education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

NOVEMBER 2007

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 1 diagram sheet and a 1-page formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 10 questions. Answer ALL the questions.

2. Show clearly ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. It is in your own interest to write legibly and to present the work neatly.

8. An information sheet with formulae is attached.

QUESTION 1

1.1 Solve for x (correct to TWO decimal places where necessary):

1.1.1 (a) $(x+3)(x-1) = -x+1$ (4)

(b) Hence or otherwise, solve for x if $x^2 + 3x - 4 < 0$ (3)

1.1.2 $x^2 + 3x = 1$ (5)

1.2 Solve simultaneously for x and y in the following system of equations:

$x + y = 3$ and $2x^2 + 2y^2 = 5xy$ (9)

1.3 If $f(x) = x^2 - 2x$, show by completing the square that $f(x-1) = (x-2)^2 - 1$. (4)
[25]

QUESTION 2

2.1 Simplify: $\sqrt[3]{125x^6} - \sqrt[4]{81x^8} + \sqrt{36x^4}$ (4)

2.2 Given: $M = \sqrt{\dfrac{2}{2x+5} + \dfrac{1}{2x}}$

2.2.1 Show that M is a rational number if $x = 1{,}5$ (3)

2.2.2 Determine the values of x for which M is a real number. (3)

2.3 Erin had to find the product of 2^{2007} and 5^{2000} and then calculate the sum of the digits of the answer. Erin arrived at an answer of 11.

Is she correct? Show ALL the calculations to motivate your answer. (5)
[15]

QUESTION 3

The number pattern 1, 5, 11, 19, … is such that the sequence of 'second differences' is a constant.

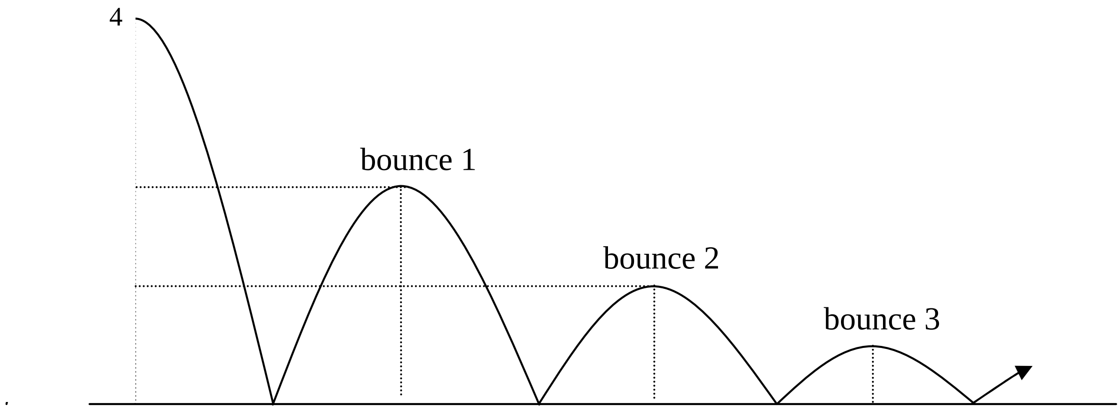
3.1 Determine the 5^{th} number in the pattern. (1)

3.2 Derive a formula for the n^{th} number in the pattern. (7)

3.3 What is the 100^{th} number in the pattern? (2)
[10]

QUESTION 4

A rubber ball is bounced from a height of 4 metres and bounces continuously as shown in the diagram below. Each successive bounce reaches a height that is half the previous height.



4.1 If the pattern of the maximum height reached during each bounce continues, what maximum height will the ball reach during the 6^{th} bounce? (2)

4.2 Determine an algebraic expression for the maximum height reached in the n^{th} bounce. (4)

4.3 After how many bounces will the ball reach a maximum height of $\frac{1}{512}$ metres? (4)
[10]

QUESTION 5

5.1 After 4 years of reducing balance depreciation, an asset has a $\frac{1}{4}$ of its original value. The original value was R86 000.

Calculate the depreciation interest rate, as a percentage.
(Correct your answer to 1 decimal place.) (5)

5.2 Jabu invests a certain sum of money for 5 years. She receives interest of 12% per annum compounded monthly for the first two years. The interest rate changes to 14% per annum compounded semi-annually for the remaining term. The money grows to R75 000 at the end of the 5-year period.

5.2.1 Calculate the effective interest rate per annum during the first year. (4)

5.2.2 Calculate how much money Jabu invested initially. (6)

5.3 The expenditure of the Department of Health (in billions of rands) is indicated in the following table. (We take 2003 as $t = 0$, 2004 as $t = 1$ and so on.)

Year	2003	2004	2005	2006
Time (t), in years	0	1	2	3
Expenditure (E), in billions of rands	2	2,5	3	3,5

5.3.1 Plot the four data points in your answer book, as accurately as you can. (2)

5.3.2 Make a conjecture about the relationship between the expenditure and time. (1)

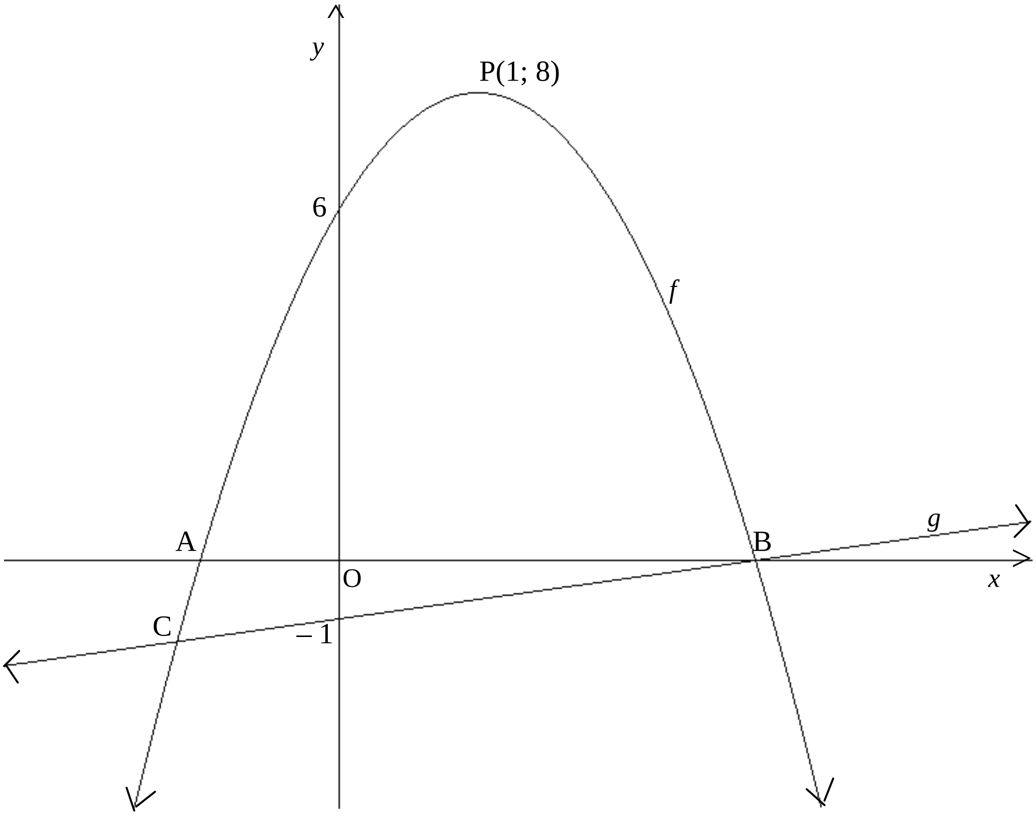
5.3.3 Use your conjecture to write down the equation of E as a function of t. (2)

5.3.4 Use your equation to predict the expenditure of the Department of Health in 2010 (in billions of rands) (1)

[21]

QUESTION 6

Below is a sketch graph of parabola, f, and straight line, g.
P(1; 8) is the turning point of f.
f cuts the y-axis at (0; 6) and g cuts the y-axis at (0; –1).
f and g intersect at B and C.
B is a point on the x-axis



6.1 Show that $f(x) = -2x^2 + 4x + 6$. (6)

6.2 Calculate the average gradient of $f(x)$ between $x = 1$ and $x = 3$. (3)

6.3 Show that the equation of g is $g(x) = \frac{1}{3}x - 1$. (3)

6.4 Calculate the coordinates of C. (6)

6.5 If $h(x) = f(-x)$, explain how the graph of h may be obtained from the graph of f. (2)

6.6 Write down the equation of h. (2)

[22]

QUESTION 7

Given: $f(x) = \dfrac{8}{x-8} + 4$

7.1 Write down the domain of f. (1)

7.2 For what value of x is $f(x) = 0$? (2)

7.3 Determine the value of p, if A $(0;\ p)$ lies on the graph of f. (2)

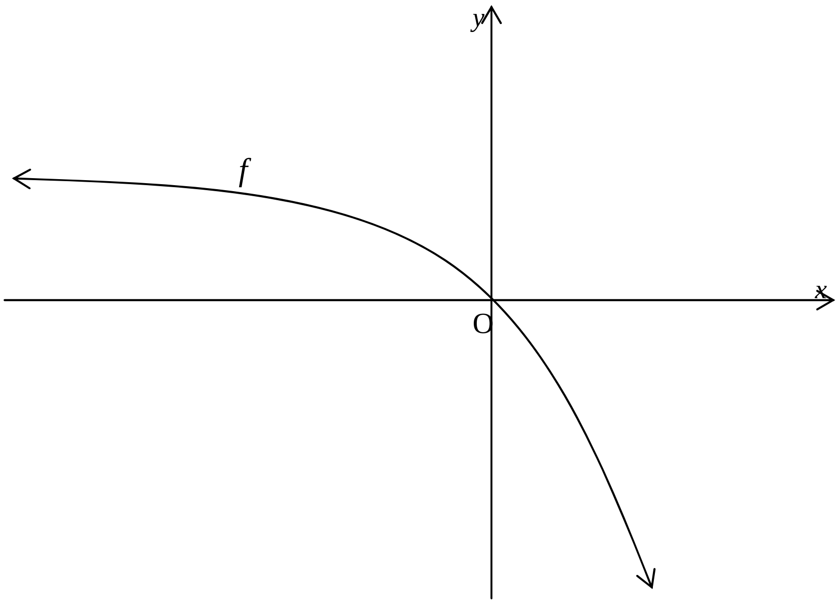
7.4 Write down the equations of the asymptotes of f. (2)

7.5 Draw a neat sketch graph of f, indicating the asymptotes and intercepts with the axes, on the diagram sheet provided. (4)

[11]

QUESTION 8

The graph of $f(x) = 1 + a.2^x$ (a is a constant) passes through the origin as shown below.



8.1 Show that $a = -1$. (2)

8.2 Determine the value of $f(-15)$ correct to FIVE decimal places. (2)

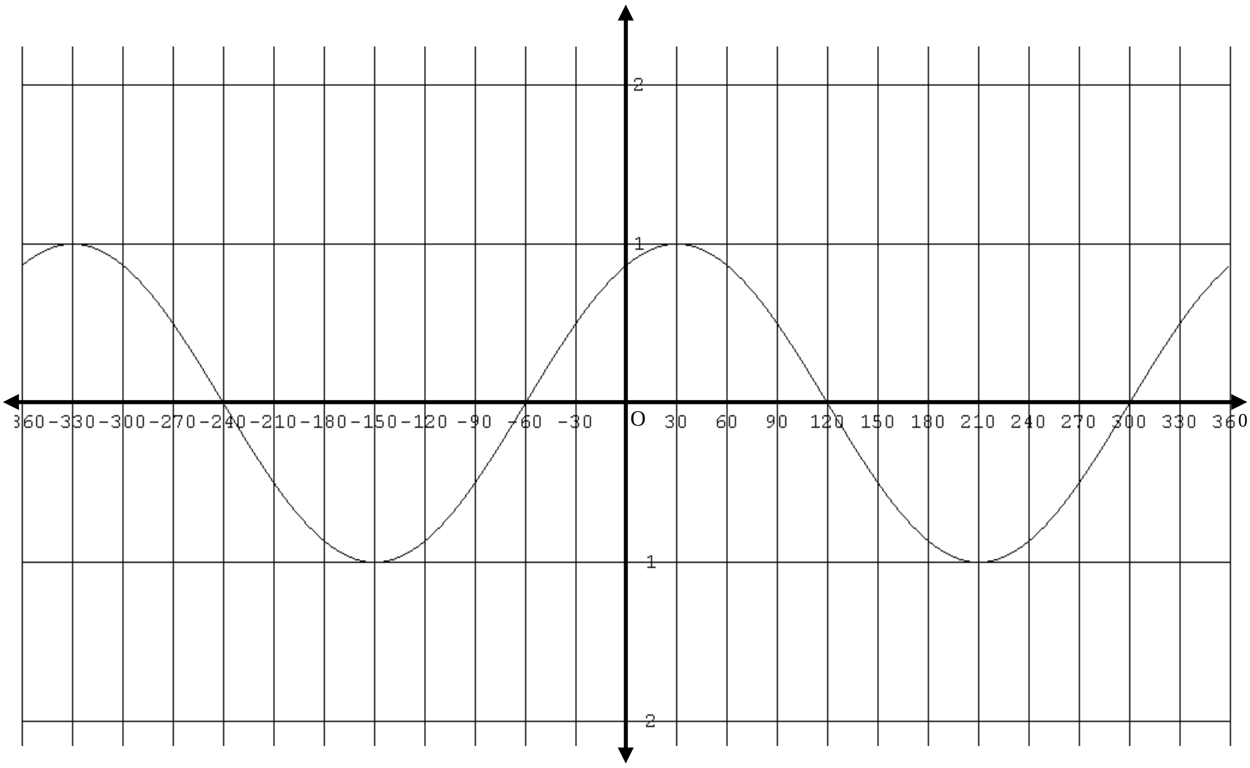
8.3 Determine the value of x, if P $(x;\ 0{,}5)$ lies on the graph of f. (3)

8.4 If the graph of f is shifted 2 units to the right to give the function h, write down the equation of h. (2)

[9]

QUESTION 9

Given the function $f(x) = \cos(x - 30°)$ for $x \in [-360°;\ 360°]$.



Determine:

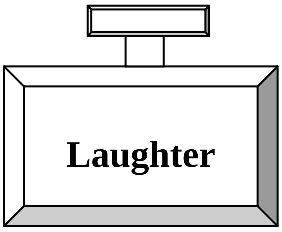
9.1 The period of the function g, if $g(x) = f(2x)$ (2)

9.2 The range of the function h, if $h(x) = f(x) - 1$ (2)

9.3 The amplitude of the function q, if $q(x) = \frac{1}{2} f(x) + 2$ (2)
[6]

QUESTION 10

Two fragances A and B are used to make the perfumes *Laughter* and *Joy*.



- You require 3 g of fragrance A and 4 g of fragrance B to produce 1 litre of *Laughter*.
- One litre of *Joy* requires 9 g of fragrance A and 6 g of fragrance B.
- At least 3 litres of *Laughter* needs to be produced per week.

At the beginning of a particular week the company has 27 g of fragrance A and 30 g of fragrance B.

Let x and y be the number of litres of *Laughter* and *Joy* respectively that are produced per week.

10.1 State algebraically, in terms of x and y, the constraints that apply to this problem for this week. (5)

10.2 Represent the constraints graphically on the graph paper provided and shade the feasible region. (8)

10.3 If the profit on 1 ℓ of *Laughter* is R30 and the profit on 1 ℓ of *Joy* is R50, express the profit, P, in terms of x and y. (2)

10.5 Calculate the maximum possible profit. (2)

[21]

TOTAL: 150

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET

QUESTION 10

10.2

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$

$A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad$ $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\, \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\alpha$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\alpha$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\partial^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$