education

Department:
Education
REPUBLIC OF SOUTH AFRICA

# NATIONAL SENIOR CERTIFICATE

## GRADE 12

## MATHEMATICS P1

## FEBRUARY/MARCH 2009

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 10 pages, an information sheet and 2 diagram sheets.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 14 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. TWO diagram sheets for answering QUESTION 7.4, QUESTION 8.1 and QUESTION 14.2 are included at the end of this question paper. Write your examination number on these sheets in the spaces provided and hand them in together with your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

**QUESTION 1**

1.1 Solve for $x$:

1.1.1 $3x+\frac{1}{x}=4$ (4)

1.1.2 $5x(x-3)=2$ (5)

1.1.3 $x^2-2x>3$ (4)

1.2 Solve simultaneously for $x$ and $y$:

$x-3y=1$
$x^2-2xy+9y^2=17$ (7)

1.3 Calculate the value of $1234567893 \times 1234567894 - 1234567895 \times 1234567892$ (3)
**[23]**

**QUESTION 2**

Consider the series: $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+...$

2.1 Express each of the following sums as a fraction of the form $\frac{a}{b}$ :

2.1.1 The sum of the first two terms of the series (1)

2.1.2 The sum of the first three terms of the series (1)

2.1.3 The sum of the first four terms of the series (1)

2.2 Make a conjecture about the sum of the first $n$ terms of the given series. (2)

2.3 Use your conjecture to predict the value of the following:

$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+...+\frac{1}{2008\times 2009}$ (1)
**[6]**

## QUESTION 3

The following is an arithmetic sequence:

$$1 - p \ ; \ 2p - 3 \ ; \ p + 5 \ ; \ ...$$

3.1 Calculate the value of $p$. (3)

3.2 Write down the value of:

3.2.1 The first term of the sequence (1)

3.2.2 The common difference (1)

3.3 Explain why none of the numbers in this arithmetic sequence are perfect squares. (2)
**[7]**

## QUESTION 4

Consider the sequence: $6 \ ; \ 6 \ ; \ 2 \ ; \ -6 \ ; \ -18 \ ; \ ...$

4.1 Write down the next term of the sequence, if the sequence behaves consistently. (1)

4.2 Determine an expression for the $n^{\text{th}}$ term, $T_n$. (5)

4.3 Show that $-6838$ is in this sequence. (4)
**[10]**

## QUESTION 5

A sequence of squares, each having side 1, is drawn as shown below. The first square is shaded, and the length of the side of each shaded square is half the length of the side of the shaded square in the previous diagram.

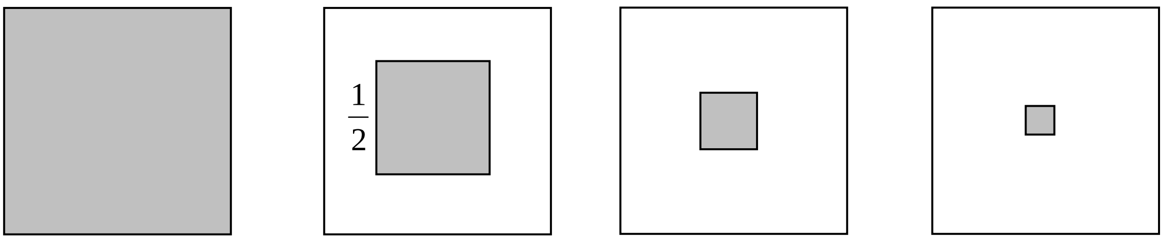



**DIAGRAM 1** **DIAGRAM 2** **DIAGRAM 3** **DIAGRAM 4**

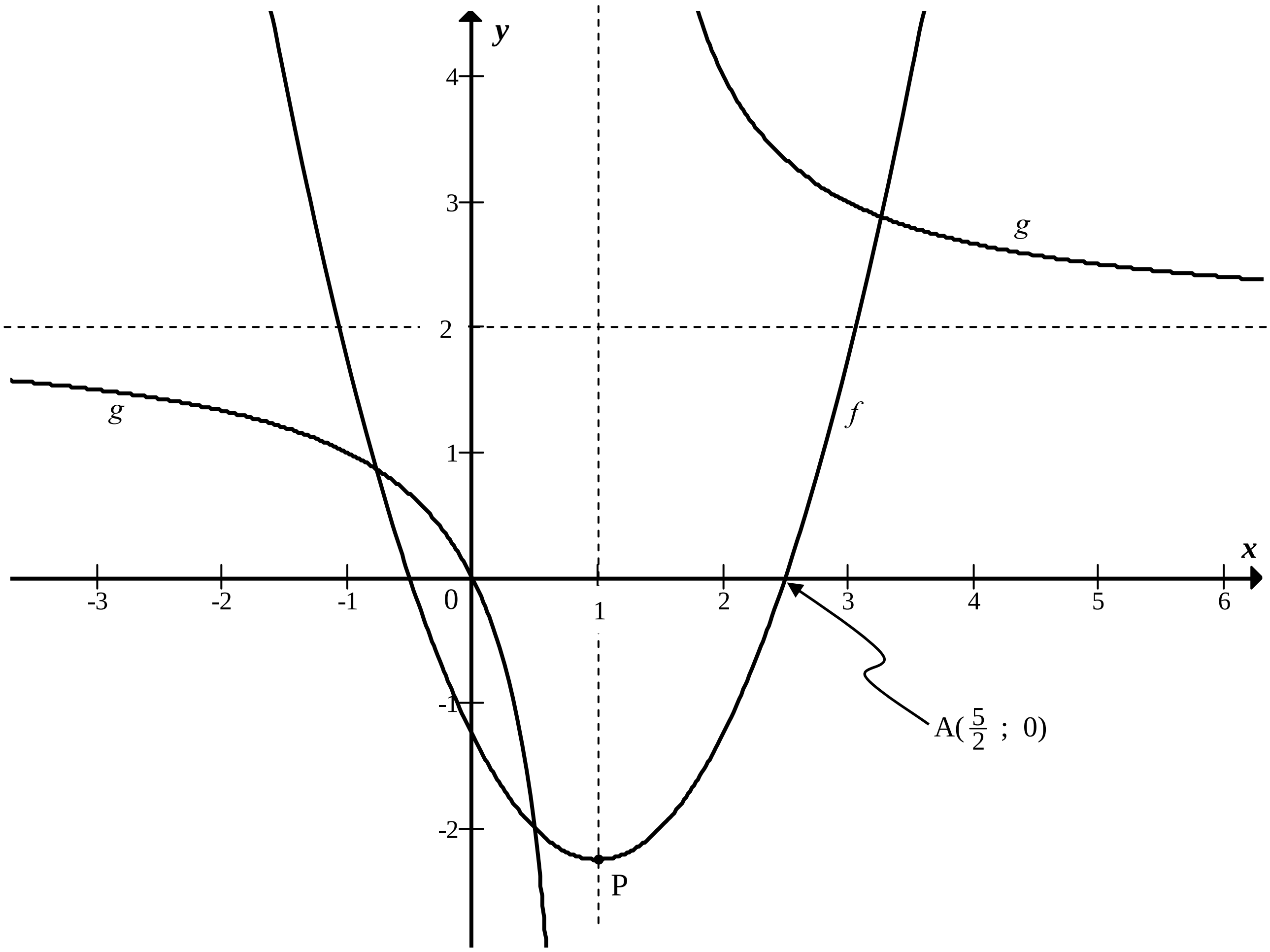
5.1 Determine the area of the unshaded region in DIAGRAM 3. (2)

5.2 What is the sum of the areas of the unshaded regions on the first seven squares? (5)
**[7]**

**QUESTION 6**

Sketched below are the graphs of $f(x) = (x - p)^2 + q$ and $g(x) = \dfrac{a}{x - b} + c$.

A(2½ ; 0) is a point on the graph of *f*. P is the turning point of *f*. The asymptotes of *g* are represented by the dotted lines. The graph of *g* passes through the origin.




6.1 Determine the equation of *g*. (4)

6.2 Determine the coordinates of P, the turning point of *f*. (4)

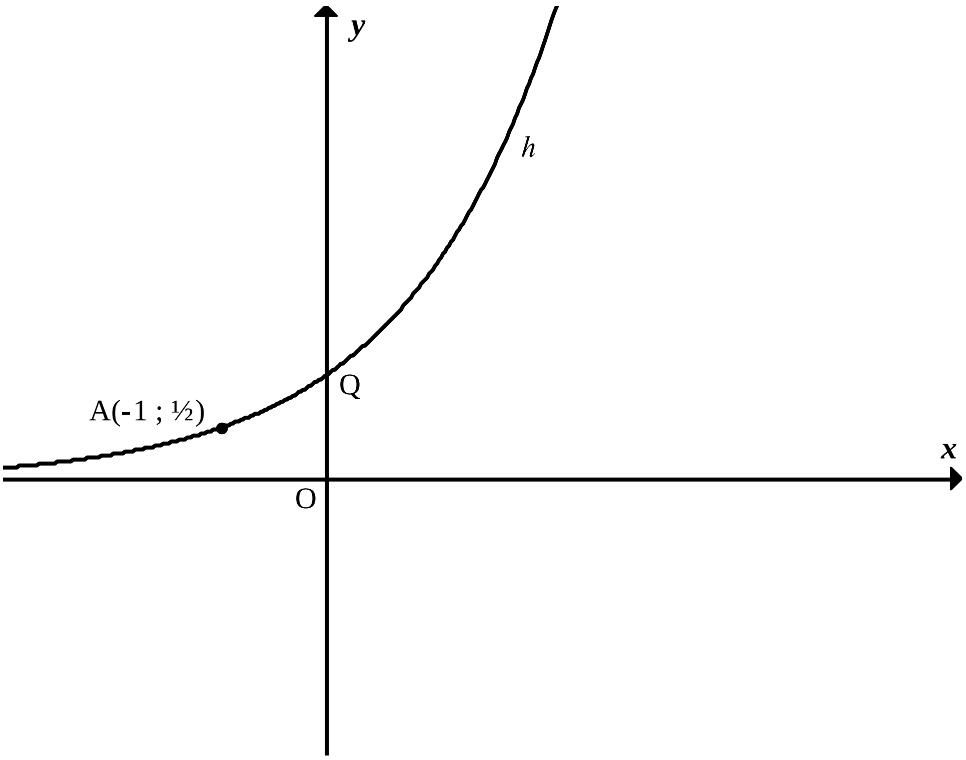
6.3 Write down the equations of the asymptotes of $g(x - 1)$. (2)

6.4 Write down the equation of *h*, if *h* is the image of *f* reflected in the *x*-axis. (1)

**[11]**

## QUESTION 7

The graph of $h(x) = a^x$ is sketched below.
$A\left(-1\,;\,\frac{1}{2}\right)$ is a point on the graph of $h$.




| | | |
|---|---|---|
| 7.1 | Explain why the coordinates of Q are (0 ; 1). | (2) |
| 7.2 | Calculate the value of $a$. | (2) |
| 7.3 | Write down the equation for the inverse function, $h^{-1}$, in the form $y = ...$ | (2) |
| 7.4 | Draw a sketch graph, on DIAGRAM SHEET 1, of $h^{-1}$. Indicate on this graph the coordinates of two points that lie on this graph. | (3) |
| 7.5 | Read off from your graph the values of $x$ for which $\log_2 x > -1$. | (2) |
| 7.6 | If $g(x) = (100).3^x$, determine the value of $x$ for which $h(x) = g(x)$. | (3) |
| | | **[14]** |

**QUESTION 8**

Consider: $f(x) = 2\sin x$

8.1 Draw a sketch graph of $f$ on DIAGRAM SHEET 1, for $x \in [-180° ; 360°]$. (2)

8.2 Write down the range of $h(x) = 2f(x)$. (2)

8.3 Write down the period of $h(x) = f\left(\frac{x}{2}\right)$. (2)

8.4 Give a value of $\theta$ if $f(x+\theta) = 2\cos x$. (2)
**[8]**

**QUESTION 9**

9.1 R2 000 was invested in a fund paying *i*% interest compounded monthly. After 18 months the value of the fund was R2 860,00. Calculate $i$, the interest rate. (4)

9.2 On 31 January 2008 Farouk banked R100 in an account that paid 8% interest per annum, compounded monthly. He continued to deposit R100 on the last day of every month until 31 December. He was hoping to have enough money on 1 January 2009 to buy a bike for R1 300. Determine whether he will be able to do so, or not. (5)
**[9]**

**QUESTION 10**

Rowan plans to buy a car for R125 000,00. He pays a deposit of 15% and takes out a bank loan for the balance. The bank charges 12,5% p.a. compounded monthly.

Calculate:

10.1 The value of the loan borrowed from the bank (1)

10.2 The monthly repayment on the car if the loan is repaid over 6 years (5)
**[6]**

**QUESTION 11**

11.1 Differentiate $f$ by first principles where $f(x) = x^2 - 2x$. (5)

11.2 Evaluate:

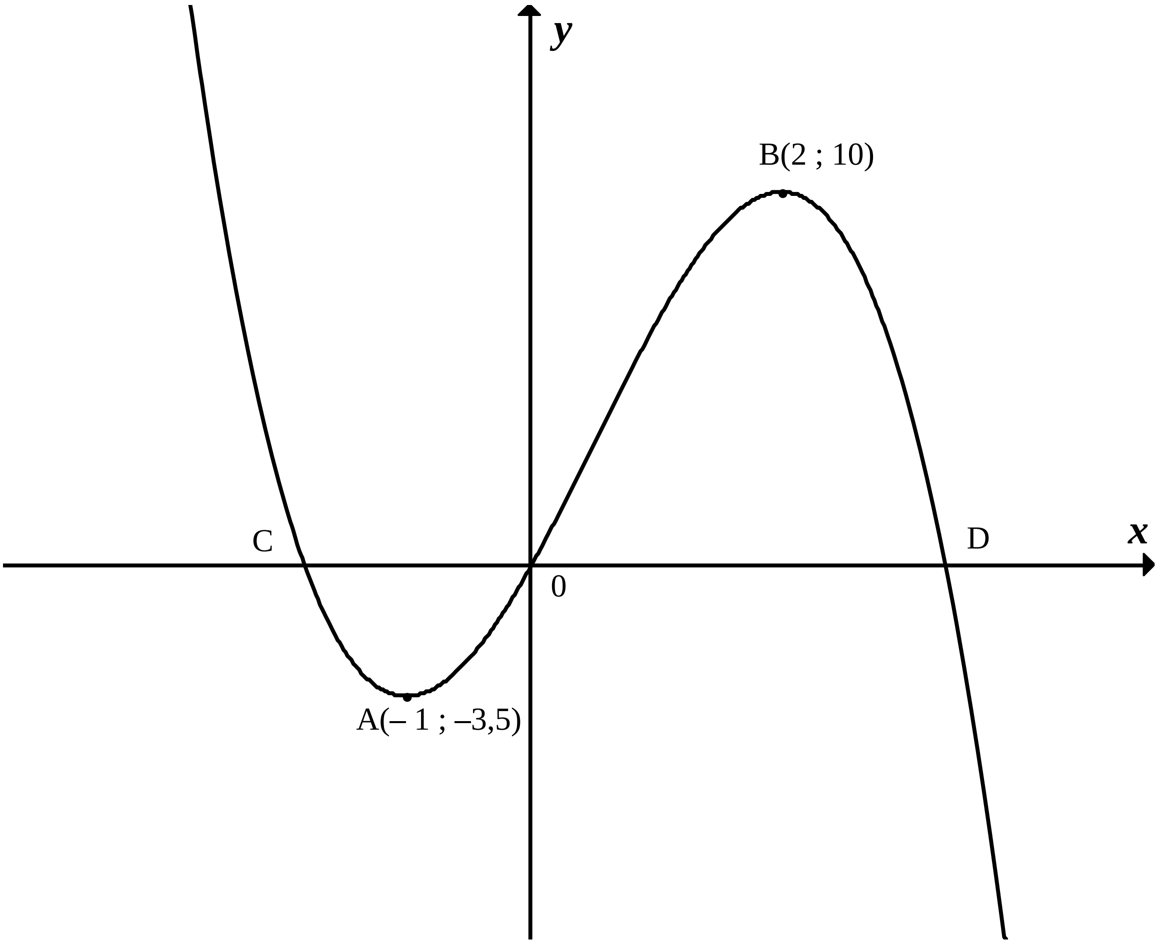
11.2.1 $D_x[(x^3 - 3)^2]$ (3)

11.2.2 $\frac{dy}{dx}$ if $y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ (3)
**[11]**

**QUESTION 12**

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. A(– 1 ; 3,5 ) and B(2 ; 10 ) are the turning points of $h$. The graph passes through the origin and further cuts the $x$-axis at C and D.




12.1 Show that $a = \frac{3}{2}$ and $b = 6$. (6)

12.2 Calculate the average gradient between A and B. (2)

12.3 Determine the equation of the tangent to $h$ at $x = -2$. (5)

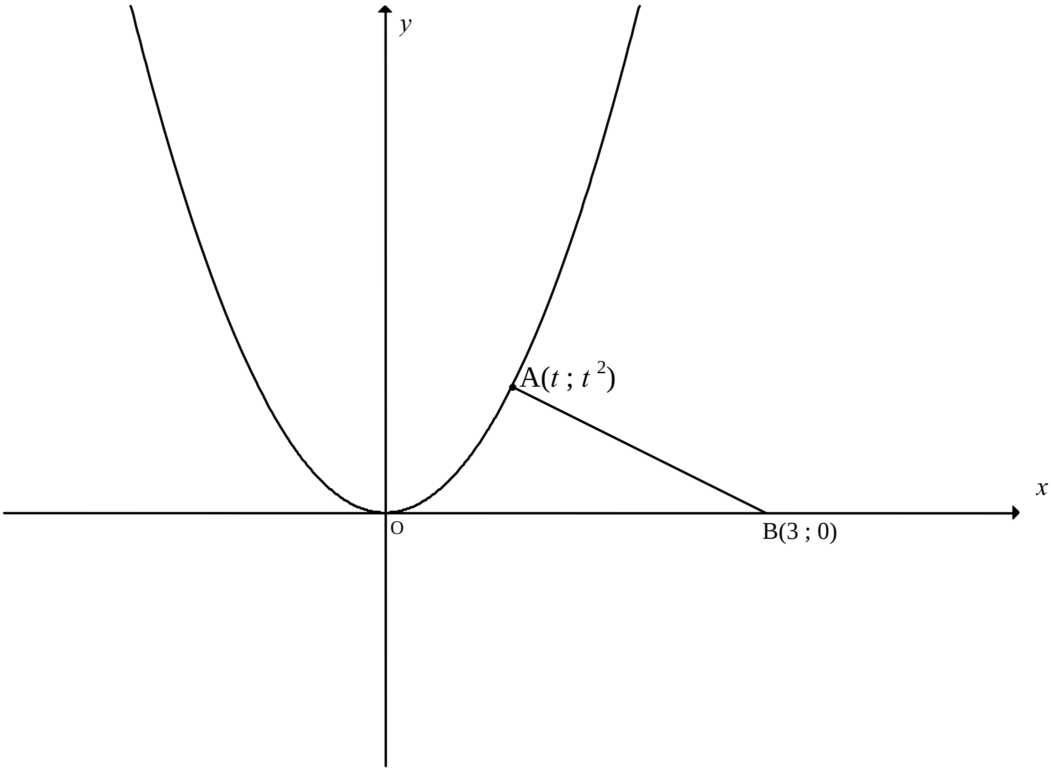
12.4 Determine the $x$-value of the point of inflection of $h$. (3)

12.5 Use the graph to determine the values of $p$ for which the equation $-x^3 + \frac{3}{2}x^2 + 6x + p = 0$ will have ONE real root. (2)

**[18]**

**QUESTION 13**

Sketched is the graph of $y = x^2$. A($t$ ; $t^2$) and B(3 ; 0) are shown.




13.1 A($t$ ; $t^2$) is a point on the curve $y = x^2$ and the point B(3 ; 0) lies on the $x$-axis. Show that $AB^2 = t^4 + t^2 - 6t + 9$. (2)

13.2 Hence, determine the value of $t$ which minimises the distance AB. (5)

**[7]**

**QUESTION 14**

A clothing company manufactures white shirts and grey trousers for schools.

- A minimum of 200 shirts must be manufactured daily.
- In total, not more than 600 pieces of clothing can be manufactured daily.
- It takes 50 machine minutes to manufacture a shirt and 100 machine minutes to manufacture a pair of trousers.
- There are at most 45 000 machine minutes available per day.

Let the number of white shirts manufactured in a day be $x$.
Let the number of pairs of grey trousers manufactured in a day be $y$.

14.1 Write down the constraints, in terms of $x$ and $y$, to represent the above information. (You may assume: $x \geq 0$, $y \geq 0$ ) (3)

14.2 Use the attached graph paper (DIAGRAM SHEET 2) to represent the constraints graphically. (5)

14.3 Clearly indicate the feasible region by shading it. (1)

14.4 If the profit is R30 for a shirt and R40 for a pair of trousers, write down the equation indicating the profit in terms of $x$ and $y$. (2)

14.5 Using a search line and your graph, determine the number of shirts and pairs of trousers that will yield a maximum daily profit. (2)
**[13]**

**TOTAL: 150**

# INFORMATION SHEET: MATHEMATICS
# INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1}$ ; $\; r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$ ; $\; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

*In ΔABC:*

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

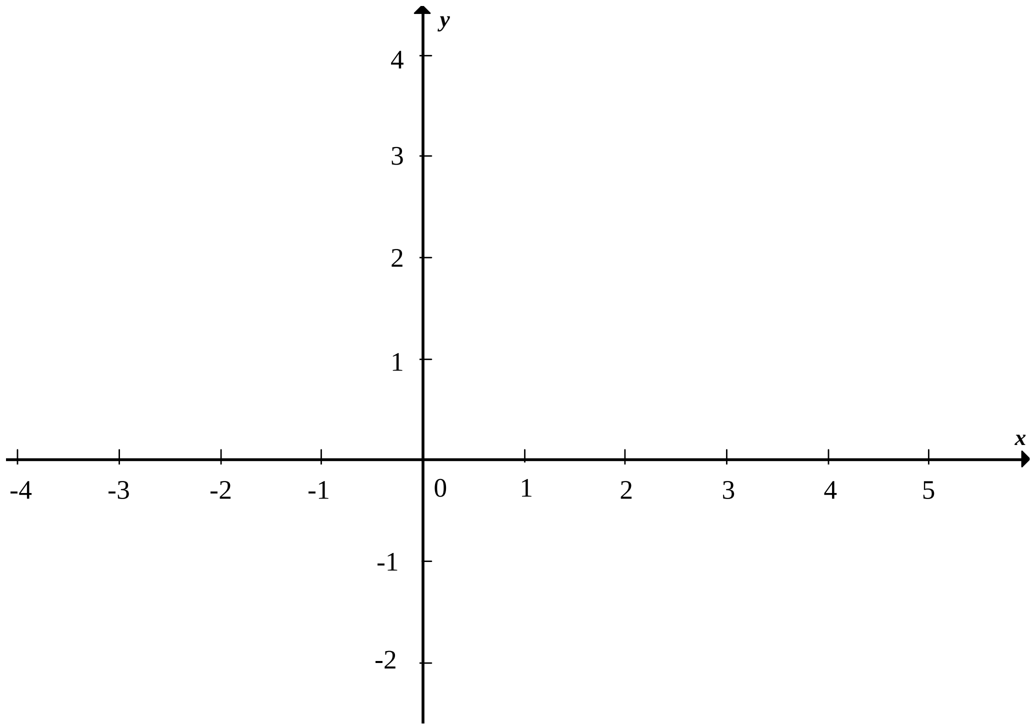
$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

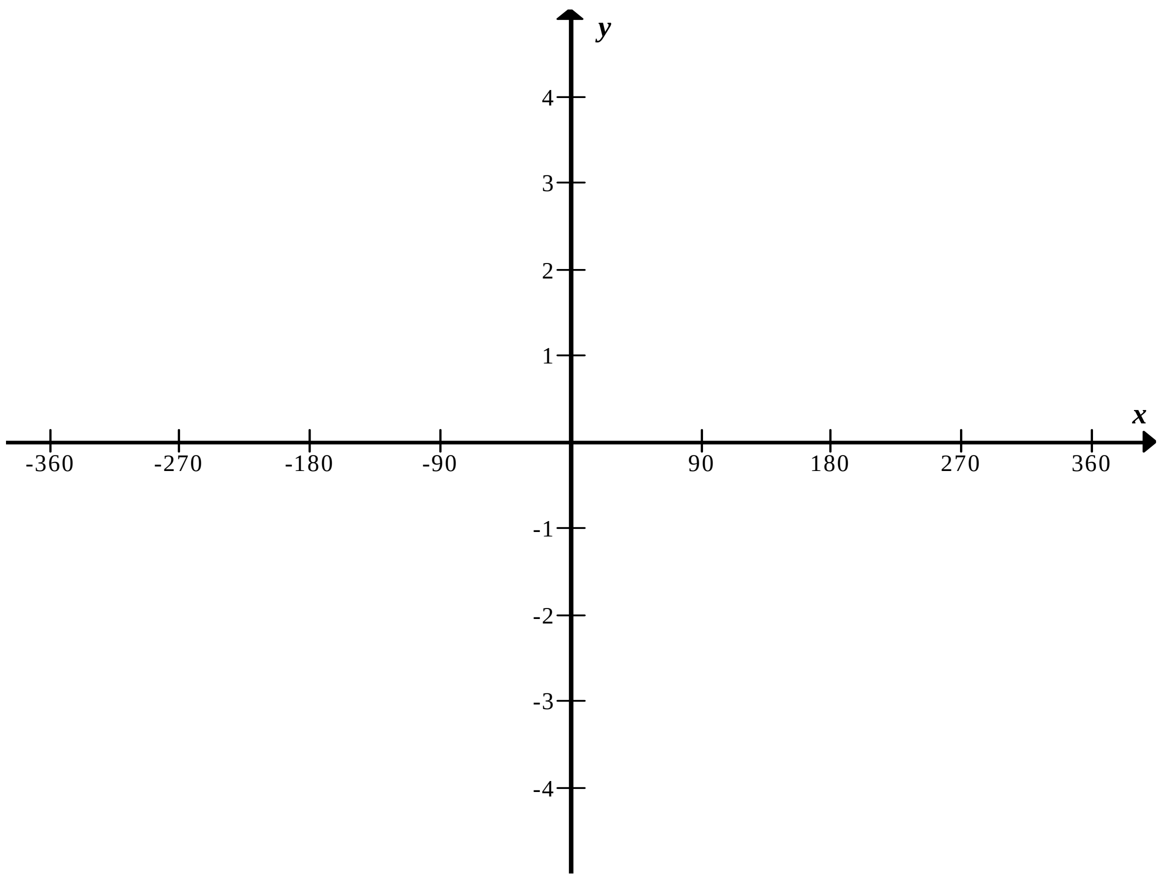
| EXAMINATION NUMBER: | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

**DIAGRAM SHEET 1**

**QUESTION 7.4**




**QUESTION 8.1**



| EXAMINATION NUMBER: | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

## DIAGRAM SHEET 2

## QUESTION 14.2