

MATHEMATICS

(PAPER 1)

JUNE 2008

TIME: 3 HOURS
MARKS: 150



education

Western Cape Education Department

NATIONAL STRATEGY FOR LEARNER ATTAINMENT
NATIONAL SENIOR CERTIFICATE

JUNE EXAMINATION - 2008

This question paper consists of **8** pages and
1 Diagram sheet and
1 Information sheet.

Instructions and Information

Read the following instructions carefully before answering this question paper:

- 1 This question paper consists of ...**12**... questions. Answer ALL questions.
- 2 Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
- 3 An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
- 4 If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
- 5 Number your answers correctly according to the numbering system used in this question paper.
- 6 Diagrams are not necessarily drawn to scale.
- 7 It is in your own interest to write legibly and to present your work neatly.
- 8 A diagram sheet for answering QUESTION 5.1 and QUESTION 12.3 is attached.

Question 1

1.1 Solve for x in each of the following;

1.1.1 $(2x - 1)(x + 3) = 12x + 1$ (5)

1.1.2 $3x^2 + 2x + 6 = 10$ (correct to TWO decimal digits) (5)

1.1.3 $2x^2 - x - 6 \leq 0$ (4)

1.2 Solve for both x and y in the system of equations below.

$xy + 6 = 0$ and $x + 3y + 3 = 0$ (7)

[21]

Question 2

2.1 For which value of x will the expression $\sqrt{\frac{3-x}{x+7}}$ be a real number? (4)

2.2 Simplify:

$$\frac{(3x)^{-2}}{3x^{-3}} \quad (3)$$

[7]

Question 3

3.1 Which is the better investment offer: 10,28% p.a. compounded daily (use 365 days in a year) or 10,3% p.a. compounded monthly? (6)

3.2 A pension fund investment of R576 000 earns 8,75% interest per annum, compounded monthly.
At the end of fourteen months, the interest rate changes to 9,25% per annum, compounded monthly and then remains unchanged for the rest of the investment period, which is 22 months. The investment is withdrawn after a total of three years.

3.2.1 Calculate the value of the investment at the end of three years. (5)

3.2.2 What is the effective annual compound interest rate for the investment over the three year period? (3)

3.2.3 What is the nominal annual interest rate for the investment spread over the period of the investment? (3)

[17]

Question 4

Consider the following sequence: 6; 18; 54; 162

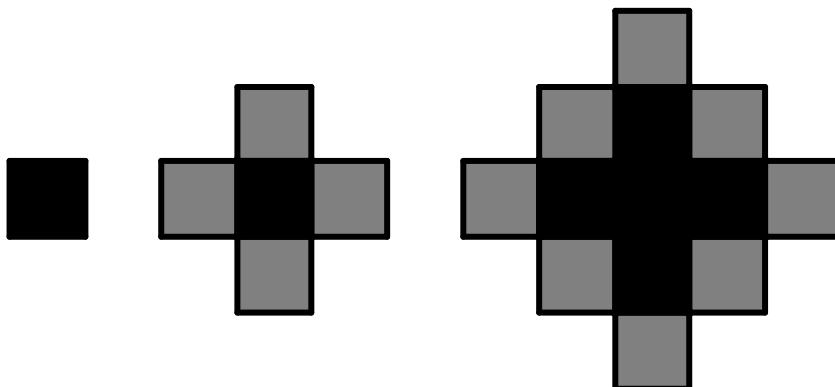
4.1 If the formula for the general term of the sequence is $T_n = ar^{n-1}$, give the value of a and r . (2)

4.2 Which term of the sequence is equal to 1458? (4)

[6]

Question 5

The diagram shows a sequence of patterns. Each one is made by surrounding the previous pattern (shade black) by squares that are shaded grey.



- 5.1 On the square grid on your diagram sheet, draw the next pattern (2)
- 5.2 There are two sequences formed. The first, is the number of squares added to each new pattern (the grey squares), the second is the total number of squares making up the pattern. Write down the first six terms of each sequence (6)

[8]

Question 6

Consider the sequence: 6 ; 10 ; 16 ; 24 ; 34 ; ...

- 6.1 If the sequence behaves consistently, determine the next two terms of the sequence. (2)
- 6.2 Calculate a formula for the n^{th} term of the sequence. (5)
- 6.3 Use your formula to calculate n if the n^{th} term in the sequence is 1264. (4)

[11]

Question 7

Given the functions $y = f(x) = -\frac{1}{2}(x + 1)^2 + 2$ and $y = g(x) = -2x - 6$:

- 7.1 Write down the co-ordinates of the turning point of f (2)
- 7.2 Calculate the roots of the equation $f(x) = 0$ (4)
- 7.3 Write down the equation of the axis of symmetry of f . (1)
- 7.4 Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same system of axes. Show ALL intercepts with the axes. (4)

[11]

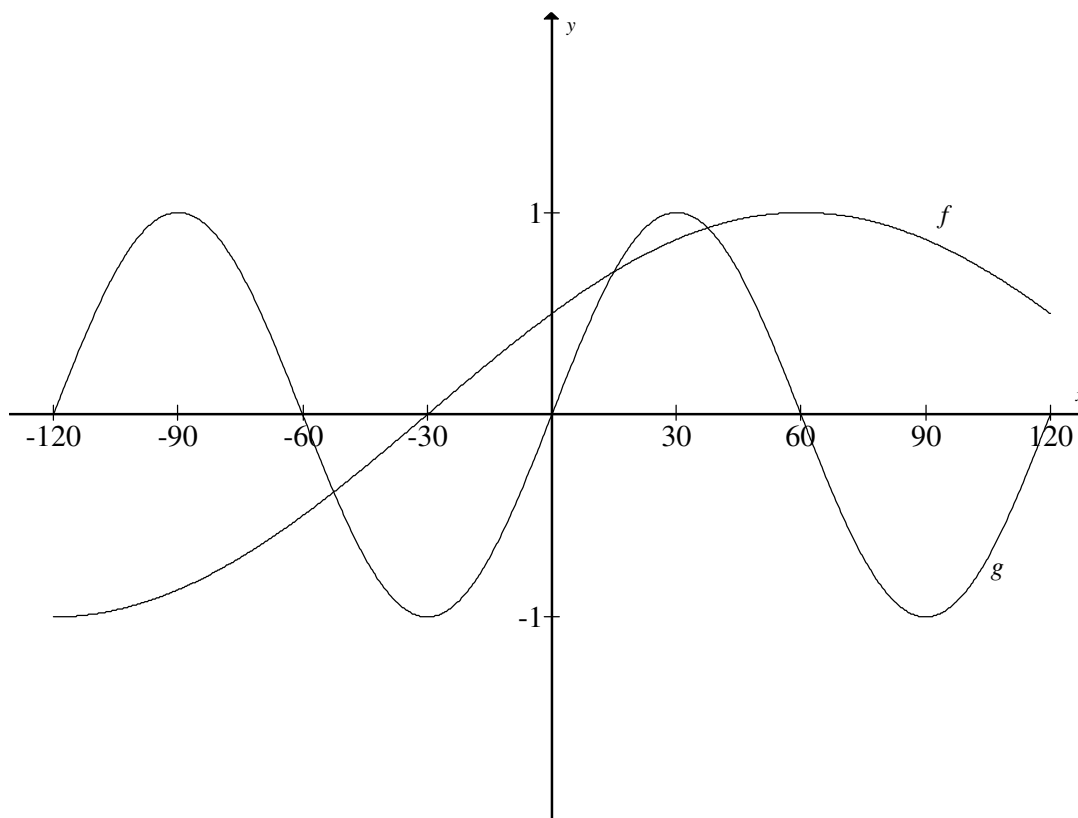
Question 8

8.1 Given: $g(x) = \frac{5}{x-2} - 4$

8.1.1 Write down the equations of the asymptotes of g . (2)

8.1.2 Calculate the intercepts of g with the axes. (3)

8.2 Sketched below are the functions $f(x) = \cos(x - 60^\circ)$ and $g(x) = \sin 3x$ for $x \in [-180^\circ ; 180^\circ]$



8.2.1 Write down the period of g . (1)

8.2.2 Write down the new equation of f if it is shifted 30° horizontally to the right. (2)

8.2.3 Write down two values of x for which $f(x) - 1 = g(x)$. (2)

[10]

Question 9

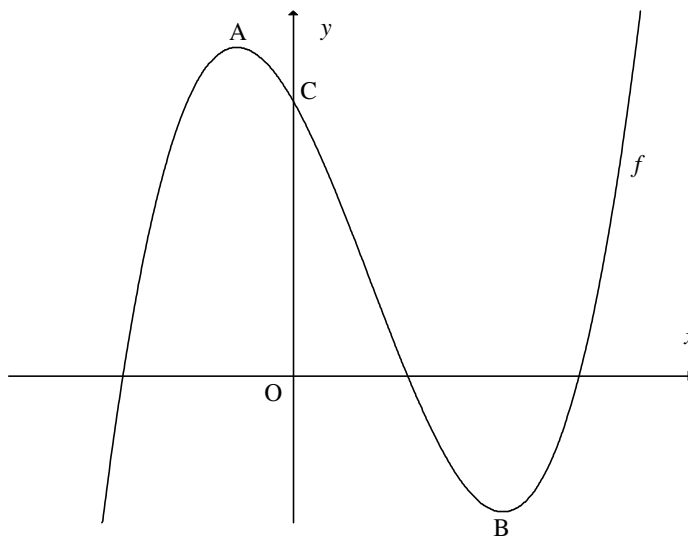
9.1 Given $f(x) = \frac{1}{x}$, determine $f'(x)$ from first principles. (5)

9.2 Determine the derivative of: $f(x) = 7x^3 - 4x + 6$ (3)

[8]

Question 10

Sketched is the graph of $f(x) = x^3 - 4x^2 - 11x + 30$
 A and B are the turning points of f .



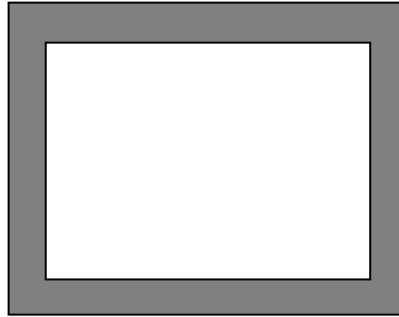
- 10.1 Determine the coordinates of A and B. (5)
- 10.2 Determine the turning points of g if $g(x) = f(x-2)$ (2)
- 10.3 Determine the average rate of change of the function f from A to B. (3)
- 10.4 Determine the equation of the tangent to the graph of f at $x = 1$. (4)
- 10.5 Determine the value(s) of k for which $x^3 - 4x^2 - 11x + 30 = k$ will have only one real root. (3)
- 10.6 Determine the point(s) of inflection of f . (4)

(6)

[22]

Question 11

A mirror is set into a wooden frame which is 2cm wide. The outside perimeter of the wooden frame is 72cm.



- 11.1 The length of the frame is x cm. Determine the breadth of the frame in terms of x . (1)
- 11.2 Determine the length and breadth of the mirror (without the frame) in terms of x . (2)
- 11.3 Show that the area of the mirror is given by the function: $A(x) = -x^2 + 36x - 128 \text{ cm}^2$ (2)
- 11.4 Calculate the dimensions of the mirror with the largest area that can fit into the frame. (4)
- [9]**

Question 12

A chocolate factory produces both chocolate coated raisons as well as chocolate coated peanuts. In a day, they can produce a maximum of 100kilos of raisons and 125kilos of peanuts. They are mixed together and sold in two different packets. The 'Nuts about Nuts' packet has one third raisons and two thirds peanuts. The other packet, 'Half-n-Half', has equal quantities of both. Packets of 'Nuts about Nuts' sell at a profit of R4 per kilogram while the 'Half-n-Half' sells at a profit of R5 per kilogram.

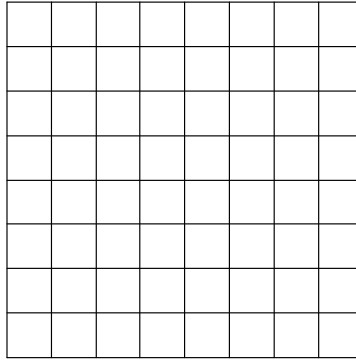
Let there be x kilos of 'Half-n-Half' produced in a day and y kilos of 'Nuts about Nuts'.

- 12.1 Give, in terms of x and y , the mathematical constraints that must be satisfied each day. (4)
- 12.2 Write down a function for the Profit (P) to be made (2)
- 12.3 Illustrate the constraints graphically, on the grid paper provided. Clearly indicate the feasible region. (8)
- 12.4 Use your graph to determine;
- 12.4.1 the values of x and y that will ensure maximum profit. (4)
- 12.4.2 the profit earned according to the values found in 12.4.1 (2)
- [20]**

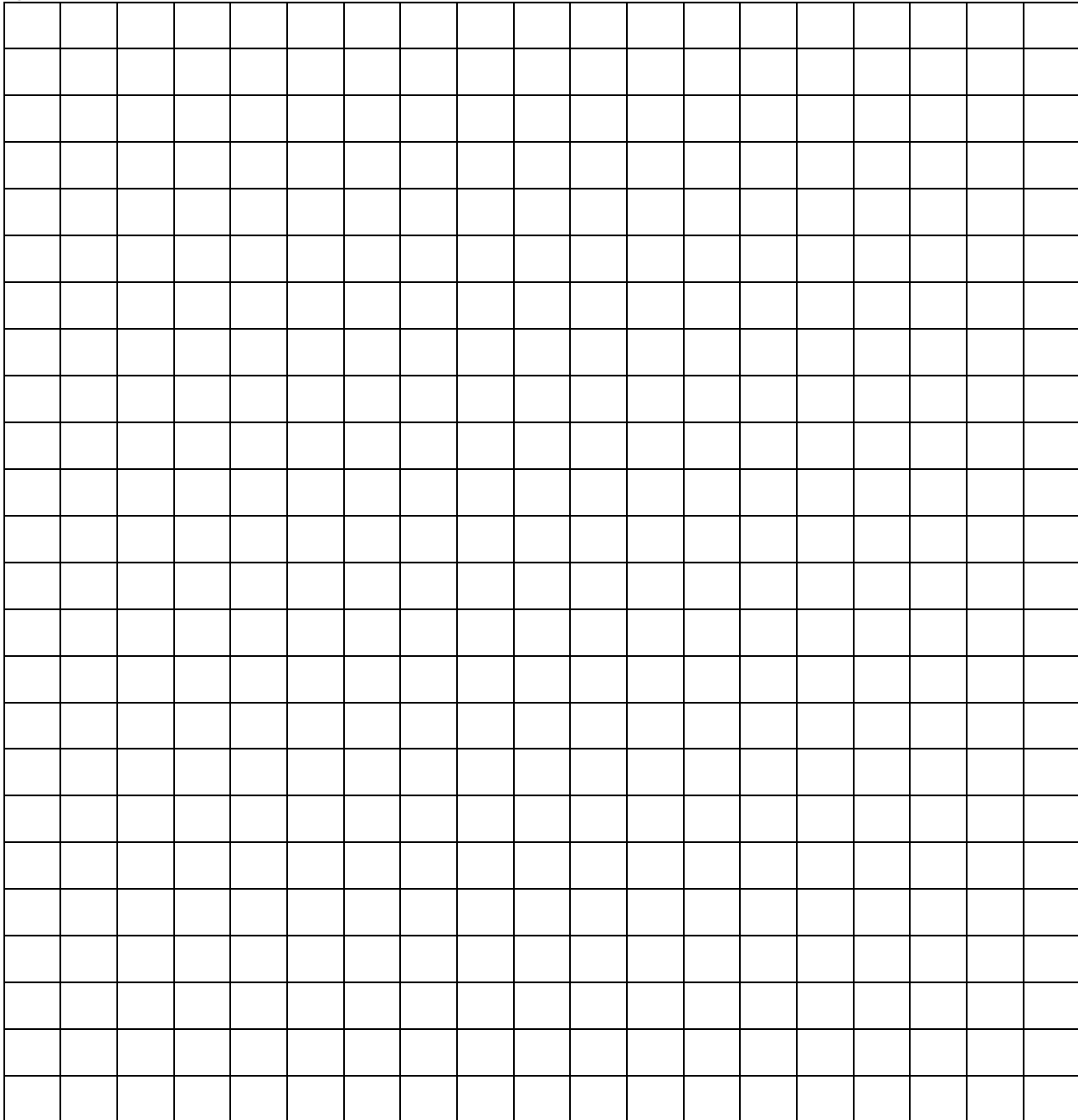
TOTAL 150

Diagram Sheet

Question 5.1



Question 12.3



Information Sheet: Mathematics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + i.n)$$

$$A = P(1 + i)^n$$

$$A = P(1 - i)^n$$

$$A = P(1 - i.n)$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$$

$$\sum_{i=1}^n ar^{i-1} = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$\sum_{i=1}^n ar^{i-1} = \frac{a}{r - 1} ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC ; \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\text{var} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$SD = \sqrt{\frac{\sum_{i=1}^n (x - \bar{x})^2}{n}}$$

$$P(A) = \frac{n(A)}{n(s)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$