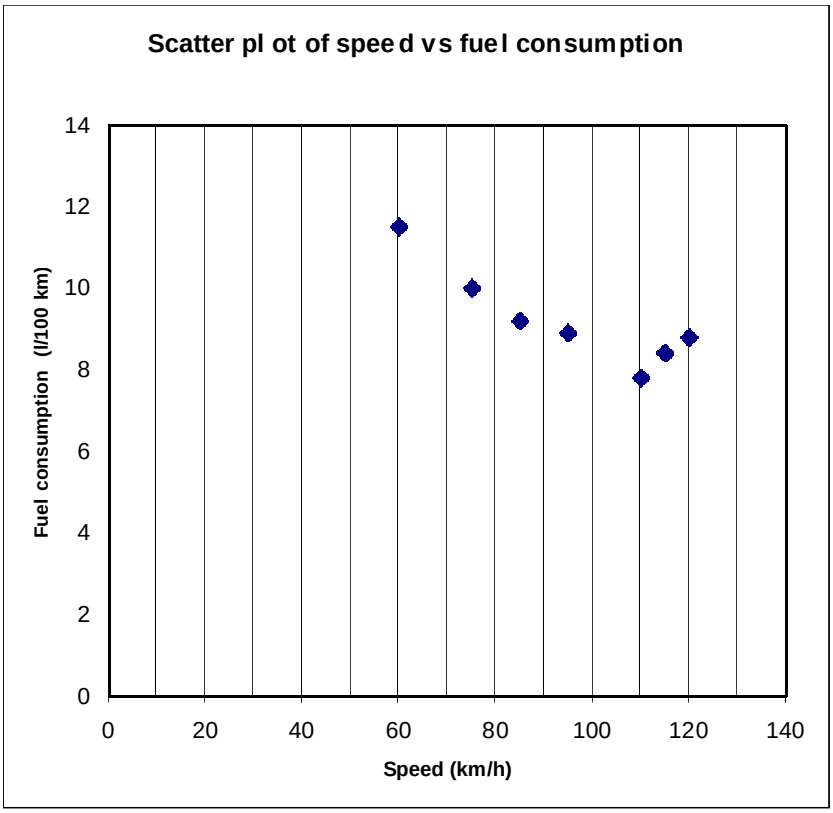
11.6 100 players fall in this height interval.

ü 100 (1)

[13]

QUESTION 12

12.1



ü üplotting points
ü labels

(3)

12.2

ü quadratic

(1)

12.3 Quadratic

ü ü answer

(2)

The quadratic pattern shows that the best fuel consumption occurs when the car is driven at 110 km/h. In order for the company to keep its fuel bill to a minimum, drivers should be urged to travel at this speed where possible.

[6]

TOTAL: 150