



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2008

MATHEMATICS: PAPER II

Time: 3 hours

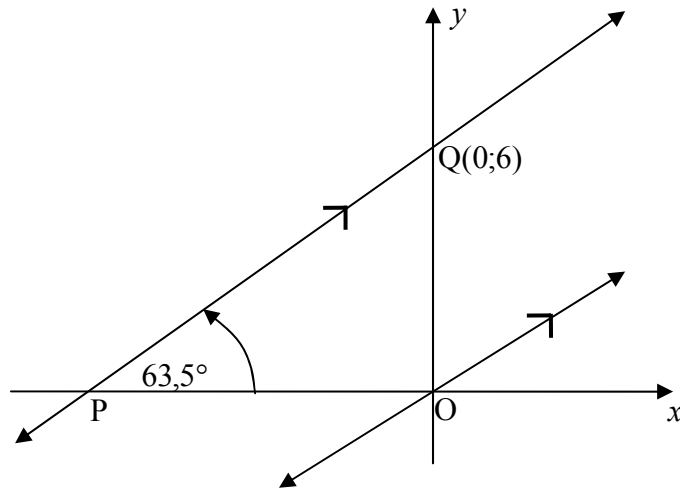
150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 16 pages, a Formula Sheet, and an Answer Booklet of 4 pages (i – iv). Please check that your paper is complete.
 2. Write your examination number in the space provided in your Answer Booklet.
 3. Answer ALL the questions. Questions 1(d), 2 and 4 should be answered in the Answer Booklet. Make sure that you hand this in with your examination script.
 4. Please note that diagrams are not necessarily drawn to scale.
 5. All necessary working details must be shown.
 6. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
 7. Answers must be rounded off to one decimal digit, unless otherwise stated.
 8. It is in your own interest to write legibly and to present your work neatly.
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SECTION A**QUESTION 1**

- (a) In the diagram below, a line segment joins P to Q and makes an angle of $63,5^\circ$ with the x -axis. A second line parallel to PQ passes through the origin.



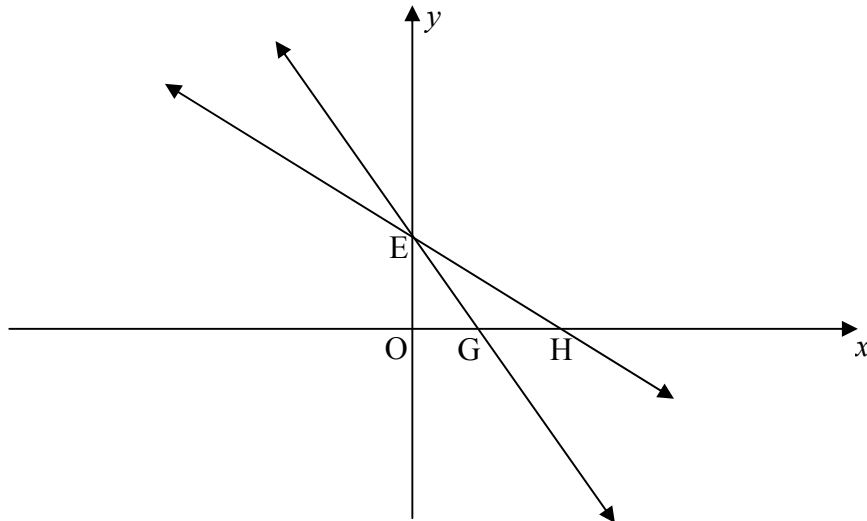
- (1) Determine the gradient of the line segment PQ, correct to the nearest whole number. (2)
- (2) Write down the equation of the line passing through P and Q. (1)
- (3) Write down the equation of line passing through the origin. (1)

(b) In the diagram the following lines have been sketched:

(i) $y + 2x = p$ and

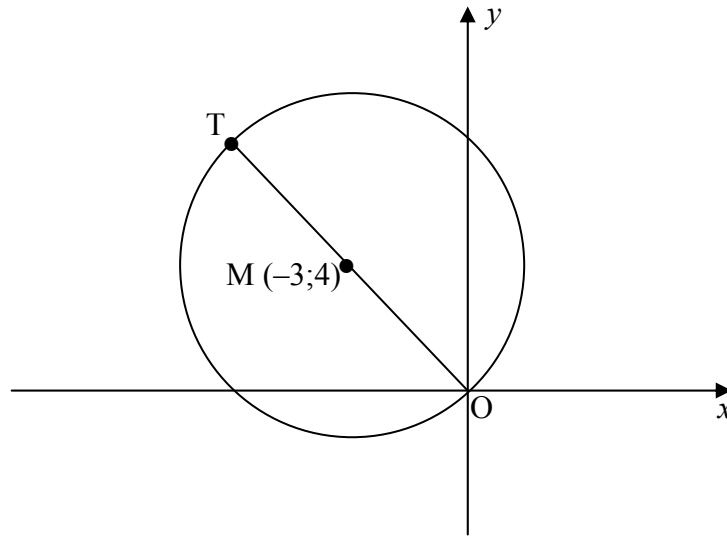
(ii) $2y + x = 2p$

where p is a constant.



- (1) Write down, in terms of p , the equation of line passing through E and H. (1)
- (2) Write down, in terms of p , the equation of line passing through E and G. (1)
- (3) Determine, in terms of p , the co-ordinates of E, G and H. (3)
- (4) Hence, write down the area of $\triangle EGH$ in terms of p . (2)

(c) In the diagram below, the circle has centre $M(-3;4)$ and passes through the origin.



- (1) Determine the length of the radius of the circle. (2)
- (2) Write down the equation of the circle. (2)
- (3) Determine the co-ordinates of T, the end point of diameter OT. (2)

ANSWER THIS QUESTION IN THE ANSWER BOOKLET

- (d) Two circles with equations $(x-3)^2 + (y-5)^2 = 25$ and $(x-7)^2 + (y-5)^2 = 9$ intersect in two points. Sketch the circles in the grid shown in the Answer Booklet and hence determine the equation of the line passing through these two points. As a scale, use one block to represent one unit. The circles can be drawn freehand.

(5)

22 marks**QUESTION 2****ANSWER THIS QUESTION IN THE ANSWER BOOKLET**

Given: $M(-2; 1)$ and $H(1; 3)$. (a) and (b) below are two given transformations.

- On each set of axes given, draw the image of ΔMHO according to the rule given. Label your image $\Delta M'H'O'$, and include the co-ordinates of the transformed points.
- Identify the type of transformation.
- Give the value of $\frac{\text{Perimeter of } \Delta MHO}{\text{Perimeter of } \Delta M'H'O'}$

(a) $(x; y) \rightarrow (-y; x)$ (4)

(b) $(x; y) \rightarrow (2x; 2y)$ (4)

8 marks

QUESTION 3

- (a) If $4\sin P = 1$ and $\cos P < 0$, use a sketch to find the value of

$$\tan(180^\circ - P) \times \cos(90^\circ - P)$$

Answers must be left in surd form. (5)

- (b) Calculate the value of each of the following correct to three decimal digits.

(1) $-\sin^2 343,6^\circ$ (1)

(2) $\frac{\tan 88,8^\circ}{\cos 44,4^\circ - 2}$ (2)

- (c) Given:

$$\sin 22^\circ \cdot \cos 12^\circ = a$$

$$\sin 12^\circ \cdot \cos 22^\circ = b$$

$$\cos 22^\circ \cdot \cos 12^\circ = c$$

$$\sin 22^\circ \cdot \sin 12^\circ = d$$

Express the following in terms of a, b, c and d.

(1) $\sin 34^\circ$ (2)

(2) $\cos 10^\circ$ (2)

- (d) Prove that :

$$\tan \theta + \frac{\cos \theta}{\sin \theta} = \frac{2}{\sin 2\theta}$$
 (4)

- (e) Solve for $\theta \in [0^\circ; 360^\circ]$, correct to one decimal digit, if $\cos \frac{\theta}{2} = -0,888$. (3)

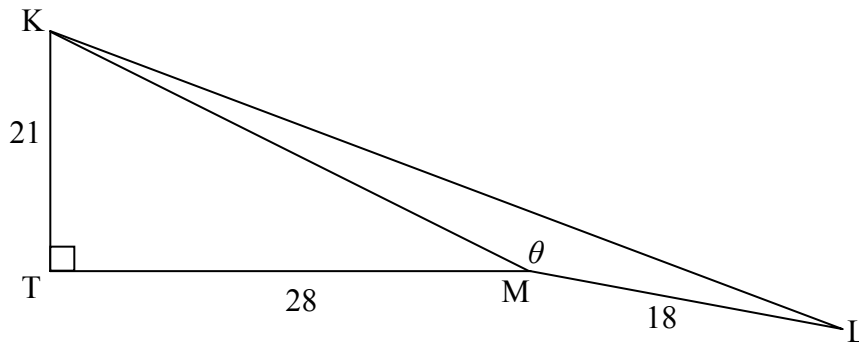
- (f) Given the equation: $\tan(5\theta) = \tan \theta$

(1) Write down the general solution. (3)

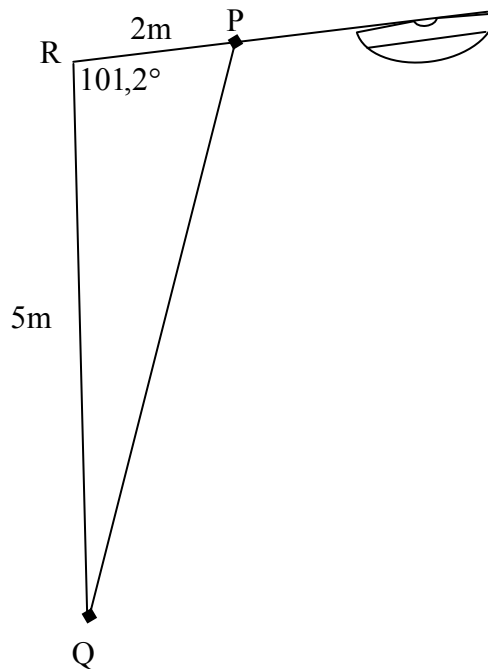
(2) Write down the value(s) of $\theta \in [-90^\circ; 90^\circ]$ for which $\tan \theta$ is undefined. (1)

(3) Hence, or otherwise, write down the values of $\theta \in [-90^\circ; 90^\circ]$ which satisfy the equation. (2)

- (g) In the diagram below, the area of $\triangle KLM$ is equal to the area of $\triangle TKM$.
 $\hat{KML} = \theta$ and θ is an obtuse angle.



- (1) Determine the area of $\triangle TKM$. (2)
 - (2) Show that $\sin \theta = \frac{14}{15}$. (4)
 - (3) Determine the value of θ , correct to the nearest degree. (1)
- (h) In the diagram below, a streetlight is depicted. The light is attached to a pole that is supported by a brace, PQ. The length of the pole RQ is 5 metres, RP is 2 metres and $\hat{PRQ} = 101,2^\circ$.



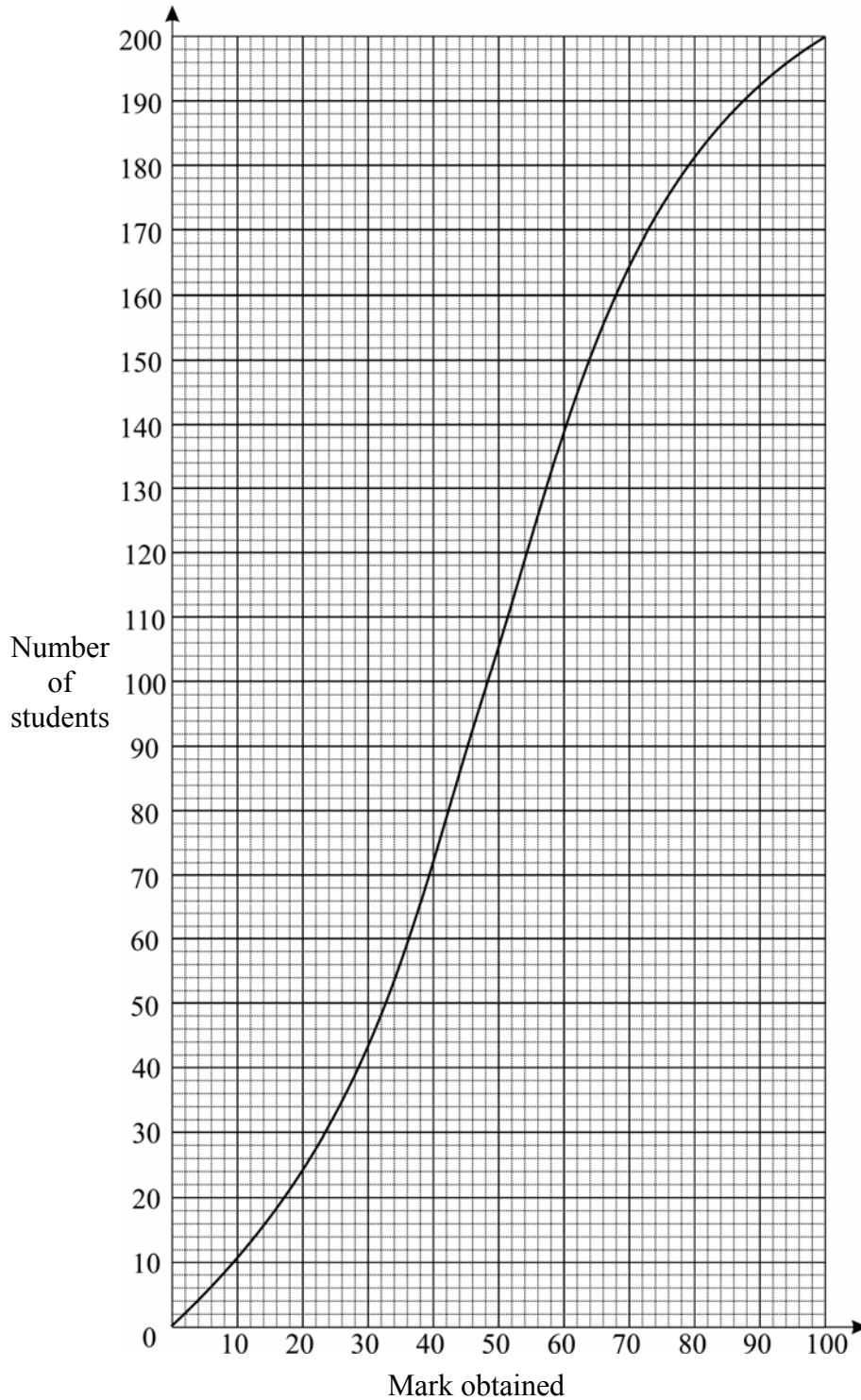
- Find the length of the brace PQ, correct to the nearest tenth of a metre. (3)

35 marks

QUESTION 4

ANSWER ALL PARTS OF THIS QUESTION IN THE ANSWER BOOKLET

- (a) The cumulative frequency curve below shows the marks obtained in an examination by a group of 200 students.



- (1) Use the cumulative frequency curve to complete the frequency table in the Answer Booklet. (4)
- (2) Forty percent of the students failed. Find the pass mark. (1)

(b) A student is investigating the sizes of apples in 2 kg bags. His findings are given in the table below:

Number of apples in the bag(N)	20	16	18	14	14	15	13	10
Median mass of apples to the nearest gram(M)	72	105	110	125	136	142	174	192

- (1) On the grid in the Answer Booklet, draw a scatter diagram to illustrate the data in the table. (4)
- (2) State the type of relationship (linear, quadratic or exponential) that exists between the number of apples in a bag and the mass of the apples. (1)

10 marks

SECTION B

QUESTION 5

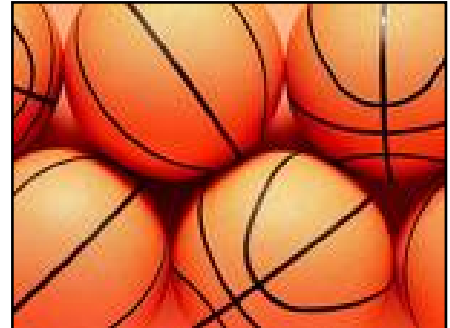
The heights in cm of players in two different basketball teams A and B are recorded below.

The heights for team A are 203, 214, 187, 188, 196, 199, 205, 203, 199 and 206.

For team B, the sum of the heights is 2388 and the sum of the squares of the heights is 475 770.

The mean height for team B is 199 cm, i.e.:

$$\sum h = 2\,388, \sum h^2 = 475\,770 \text{ and } \bar{h} = 199.$$



The number of players in each team is different.

- (a) Calculate, correct to one decimal digit, the standard deviation of the heights of team A using the formula.

$$\text{standard deviation} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}.$$

Show all your working by using a table, such as the one shown below. Draw the table in your Answer Book.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
203		
214		
187		
188		
196		
199		
205		
203		
199		
206		
		$\sum_{i=1}^n (x_i - \bar{x})^2 =$

(6)

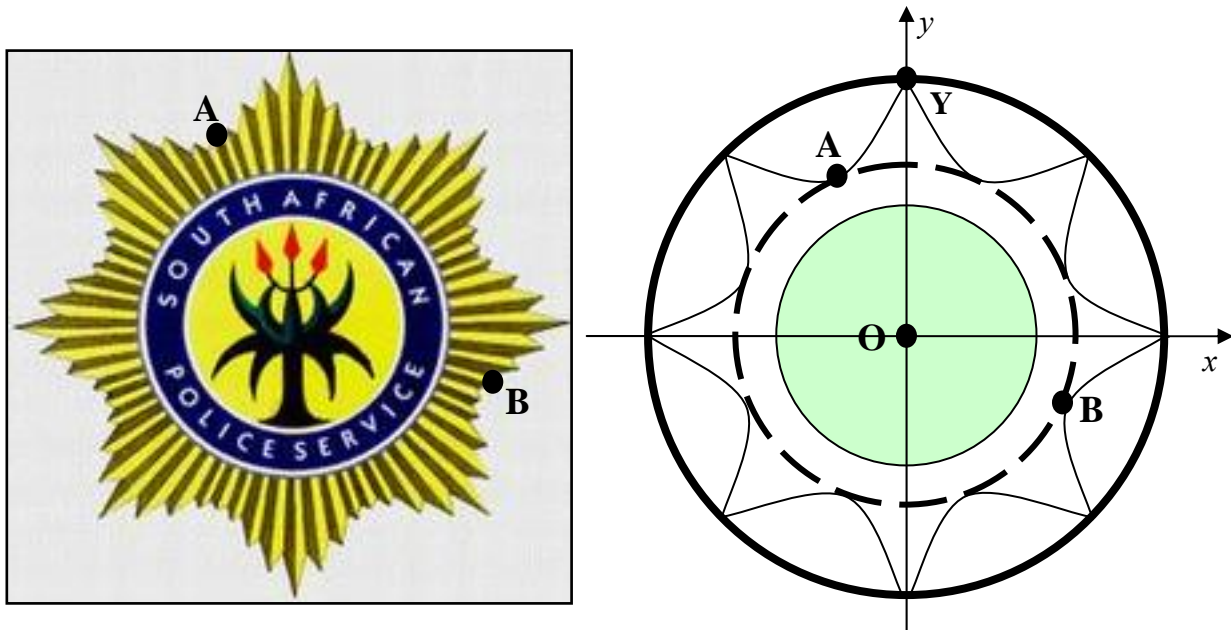
- (b) The formula: $\text{Variance} = \frac{\sum_{i=1}^n x_i^2}{n} - (\bar{x})^2$ can also be used to find the standard deviation for a set of data. Use this formula to find, correct to one decimal digit, the standard deviation for team B. (4)
- (c) Make a valid comment about the dispersion of the heights of the players in the different teams based on the answers obtained in 5(a) and 5(b). (1)

11 marks

QUESTION 6

(a) Without the use of a calculator, show that the co-ordinates of the image A' of the point $A(p; q)$ after rotation about the origin, through an angle of 135° in a clockwise direction, will be $\left(-\frac{\sqrt{2}}{2}(p - q); -\frac{\sqrt{2}}{2}(p + q)\right)$. (6)

(b) The South African Services Emblem is given below. Alongside the actual photograph is a sketch in the Cartesian plane illustrating the emblem. The points A and B are on the emblem as shown. The centre of the emblem is at the origin, O .

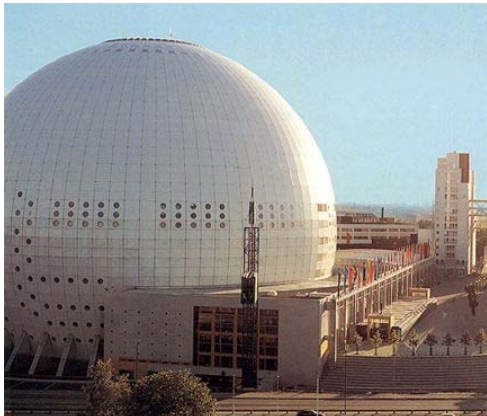


- (1) Calculate, by inspection, the size of $\hat{A}OY$ where Y is a point on the y -axis. (2)
- (2) Hence, if $A(-3; y)$, show, correct to one decimal digit, that $y = 7,2$. (3)
- (3) Determine the co-ordinates of B , correct to one decimal digit. (4)

15 marks

QUESTION 7

- (a) If $\sin 24^\circ = p$, express, without using a calculator, the following in terms of p .
- (1) $\cos 24^\circ$ (2)
- (2) $\sin 168^\circ \cdot \sin(-78^\circ)$ (5)
- (b) (1) Show that $\frac{\cos \theta + \cos 3\theta}{\cos 2\theta} = 2 \cos \theta$. (4)
- (Hint: $\theta = 2\theta - \theta$ and $3\theta = 2\theta + \theta$)
- (2) Hence write down one possible pair of values for A and B if $\frac{\cos 70^\circ + \cos A}{\cos B} = 2 \cos 70^\circ$. (2)
- (3) Hence, write down a second pair of values for A and B if $\frac{\cos 70^\circ + \cos A}{\cos B} = 2 \cos 70^\circ$. (2)
- (c) Buildings in the shape of spheres are common architectural marvels around the world.



The Globe Arena in Stockholm

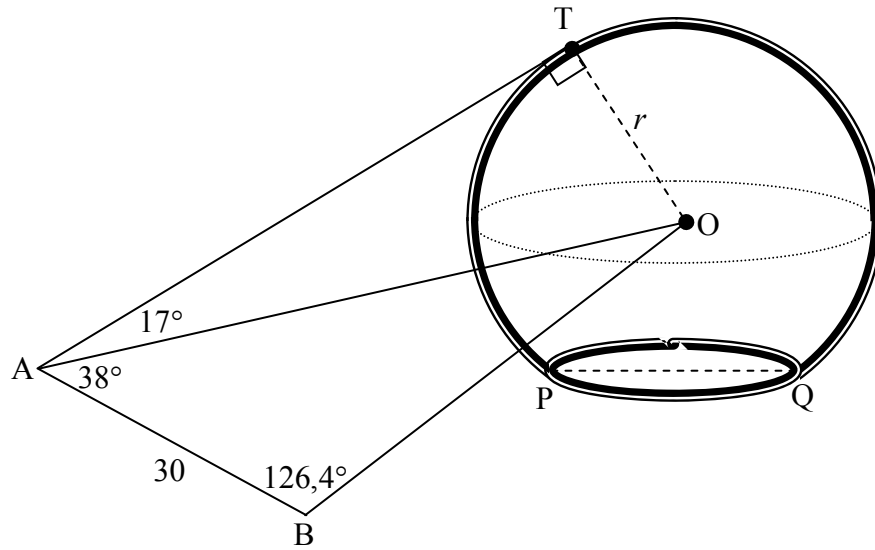


The Matrimandir in India

Shlomo, a tourist, comes across a building in the shape of a sphere that is made of steel. He is immediately curious about the amount of steel making up the surface of the sphere.

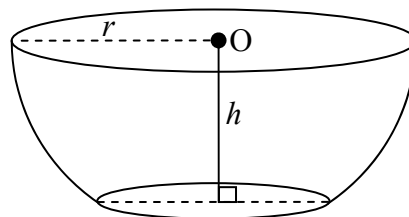
He decides to use the trigonometry that he has studied at school. (Refer to diagram on page 14). He stands at point A and is able to calculate that $\hat{TAO} = 17^\circ$ and $\hat{OAB} = 38^\circ$, where O is the centre of the sphere. Then he walks in a straight line, 30 metres away from A to a point B where he calculates that $\hat{ABO} = 126,4^\circ$. OT is the radius of the sphere and is given by r .

The sphere is cut off by the horizontal ground at P, Q and P, Q, A and B are all in the same horizontal plane.



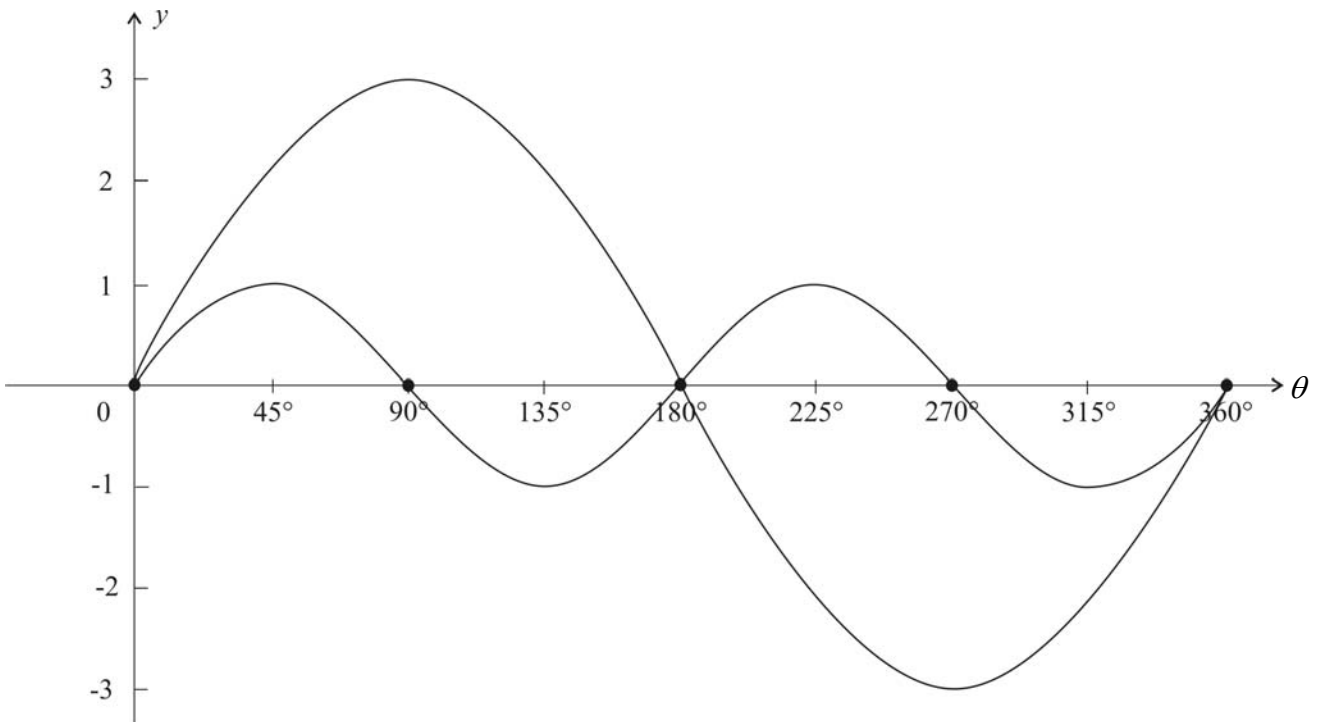
$\hat{A}TO = 90^\circ$

- (1) Show that $r \approx 26,25$ m. (5)
- (2) The surface area of a sphere is given by $4\pi r^2$.
Calculate the surface area, S, of a sphere with radius of 26,25 m. (2)
- (3) The volume of a truncated hemisphere is given by $V = \pi r^2 h - \pi \frac{h^3}{3}$ and the surface area (excluding top and bottom bases) is given by $S = 2\pi r h$ where r is the radius of the sphere and h is the perpendicular height of the truncated solid.



- (i) Show that the volume of the building visited by Shlomo is given by $V = \frac{2}{3}\pi r^3 + 18\pi r^2 - 1944\pi$ if $h = 18$ m.
You may use the formula for the volume of a sphere which is $V = \frac{4}{3}\pi r^3$. (3)
- (ii) Calculate the amount of steel (in m^2) required to build the surface of the building visited by Shlomo if $h = 18$ m and $r = 26,25$ m. (4)

(d) The diagram below shows the graphs of $f(\theta) = 3\sin\theta$ and $g(\theta) = \sin 2\theta$.



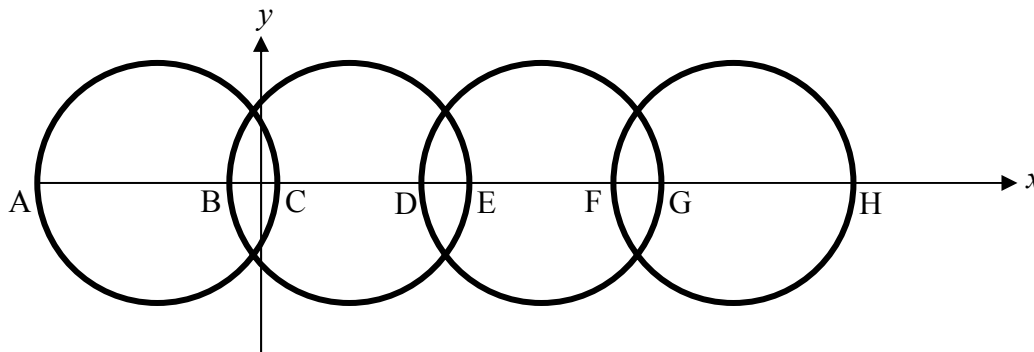
Use the graph to answer the following questions:

- (1) How many solutions does the equation $3\sin\theta = -\sin 2\theta$ have in the interval $\theta \in [0^\circ; 360^\circ]$? (1)
- (2) Write down the complete general solution to $3\sin\theta = -\sin 2\theta$. (2)

32 marks

QUESTION 8

- (a) The diagram below is a sketch of the Audi logo.
 Audi is a car manufacturer.
 The logo consists of four equal circles intersecting as shown.
 $BC = DE = FG$.
 AC, BE, DG and FH are diameters of the circles.
 A, B, C, D, E, F, G and H lie on the same horizontal line.



The equations of the two middle circles are given by $x^2 - 12x + y^2 = 64$ and $(x - 20)^2 + y^2 = r^2$

- (1) Find the value of r . (3)
 (2) Write down the equation of the circle with diameter AC . (2)
 (3) Determine the length of BC . (2)
- (b) (1) (i) Find, correct to two decimal digits, the acute angle of inclination of the line $y = 2x$ to the x -axis. (2)
 (ii) Find, correct to two decimal digits, the acute angle of inclination of the line $y = \frac{1}{2}x$ to the x -axis. (1)
- (2) Study the tables below which contain the values of the inclinations of lines with equations of the form $y = ax$ and $y = \frac{1}{a}x$ for $a \in \{3,4,5\}$.

Equation	$y = 3x$	$y = \frac{1}{3}x$
Inclination	$71,6^\circ$	$18,4^\circ$

Equation	$y = 4x$	$y = \frac{1}{4}x$
Inclination	$76,0^\circ$	$14,0^\circ$

Equation	$y = 5x$	$y = \frac{1}{5}x$
Inclination	$78,7^\circ$	$11,3^\circ$

Based on the information in the tables above, formulate a conjecture by completing the following sentence:

- The inclinations of the lines $y = ax$ and $y = \frac{1}{a}x$ where $a > 0$ (2)
 (3) Prove the conjecture that you formulated in (2). (5)

17 marks

Total: 150 marks