education

Department:
Education
REPUBLIC OF SOUTH AFRICA

# NATIONAL SENIOR CERTIFICATE

## GRADE 12

## MATHEMATICS P2

## NOVEMBER 2008

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 9 pages, an information sheet and 4 diagram sheets.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

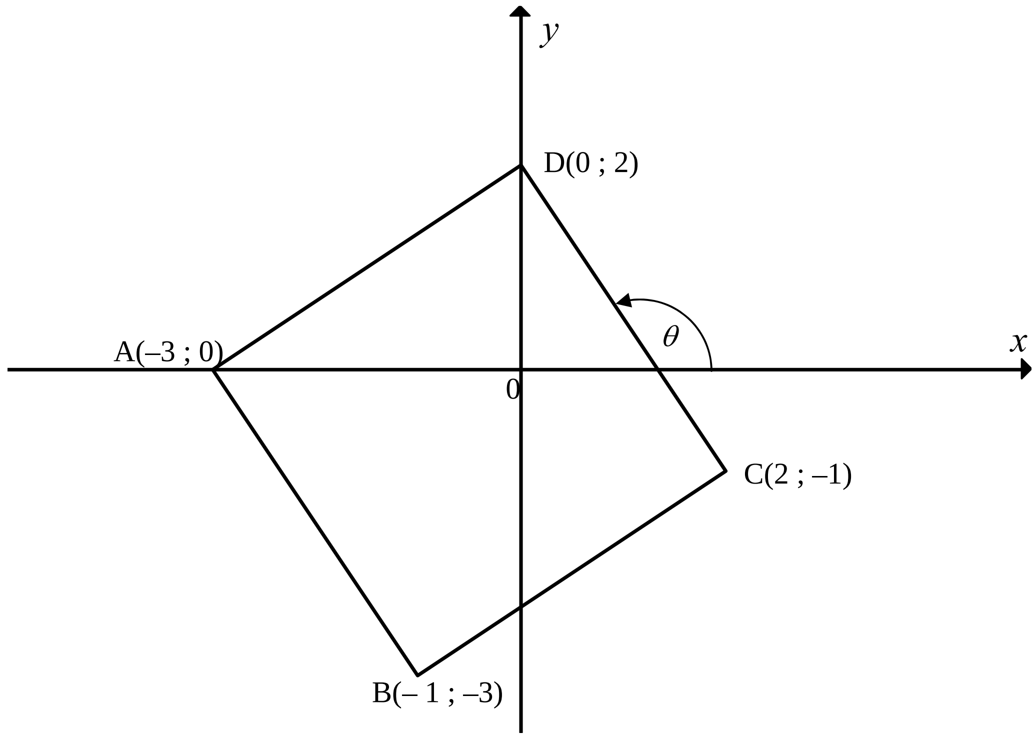
6. FOUR diagram sheets for answering QUESTION 3.2, QUESTION 8.2, QUESTION 10.1, QUESTION 10.2 and QUESTION 11.1 are included at the end of this question paper. Write your examination number on these sheets in the spaces provided and hand them in together with your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

**QUESTION 1**

ABCD is a quadrilateral with vertices A(– 3 ; 0), B(– 1 ; – 3), C(2 ; –1) and D(0 ; 2).

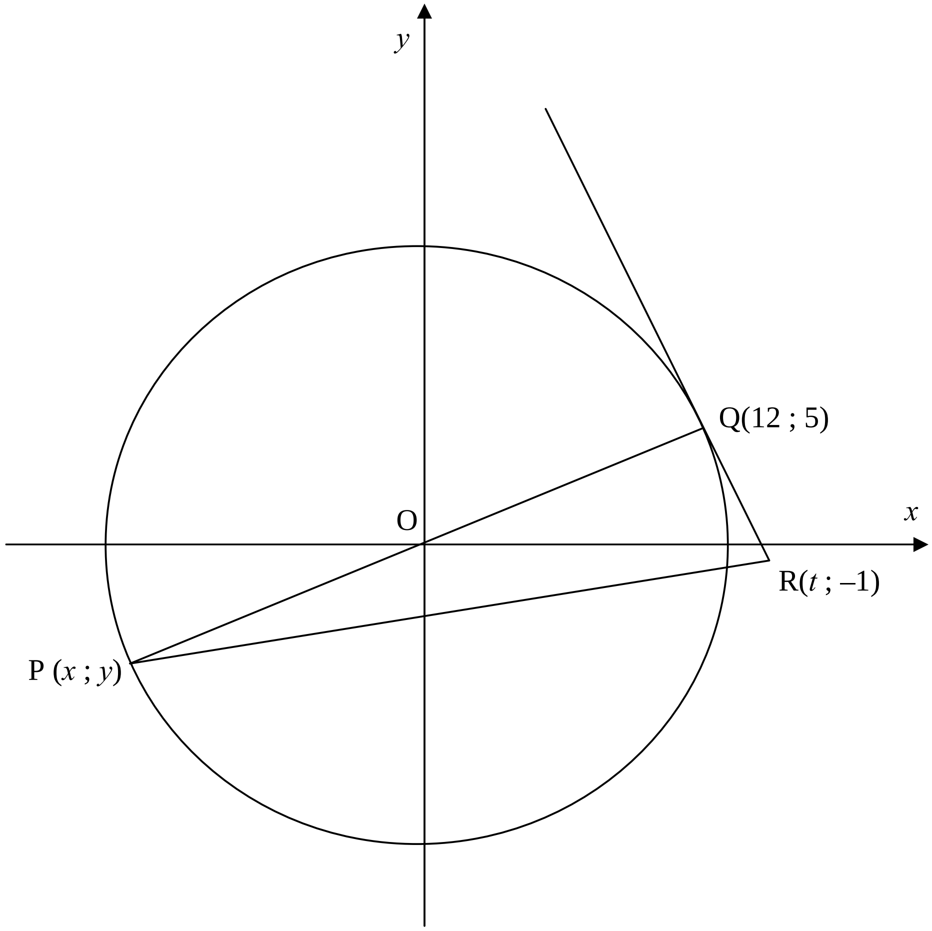



| | | |
|---|---|---|
| 1.1 | Determine the coordinates of M, the midpoint of AC. | (2) |
| 1.2 | Show that AC and BD bisect each other. | (3) |
| 1.3 | Prove that $A\hat{D}C = 90°$. | (4) |
| 1.4 | Show that ABCD is a square. | (6) |
| 1.5 | Determine the size of $\theta$, the angle of inclination of DC, correct to ONE decimal place. | (3) |
| 1.6 | Does C lie inside or outside the circle with centre (0 ; 0) and radius 2? Justify your answer. | (2) |

**[20]**

**QUESTION 2**

O is the centre of the circle in the figure below. P($x$ ; $y$) and Q (12 ; 5) are two points on the circle. POQ is a straight line. The point R($t$ ; – 1) lies on the tangent to the circle at Q.




2.1 Determine the equation of the circle. (3)

2.2 Determine the equation of the straight line through P and Q. (2)

2.3 Determine $x$ and $y$, the coordinates of P. (2)

2.4 Show that the gradient of QR is $-\frac{12}{5}$. (2)

2.5 Determine the equation of the tangent QR in the form $y = \ldots$ (3)

2.6 Calculate the value of $t$. (2)

2.7 Determine an equation of the circle with centre Q(12 ; 5) and passing through the origin. (3)

**[17]**

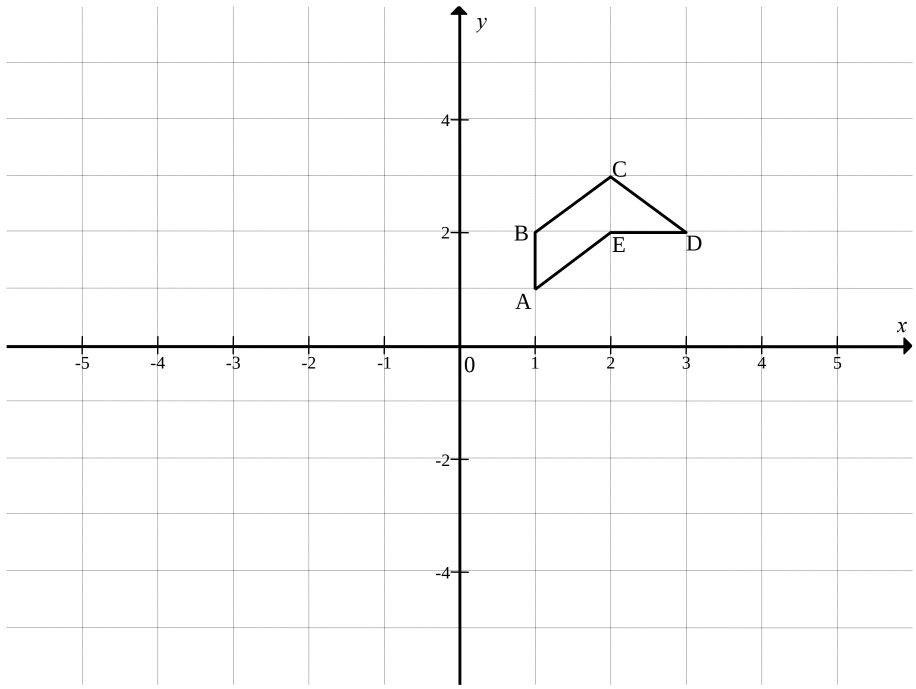
**QUESTION 3**

3.1 The point P($-\sqrt{2}\,;\sqrt{3}$) lies in a Cartesian plane. Determine the coordinates of the image of P if:

3.1.1 P is reflected in the line $y = x$. (2)

3.1.2 P is rotated about the origin through 180°. (2)

3.2 The vertices of the polygon ABCDE are shown in the grid. The coordinates are: A(1 ; 1), B(1 ; 2), C(2 ; 3), D(3 ; 2) and E(2 ; 2). Each of the points of ABCDE in the grid below is rotated about the origin in a clockwise direction through an angle of 90°.




3.2.1 Write down the coordinates of $D^{/}$, the image of D. (1)

3.2.2 Sketch and label the vertices of $A^{/}B^{/}C^{/}D^{/}E^{/}$, the image of ABCDE on DIAGRAM SHEET 1. (5)

3.2.3 The polygon $A^{/}B^{/}C^{/}D^{/}E^{/}$ is then enlarged through the origin by a factor of 3 in order to give the polygon $A^{//}B^{//}C^{//}D^{//}E^{//}$. Write down the coordinates of $D^{//}$, the image of $D^{/}$. (2)

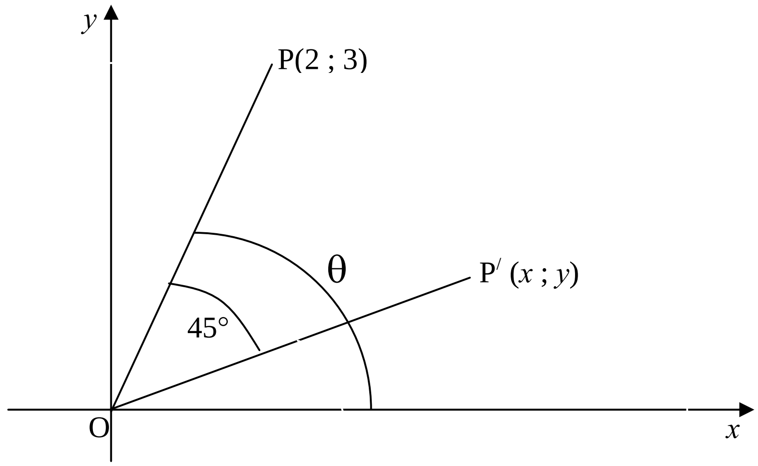
3.2.4 Write down the general transformation of a point ($x$ ; $y$) in ABCDE to ($x^{//}$ ; $y^{//}$) after ABCDE has undergone the above two transformations; that is, rotation in a clockwise direction through an angle of 90° followed by an enlargement through the origin by a factor of 3. (4)

3.2.5 Calculate the ratio of Area ABCDE : Area $A^{//}B^{//}C^{//}D^{//}E^{//}$. (2)

**[18]**

**QUESTION 4**

Determine the coordinates $x$ and $y$ of $P^{/}$, the image of P(2 ; 3) when OP is rotated about the origin through an angle of 45° in the clockwise direction.




**[7]**

**QUESTION 5**

5.1 Do NOT use a calculator to answer this question. Show ALL calculations.

Prove that:

5.1.1 $$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°} = \frac{3}{2}.$$ (6)

5.1.2 $$\cos 75° = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$ (4)

5.2 Prove that $\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x) = 3\sin^2 x$ (6)

**[16]**

**QUESTION 6**

6.1 6.1.1 Prove that $(\tan x - 1)(\sin 2x - 2\cos^2 x) = 2(1 - 2\sin x\cos x)$ (5)

6.1.2 Determine the general solution for: $\frac{\tan x - 1}{2} = -3$ correct to ONE decimal place. (5)

6.2 If $\cos\beta = \frac{p}{\sqrt{5}}$ where $p < 0$ and $\beta \in [180°; 360°]$, determine, using a diagram, an expression in terms of $p$ for:

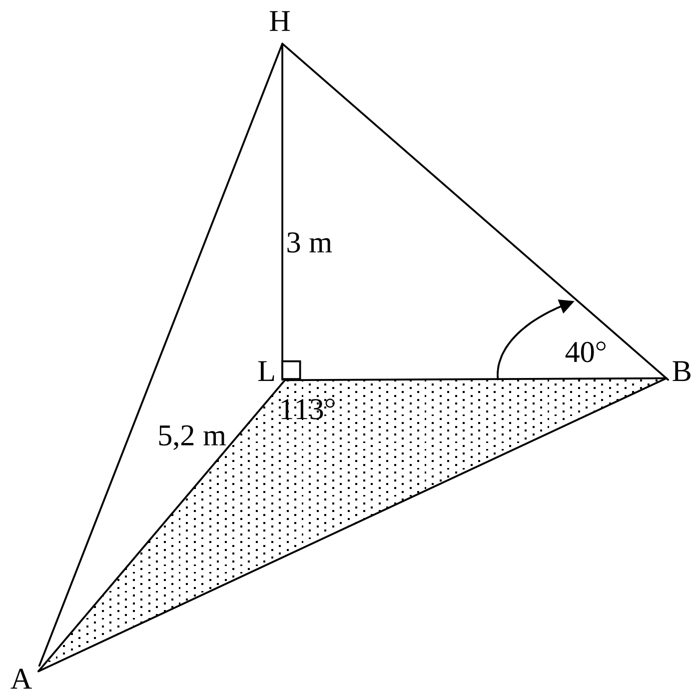
6.2.1 $\tan\beta$ (4)

6.2.2 $\cos 2\beta$ (3)

**[17]**

## QUESTION 7

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, AL = 5,2 m, the angle $A\hat{L}B = 113°$ and the angle of elevation of H from B is 40°.




7.1 Calculate the length of LB. (2)

7.2 Hence, or otherwise, calculate the length of AB. (4)

7.3 Determine the area of $\Delta ABL$. (4)
**[10]**

## QUESTION 8

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90° ; 180°]$.

8.1 Solve for $x$ if $f(x) = g(x)$. (8)

8.2 Sketch the graphs of $f$ and $g$ on the system of axes on DIAGRAM SHEET 2 for $x \in [-90° ; 180°]$. (6)

8.3 Solve for $x$ if $f(x) \leq g(x)$ where $x \in [-90° ; 0°]$. (3)
**[17]**

## QUESTION 9

The time taken, in minutes, to complete a 5 kilometre race by a group of 10 runners is given below:

18 21 16 24 28 20 22 29 19 23

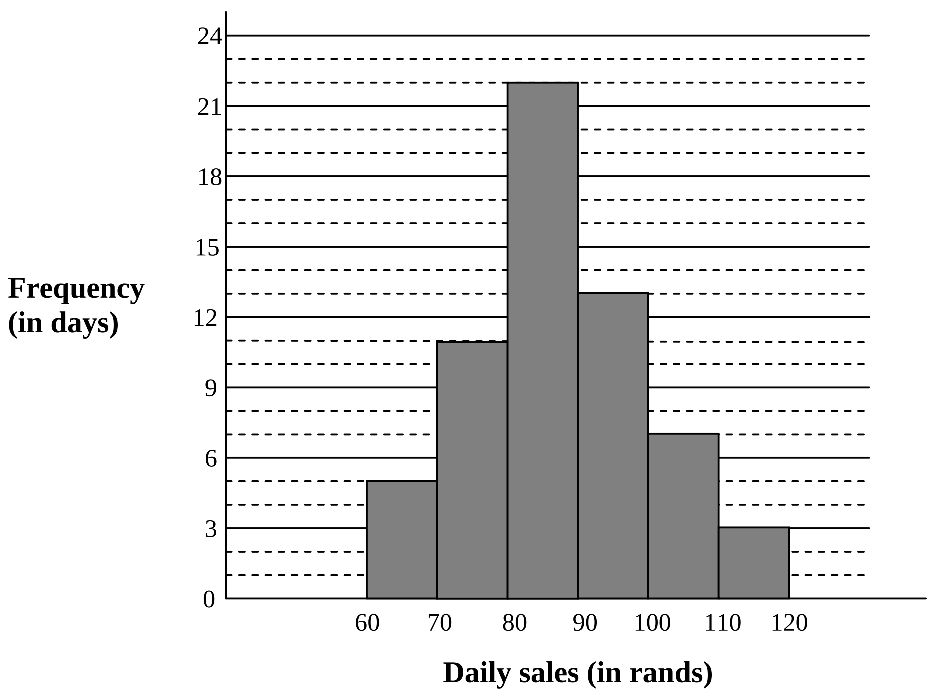
9.1 Calculate the mean time taken to complete the race. (2)

9.2 Calculate the standard deviation of the time taken to complete the race. (Use the formula on the information sheet.) (4)

9.3 How many runners completed the race within one standard deviation of the mean? (2)
**[8]**

## QUESTION 10

A street vendor has kept a record of sales for November and December 2007.
The daily sales in rands is shown in the histogram below.




10.1 On DIAGRAM SHEET 3, complete the cumulative frequency table for the sales over November and December. (3)

10.2 Draw an ogive for the sales over November and December on DIAGRAM SHEET 3. (3)

10.3 Use your ogive to determine the median value for the daily sales. Explain how you obtain your answer. (1)

10.4 Estimate the interval of the upper 25% of the daily sales. (2)
**[9]**

**QUESTION 11**

A parachutist jumps out of a helicopter and his height above ground level is estimated at various times after he opened his parachute. The following table gives the results of the observations where $y$ measures his height above ground level in metres and $t$ represents the time in seconds after he opened his parachute.

| $t$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $y$ | 500 | 300 | 200 | 120 | 70 | 40 | 20 |

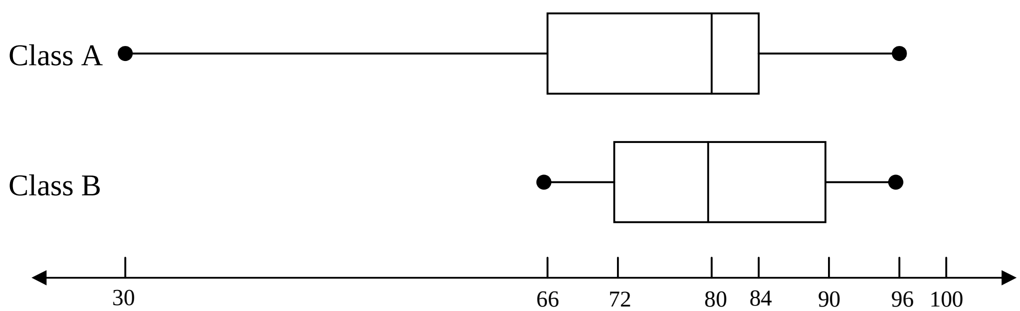
11.1 On DIAGRAM SHEET 4, draw a scatter plot for the above information. (2)

11.2 Describe the curve of best fit. (1)

11.3 Use the scatter plot to estimate the height of the parachutist 5,5 seconds after he had opened his parachute. (1)
**[4]**

**QUESTION 12**

The box and whisker plots below summarise the final test scores for two of Mr Jack's Mathematics classes from the same grade.




12.1 Describe the features in the scores that are the same for both classes. (2)

12.2 Calculate the interquartile range for Class B. (2)

12.3 Mr Jack considers the median of each class and reports that there is no significant difference in the performance between them. Is Mr Jack's conclusion valid? Support your answer with reasons. (3)
**[7]**

**TOTAL: 150**

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# INFORMATION SHEET: MATHEMATICS
# INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1}$ ; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$ ; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

*In ΔABC:*

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

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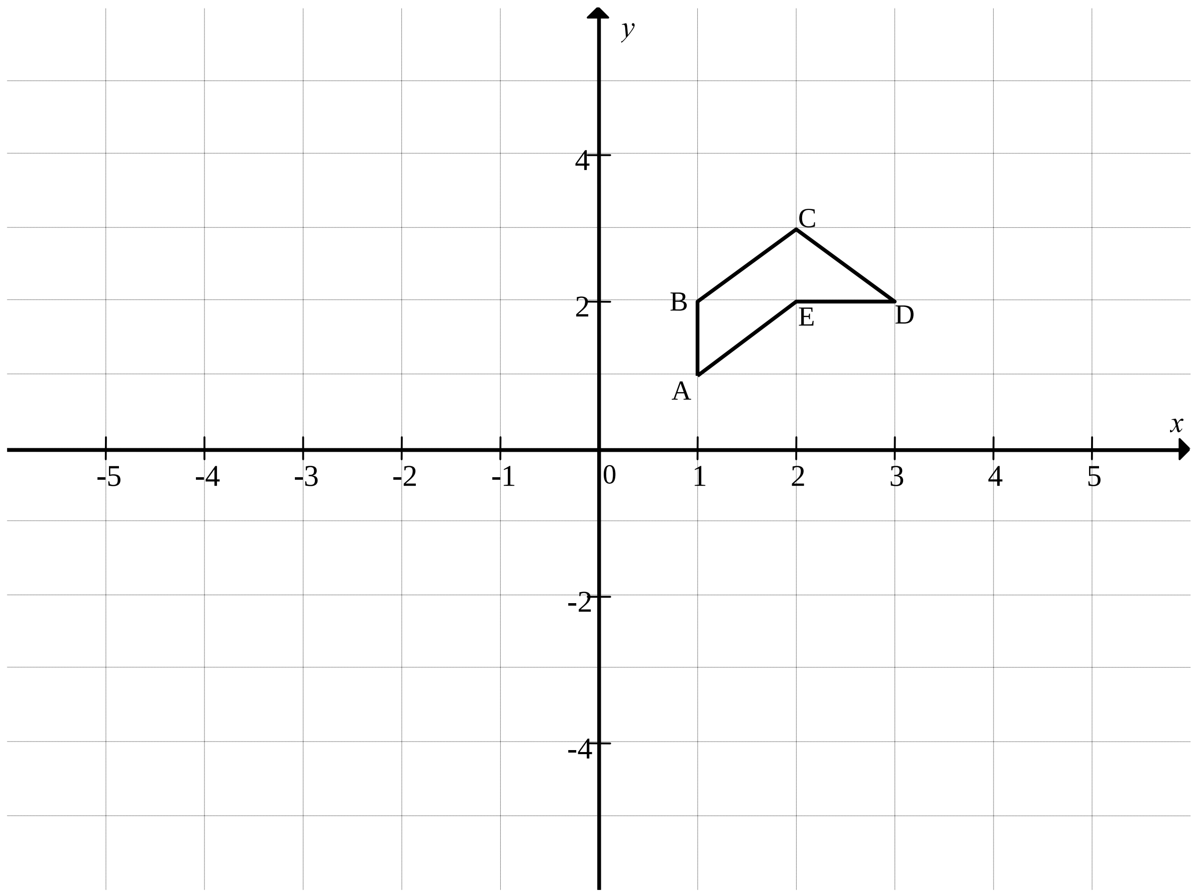
**EXAMINATION NUMBER:**

**DIAGRAM SHEET 1**

**QUESTION 3.2**



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## DIAGRAM SHEET 2

## QUESTION 8.2

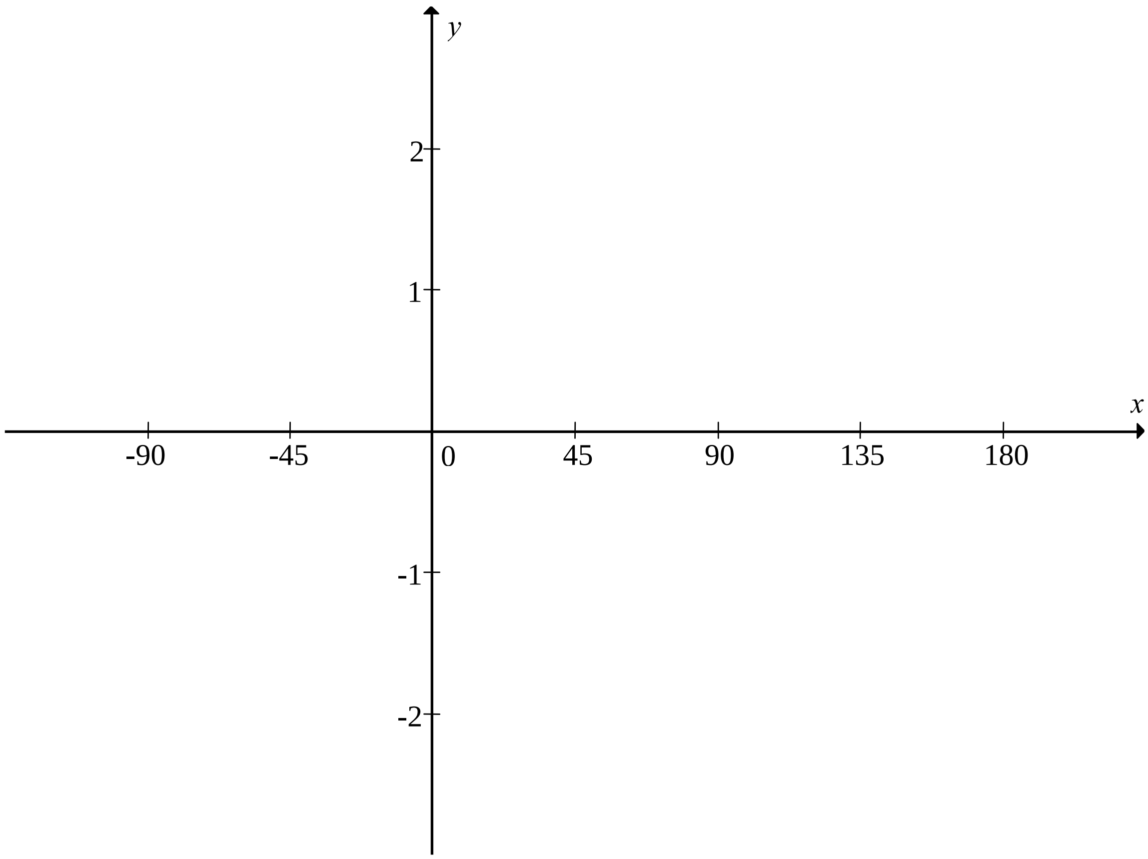

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**DIAGRAM SHEET 3**

**QUESTION 10.1**

| DAILY SALES | FREQUENCY | CUMULATIVE FREQUENCY |
|---|---|---|
| | | |
| | | |
| | | |
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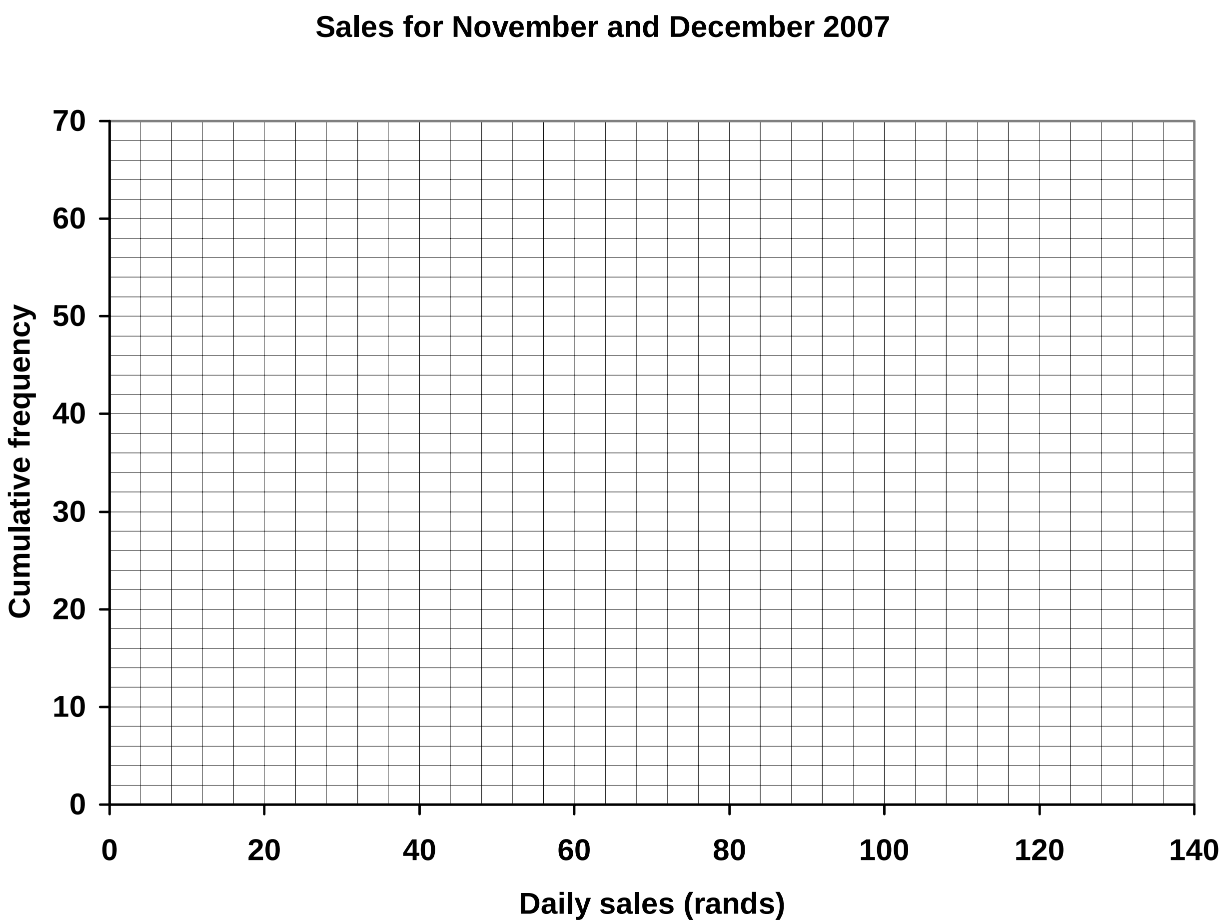
**QUESTION 10.2**

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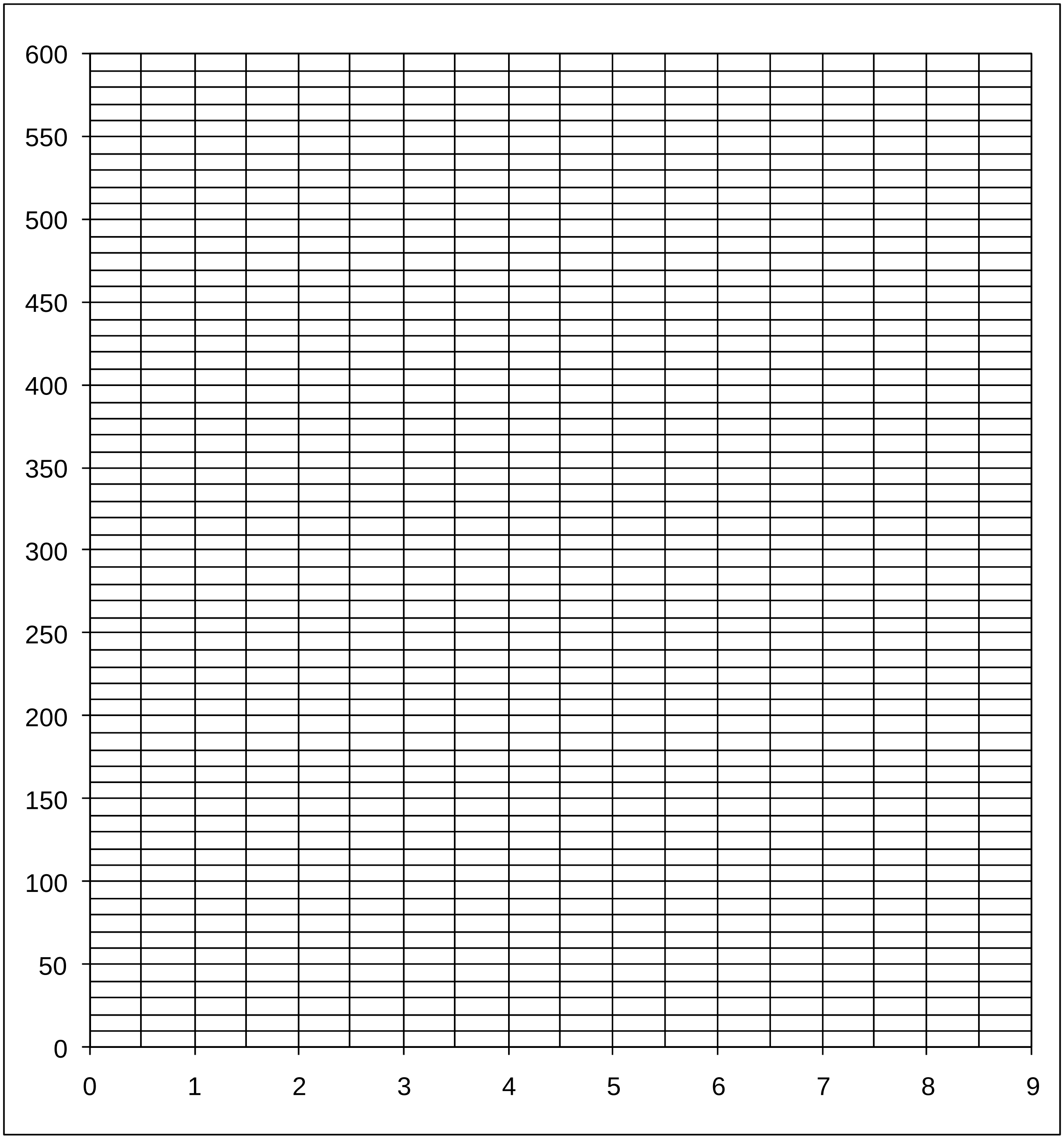
**EXAMINATION NUMBER:**

**DIAGRAM SHEET 4**

**QUESTION 11.1**

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