



NATIONAL SENIOR CERTIFICATE EXAMINATION  
NOVEMBER 2008

**MATHEMATICS: PAPER I  
(LO 1 AND LO 2)**

Time: 3 hours

150 marks

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**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 10 pages, an Answer Booklet of 5 pages (i – v) and an Information Sheet. Please check that your paper is complete.
  2. Read the questions carefully.
  3. Answer all the questions. Questions 5 and 6 should be answered in the Answer Booklet.
  4. Number your answers exactly as the questions are numbered.
  5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
  6. Round off your answers to one decimal digit where necessary.
  7. All the necessary working details must be clearly shown.
  8. It is in your own interest to write legibly and to present your work neatly.
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**SECTION A****QUESTION 1**(a) Solve for  $x$ :

(1)  $x^2 = 5x + 6$  (2)

(2)  $3^x = 33$  (correct to one decimal digit) (2)

(3)  $x^2 - x < 12$  (4)

(b) Solve for  $x$  and  $y$ :

$$y = 2x \quad \text{and} \quad y = 3x^2 - 1$$
 (4)

(c) Given  $f(x) = x^3 + x^2 - 3x - 3$ (1) Factorise  $f(x)$  into two factors. (3)(2) Hence solve  $f(x) = 0$  when

(i)  $x \in Q$  (1)

(ii)  $x \in R$  (2)

**18 marks****QUESTION 2**

(a) The table below shows some values of an arithmetic sequence.

$n$	2	3	4	7	10	$B$
$T_n$	5	7	9	$A$	21	153

Determine the value of  $A$  and  $B$ . (6)(b) Given that an arithmetic sequence has first term equal to 7 and  $T_{15} = 35$ , find the value of the eighth term. (4)

(c) Evaluate:

(1)  $\sum_{x=1}^4 (x^2 - x + 1)$  (3)

(2) the sum of the geometric series  $\frac{45}{4} + \frac{135}{16} + \frac{405}{64} + \frac{1215}{256} + \dots$  to 15 terms. (4)**17 marks**

**QUESTION 3**

- (a) The Vorsters buy a new car for R255 000.  
It depreciates in value (on a reducing balance) by 12,5% per year.
- (1) Determine the value of the car after 3 years. (2)
- (2) Determine how long (to the nearest year) it will take for the Vorsters' car to be worth R100 000. (5)
- (b) Calculate the depreciation rate (using reducing balance) for another brand of vehicle that will depreciate to be half of its original value in exactly 6 years. (4)
- (c) Nosizwe's parents are considering taking a bank loan of R40 000 to cover the costs of her first year at university. They are offered an interest rate of 14,75% p.a. compounded monthly and wish to pay back the loan in 12 equal monthly instalments starting one month after receiving the loan.  
Calculate what these monthly payments will be. (7)

<b>18 marks</b>
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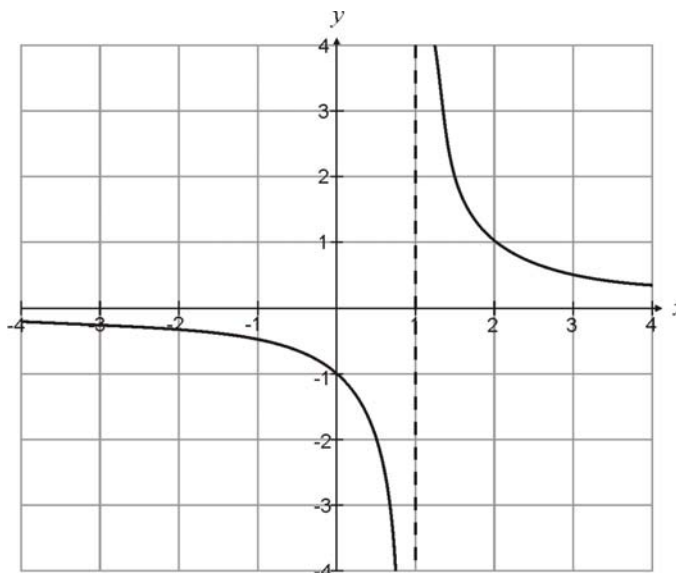
**QUESTION 4**

- (a) Find  $\frac{dy}{dx}$  if  $y = 3x^3 + x$  (2)
- (b) Find  $f'(4)$  if  $f(x) = \frac{1}{2\sqrt{x}}$  (4)
- (c) Given:  $y = x^3 - x^2 - x + 2$
- (1) Find the equation of the tangent to the curve at the point where  $x = -1$ . (7)
- (2) Find the  $x$ -coordinate of another point on the curve at which the tangent is parallel to the one found in (1). (3)

<b>16 marks</b>
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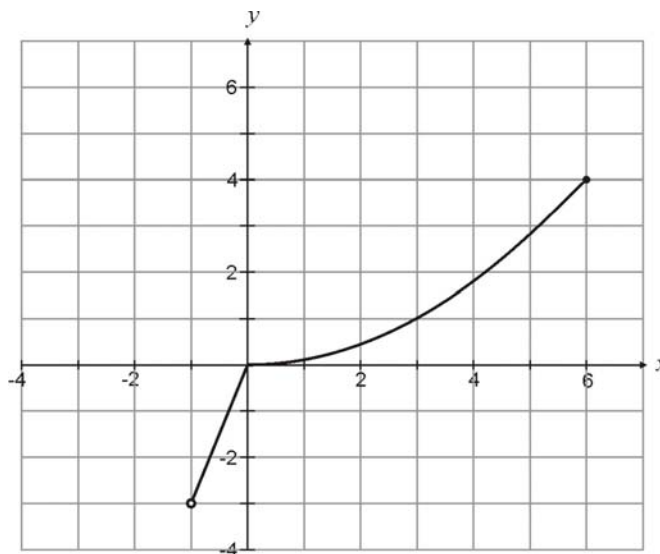
**QUESTION 5 ANSWER THIS QUESTION IN THE ANSWER BOOKLET**

(a) Part of the graph of the function with equation  $y = f(x)$  is shown below.



- (1) Give the equation of each asymptote. (2)
- (2) On this set of axes, draw the graph of  $y = f(-x)$  (2)

(b) The graph of the function  $g$  is shown below.



- (1) Give the domain and range of the function. (2)
- (2) On this set of axes, draw the graph of the inverse function of  $g$ . (4)
- (3) Explain why this inverse is a function. (1)

**11 marks**

**SECTION B****QUESTION 6 ANSWER THIS QUESTION IN THE ANSWER BOOKLET**

Nutritionists have established that vitamins A, B<sub>1</sub> and B<sub>2</sub> are the most popular vitamins that people take to supplement their diets.

Research has shown that a person's daily requirement is at least 16 units of vitamin A, 5 units of B<sub>1</sub> and 20 units of B<sub>2</sub>.

Two brands of vitamins are available: brand P and brand Q.

The table below summarises the units of each vitamin per tablet for each brand, and the cost per tablet.

	Vit. A	Vit. B <sub>1</sub>	Vit. B <sub>2</sub>	Cost per tablet
Brand P	8 units	1 unit	2 units	40c
Brand Q	2 units	1 unit	7 units	80c

Let  $x$  represent the number of tablets of brand P taken each day and  $y$  represent the number of tablets of brand Q taken each day.

The inequalities below summarise the information about a person's daily requirements of each vitamin mentioned above.

$$\text{Vit. A:} \quad 8x + 2y \geq 16$$

$$\text{Vit. B}_1: \quad x + y \geq 5$$

$$\text{Vit. B}_2: \quad 2x + 7y \geq 20$$

$$\text{Implicit constraints:} \quad x \geq 0, \quad y \geq 0$$

- (a) On the axes provided, draw the constraints and shade the feasible region. (4)
- (b) State the objective function that represents the cost of the daily requirement of vitamins. (1)
- (c) Determine the values of  $x$  and  $y$  within the feasible region that will minimise the daily cost and calculate this minimum. (4)
- (d) Suppose brand Q decreased in price to 40c per tablet. Determine how this would effect the optimal solution and calculate the new minimum. (3)
- (e) Further research has shown that the daily requirement of vitamin B<sub>1</sub> is 6 units instead of 5 units as previously stated.
- (1) Rewrite the constraint that is effected by this change. (1)
- (2) Draw this constraint on the graph used in (a) and label it 'new vitamin B<sub>1</sub>'. (1)
- (3) Determine the new optimal solution, using the objective function in (b). (2)

<b>16 marks</b>
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**QUESTION 7**

(a) Given  $a^x = 1$ , state the values of  $a$  if:

(1)  $x = 0$  (1)

(2)  $x \neq 0$  (1)

(b) Given:  $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$

(1) Write  $x^2$  in terms of  $x$ . (2)

(2) Hence determine the value of  $x$ . (3)

(c) Given:  $f(x) = 3x$ , find a simplified expression for

$$f(x) + f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + f^{-1}(x). \quad (6)$$

(d) Given a function  $f$  that satisfies the following conditions:

$$f(0) = 2, \quad f(-2) = 0, \quad f'(-1) = 0$$

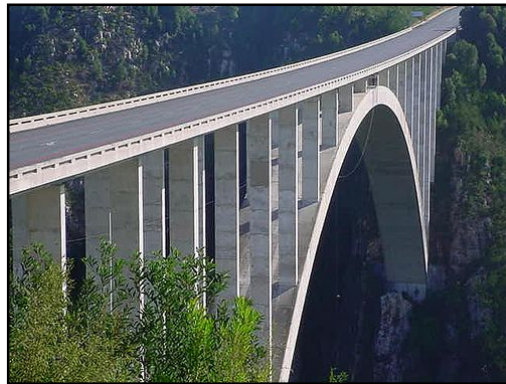
$$f'(x) > 0 \quad \text{for } x \neq -1$$

Draw a rough sketch of a possible graph of  $f$ . (5)

<b>18 marks</b>
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**QUESTION 8**

The Bloukrans River Bridge (completed in 1984) is claimed to be the fourth largest concrete bridge in the world.

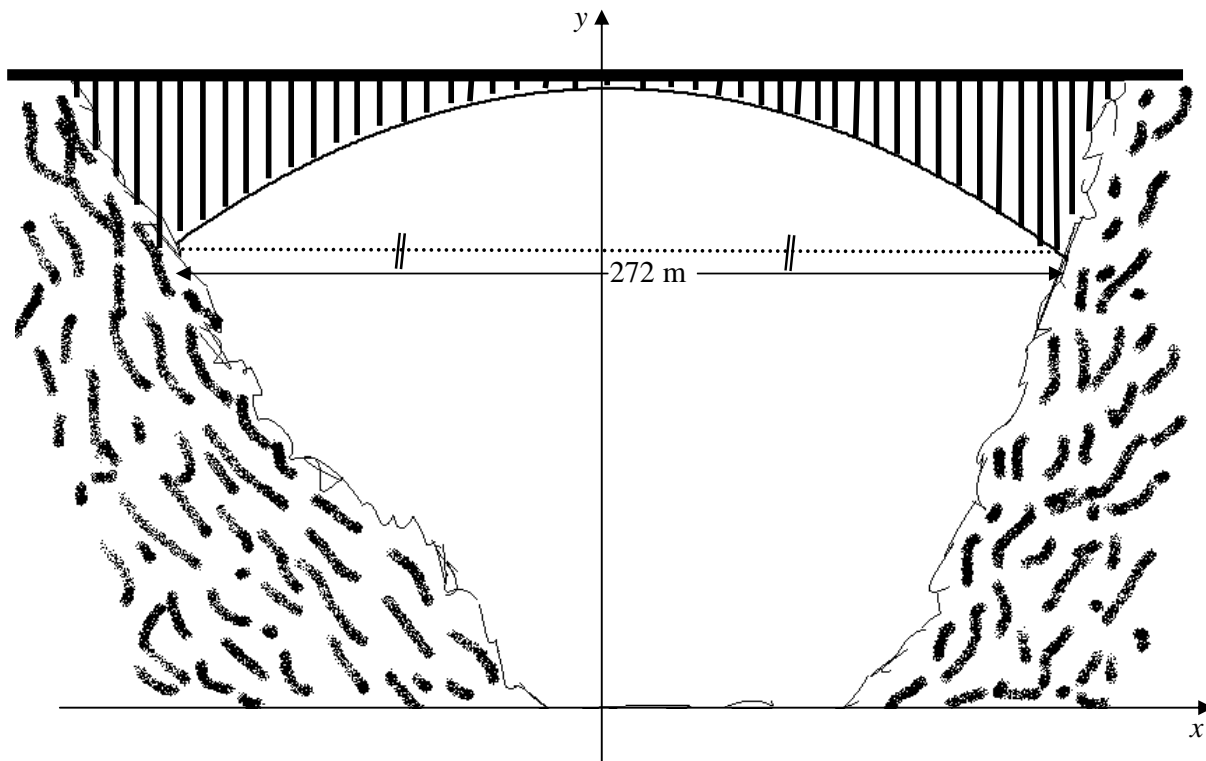


<http://www.adventureblog.org>

- (a) Suppose the arch is modelled by:

$$y = \frac{-x^2}{240} + 210$$

where  $x$  (in metres) is the horizontal distance from the centre of the arch and  $y$  (in metres) is the height above the bottom of the gorge.



The road bridge was constructed 6 m above the highest point of the arch.

(1) Show that the road is 216 m above the bottom of the gorge. (2)

(2) The concrete arch has a span of 272 m.  
Determine the height of the longest pillars between the bridge and arch. (4)

(b) Another claim to fame of the Bloukrans River Bridge is that it offers the highest commercial bungee-jumping site in the world.

At full stretch, the bungee rope is 160 m long. After the initial drop of 160 m, a person usually rises up by  $\frac{2}{3}$  of the distance that they previously fell, then falls

down to the full stretch of the rope again. A person thus covers:

$$160 + 2 \times \frac{2}{3} \times 160 + 2 \times \frac{2}{3} \times \frac{2}{3} \times 160 + \dots \text{ metres}$$

Calculate the total vertical distance covered by a jumper before they are hoisted back to the platform. (5)

<b>11 marks</b>
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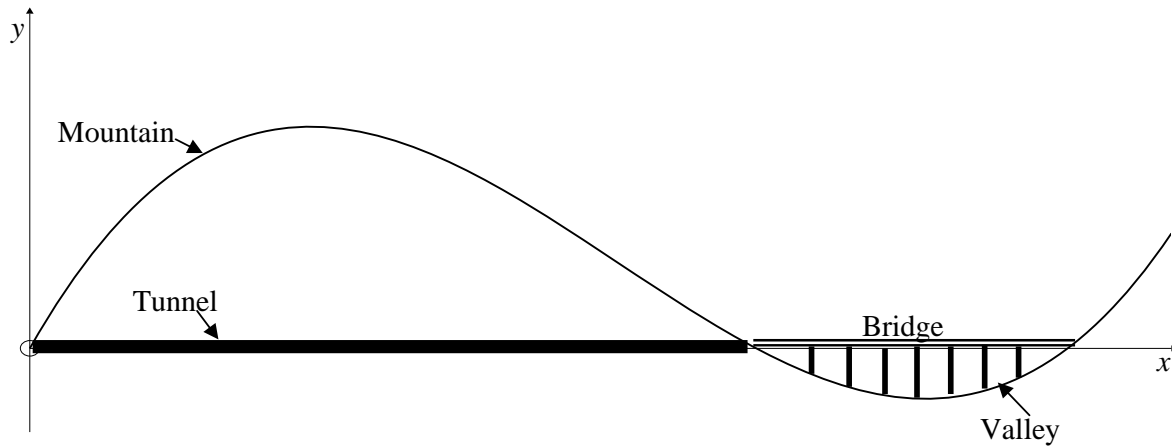


**QUESTION 9**

BMA Construction have been assigned to build a tunnel and a bridge in a mountainous area. They have determined that the cross-section can be modelled by a function:

$$y = 2x^3 - 17x^2 + 35x \quad 0 \leq x \leq 5$$

where  $x$  is the distance (in hundreds of metres) from a point where the tunnel will start and  $y$  is the height (in hundreds of metres) above the proposed bridge.



Giving answers to the nearest metre:

- (a) Determine the length of:
  - (1) the tunnel (5)
  - (2) the bridge (1)
- (b) Calculate the height (above the tunnel) of the top of the mountain. (7)
- (c) Calculate the deepest drop from the bridge to the valley below. (3)

<b>16 marks</b>
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**QUESTION 10**

Given the following sums:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

(a) Show that:  $1^3 + 2^3 + 3^3 + \dots + (2n+1)^3 = (2n+1)^2 (n+1)^2$  (3)

(b) Find a single term for:  $2^3 + 4^3 + 6^3 + \dots + (2n)^3$   
(Hint:  $(2k)^3 = 2^3 k^3$ ) (3)

(c) Hence find the sum of the series:  $1^3 + 3^3 + 5^3 + \dots + (2n+1)^3$  (3)

<b>9 marks</b>
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**Total: 150 marks**