NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2008

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1. (a) (1) $\tan 63{,}5° = 2$ ✓
(2) $y = 2x + 6$ ✓
(3) $y = 2x$ ✓ (4)

(b) (i) $y + 2x = p \Rightarrow y = -2x + p$
$2y + x = 2p \Rightarrow y = -\frac{1}{2}x + p$

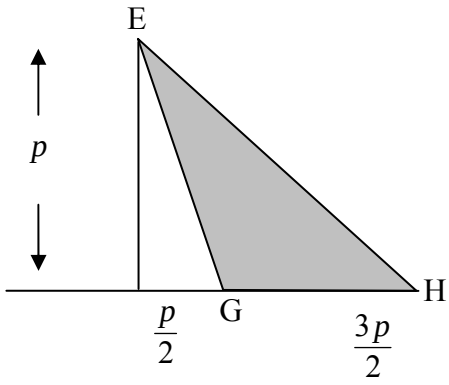
(1) $y = -\frac{1}{2}x + p$ EH ✓ or $2y + x = 2p$

(2) $y = -2x + p$ EG ✓ or $y + 2x = p$

(3) E $(0 ; p)$ ✓ H $(2p ; 0)$ ✓ G $(\frac{p}{2} ; 0)$ ✓

(4) Area $= \frac{1}{2} . \frac{3p}{2} . p$ ✓

$= \frac{3p^2}{4}$ units^2 ✓



(7)

(c) (1) $(-3)^2 + (4)^2 = r^2$ ✓
$r = 5$ units ✓
(2) $(x + 3)^2 + (y - 4)^2 = 25$ ✓✓
(3) $\left(\frac{0 + x}{2} = -3 \quad and \quad \frac{0 + y}{2} = 4\right)$
∴ T has co-ordinates $(-6 ; 8)$ ✓✓ (6)

(d) Drawing of circles ✓✓
From the graph the points of intersection are (7 ; 2) and (7 ; 8) ✓✓
Equation is: $x = 7$ ✓ (5)

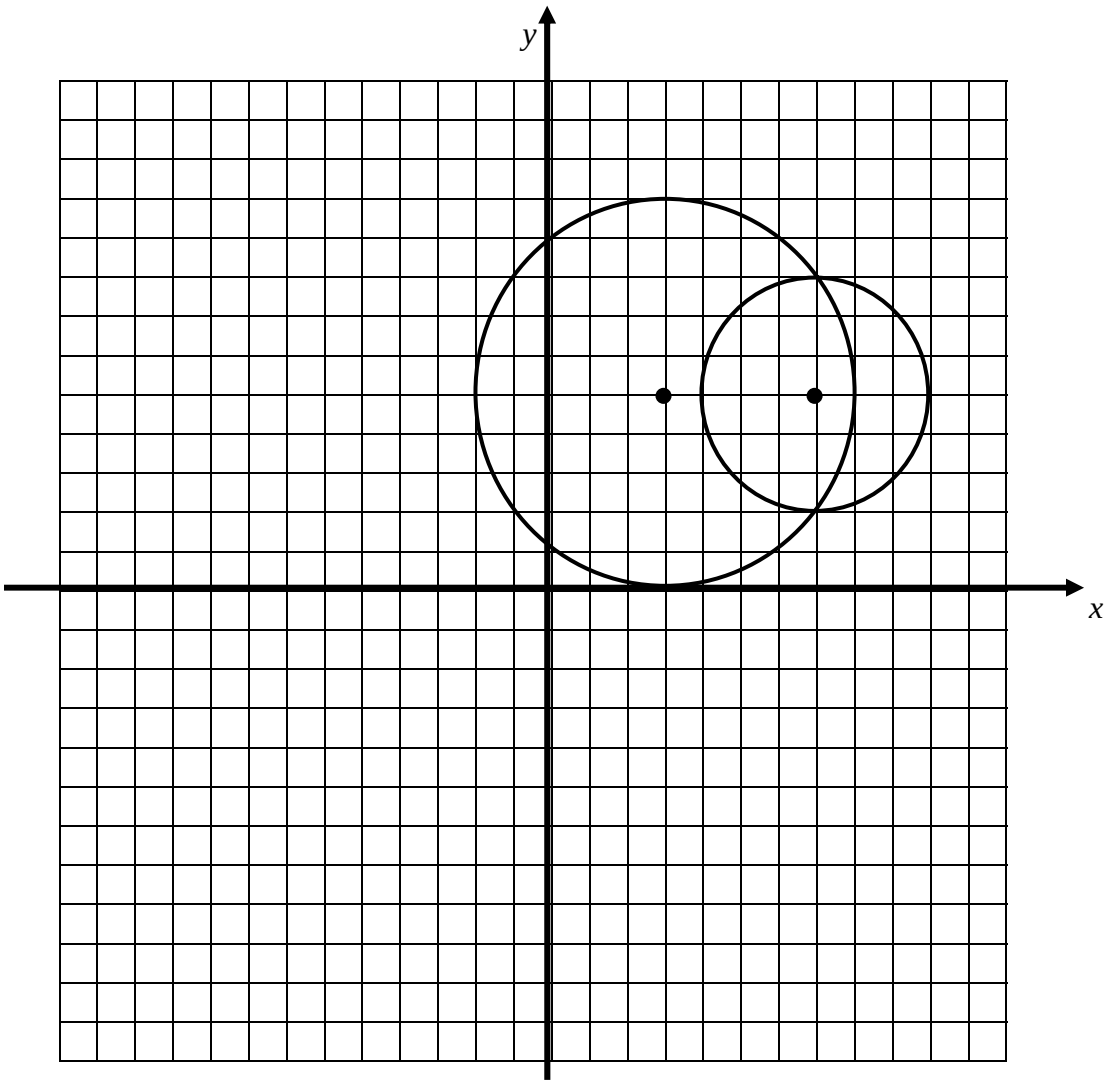
Check: (7 ; 2)

$$(x-3)^2 + (y-5)^2 = (4)^2 + (-3)^2$$
$$= 16 + 9$$
$$= 25$$
$$r = 5$$
$$(x-7)^2 + (y-5)^2 = (0)^2 + (-3)^2$$
$$= 9$$

(7 ; 2) on both circles

Check: (7 ; 8)

$(4)^2 + (3)^2 = 5^2$
$(x\ , 7)^2 + 1y - 5 = 9^2$
$(0)^2 + (3)^2 = 9$
(7,8) is on both

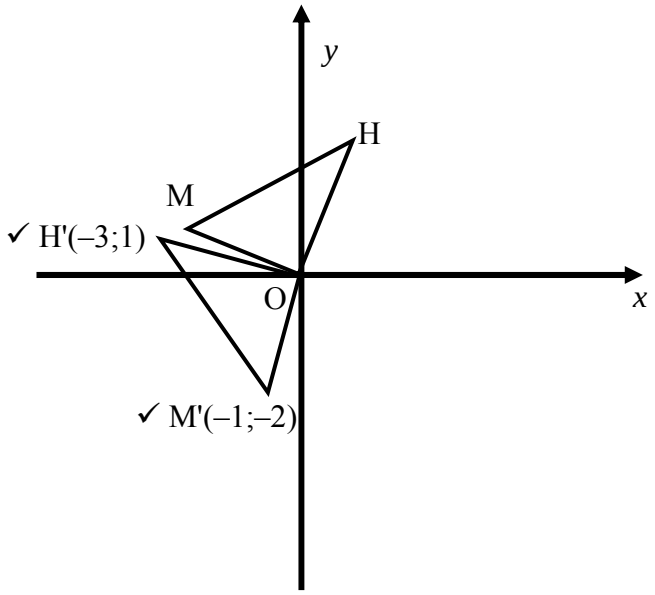


22 marks

QUESTION 2

(a) Transformation Type: 90° anticlockwise ✓ rotation through the origin.

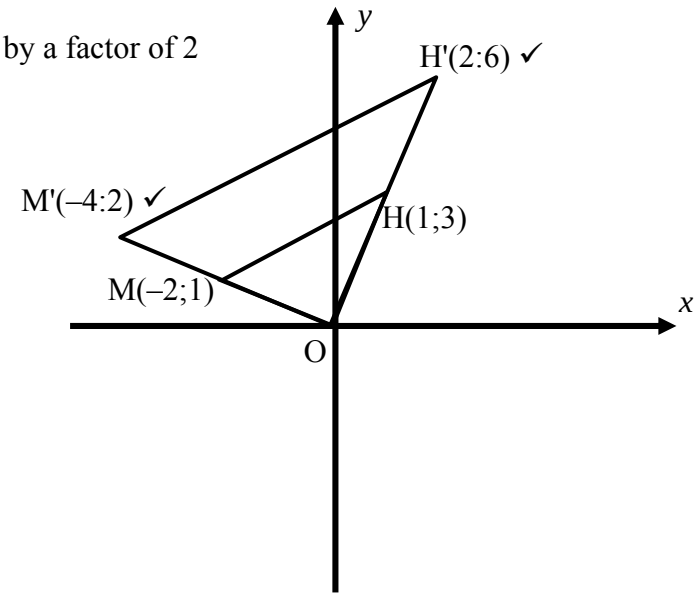
$$\frac{\text{Perimeter of } \Delta MHO}{\text{Perimeter of } \Delta M'H'O'} = 1 \checkmark$$



(b) $(x : y) \rightarrow (2x ; 2y)$

Transformation Type: enlargement ✓ by a factor of 2
Centre the origin

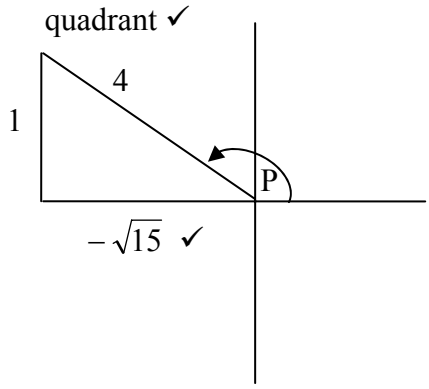
$$\frac{\text{Perimeter of } \Delta MHO}{\text{Perimeter of } \Delta M'H'O'} = \frac{1}{2} \checkmark$$



(8)

8 marks

QUESTION 3



(a) $4 \sin P = 1$

$\sin P = \frac{1}{4}$

$\tan (180° - P) \times \cos (90° - P)$ ✓

$= (-\tan P)(\sin P)$

$= \left(-\frac{1}{-\sqrt{15}}\right)\left(\frac{1}{4}\right)$ ✓ (✓)

$= \frac{1}{4\sqrt{15}}$ (5)

(b) (1) –0,080 ✓ (1)

(2) –37,136 ✓✓ (2)

(c) (1) $\sin (34°) = \sin (22° + 12°)$

$= \sin 22° \cos 12° + \cos 22° \sin 12°$ ✓

$= a + b$ ✓

(2) $\cos (10°) = \cos (22° - 12°)$

$= \cos 22° \cos 12° + \sin 22° \sin 12°$ ✓

$= c + d$ ✓ (4)

(d) $\tan\theta + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ ✓

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ ✓

$= \frac{1}{\sin\theta\cos\theta}$ ✓

$= \frac{2}{2\sin\theta\cos\theta}$ ✓

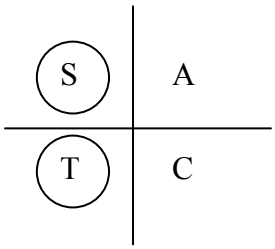
$= \frac{2}{\sin 2\theta}$ (4)

(e) $\cos\frac{\theta}{2} = -0{,}888$

$\frac{\theta}{2} = 152{,}6° + k\,360$ ✓ $\quad k \in Z$ or $\frac{\theta}{2} = 207{,}4° + k\,360°$

$\theta = 305{,}2° + k\,720°$ or $414{,}8° + k720°$

$\theta = 305{,}2°$ ✓



Ref angle = 27,37700618 ✓

(3)

OR

$\frac{\theta}{2} = \pm\cos^{-1}(-0{,}888) + 360°k$

$\frac{\theta}{2} = \pm 152{,}6°$ ✓ $+ 360°k$ ✓

$\theta = \pm 305{,}2° + 720°k$

$\therefore\ \theta = 305{,}2°$ ✓

OR

$\theta \in [0° ; 360°]\ \therefore\ \frac{\theta}{2} \in [0° ; 180°]$

since $\cos\left(\frac{\theta}{2}\right) < 0$; $\frac{\theta}{2} = 152{,}6°$

$\therefore\ \theta = 305{,}2°$

(f) (1) $\tan(5\theta) = \tan\theta$

$\therefore\ 5\theta = \theta + 180\,k$ ✓ ; $k \in Z$

$\therefore\ 4\theta = 180\,k$ ✓

$\theta = 45°\,k$ ✓ ; $k \in Z$ (3)

(2) $-90° ; 90°$ ✓ (1)

(3) $-45° ; 0° ; 45°$ ✓✓ (2)

(g) (1) Area $= \frac{1}{2} . 28 . 21$ ✓

$= 294$ units^2 ✓ (2)

(2) KM $= \sqrt{21^2 + 28^2 = 35}$ ✓

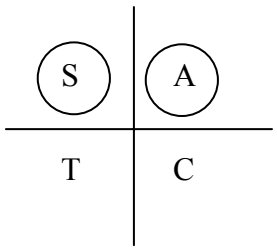
Area $\Delta KLM = \frac{1}{2} . 35 . 18 \sin\theta$ ✓

$\therefore\ = 315\sin\theta$ ✓

$\therefore\ 315\sin\theta = 294$ ✓ $\therefore\ \sin\theta = \frac{14}{15}$ (4)

(3) $\sin\theta = \frac{14}{15}$

$\theta = 111°$ ✓ (1)



Ref – 68,9605

(h) $PQ^2 = 5^2 + 2^2 - 2(5)(2)\cos 101{,}2°$ ✓

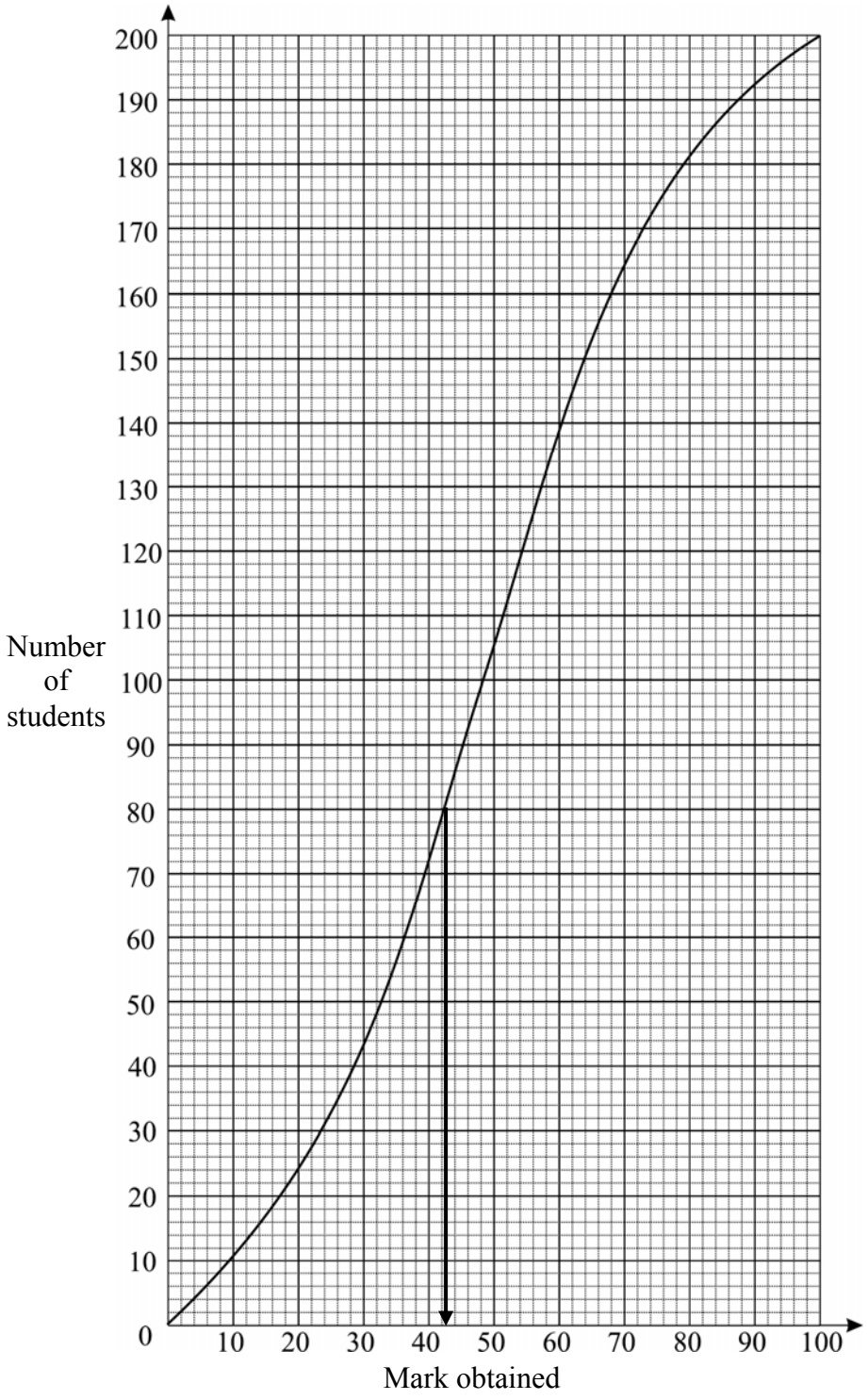
$PQ^2 = 32{,}88468702$ ✓

$PQ^2 = 5{,}7$ m ✓ (3)

35 marks

QUESTION 4

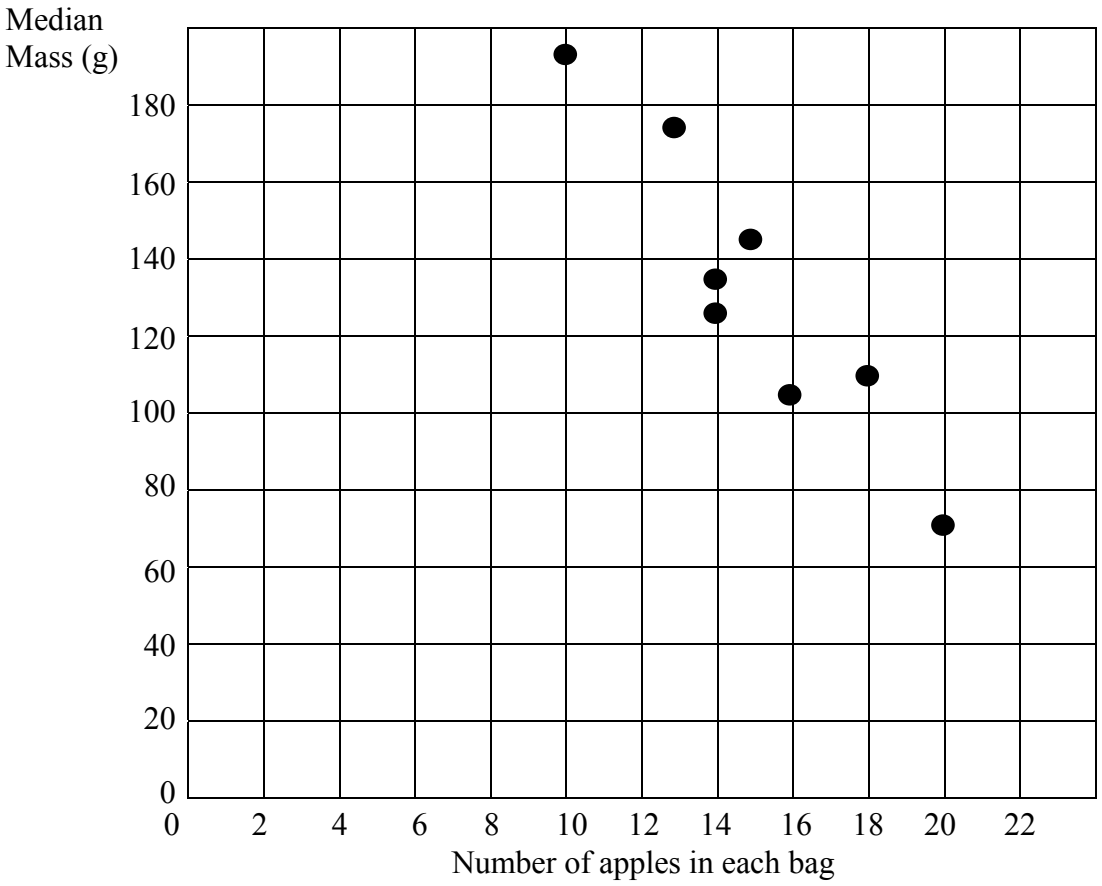
(a) The cumulative frequency curve below shows the marks obtained in an examination by a group of 200 students.



(a) (1) Use the cumulative frequency curve to complete the frequency table below.

Mark (X)	$0 \leq x < 20$	$20 \leq x < 40$	$40 \leq x < 60$	$60 \leq x < 80$	$80 \leq x < 100$
Number of Students	22	50 ✓	66 ✓	42 ✓	20 ✓

(2) 42 ✓ (4)



✓✓✓✓✓

(4)

(2) linear ✓ (1)

10 marks

SECTION B

QUESTION 5

(a) $\bar{x} = \frac{2000}{10} = 200$

	✓ ✓	✓
x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
203	3	9
214	14	196
187	–13	169
188	–12	144
196	–4	16
199	–1	1
205	5	25
203	3	9
199	–1	1
206	6	36
		$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 606$ ✓

Standard deviation = $\sqrt{\frac{606}{10}}$ ✓ = $\sqrt{60,6}$ = 7,8 ✓ (6)

(b) Variance = $\frac{\sum_{x=1}^{n} x_i^2}{n} - (\bar{x})^2$ ✓ and n = $\frac{2388}{199} = 12$ ✓

$= \frac{475770}{12} - (199)^2$

$= 46,5$ ✓

Standard deviation = $\sqrt{46,5} = 6,8$ ✓ (4)

(c) The spread of heights in team A is greater than the spread of height in team B. (1)

11 marks

QUESTION 6

(a) $(x\cos\theta - y\sin\theta \;;\; x\sin\theta + y\cos\theta)$

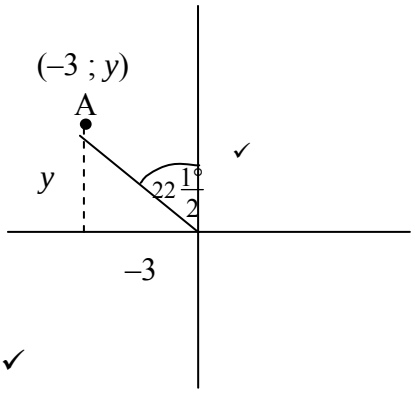
$(p\cos(-135°) - q\sin(-135°) \;;\; p\sin(-135°) + q\cos(-135°))$ ✓✓

$(p(-\cos 45°) + q((\sin 45°)\,;\, -p\sin 45° + q(-\cos 45))$ ✓✓

$\left(-\frac{\sqrt{2}}{2}p + \frac{\sqrt{2}}{2}q \;;\; \frac{-\sqrt{2}}{2}p - \frac{\sqrt{2}}{2}q\right)$ ✓✓

$\left(\frac{-\sqrt{2}}{2}(p-q) \;;\; \frac{-\sqrt{2}}{2}(p+q)\right)$ (6)

(b) (1) $A\hat{O}Y = \frac{45°}{2} = 22\frac{1}{2}°$ ✓ (2)



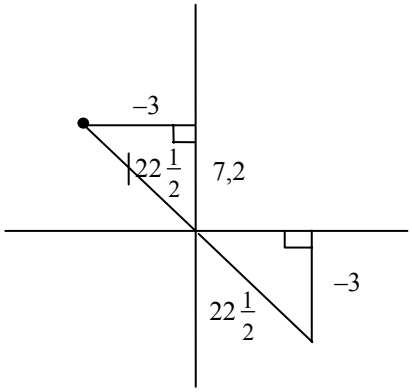
(2) $\frac{y}{-3} = \tan 112\frac{1}{2}°$ ✓

$y = -3\tan 112\frac{1}{2}°$

$= 7{,}2$ ✓ (3)

(3) B is the reflection of A about $y = x$ ∴ ✓✓

$B(y\,;-3) = B(7{,}2\,;-3)$ ✓✓



$B\left(-\frac{\sqrt{2}}{2}(-3-7{,}2) \;;\; -\frac{\sqrt{2}}{2}(-3+7{,}2)\right)$ ✓✓

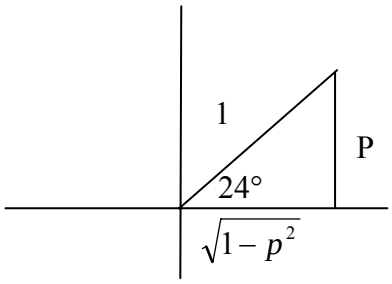
$= B(7{,}2\,;-3)$ ✓✓ (4)

Inspection gives (7,2 ; –3)

15 marks

QUESTION 7

(a) (1) $\cos 24° = \sqrt{1 - p^2}$ ✓ (2)



(2) $\sin 12° \ (-\sin 78°)$
$= -\sin 12° \cdot \cos 12°$ ✓✓
$= -\frac{1}{2}\sin 24°$
$= -\frac{1}{2}p$ (5)

(b) (1) $\frac{\cos\theta + \cos 3\theta}{\cos 2\theta}$ ✓ ✓

$= \frac{\cos(2\theta - \theta) + \cos(2\theta + \theta)}{\cos 2\theta}$ ✓ $= \frac{\cos 2\theta \ \cos\theta + \sin 2\theta \sin\theta + \cos 2\theta \cos\theta - \sin 2\theta \sin\theta}{\cos 2\theta}$

$= \frac{2\cos 2\theta \cos\theta}{\cos 2\theta}$ ✓

$= 2\cos\theta$ (4)

(2) $\theta = 70°$ $A = 3\theta = 210°$ ✓

$B = 2\theta = 140°$ ✓ (2)

(3) $A = 210° + 360° = \underline{570°}$ →

$B = 140 + 360 = \underline{500}$ → (2)

(c) (1) In ΔABO

$\frac{30}{\sin 15{,}6°} = \frac{OA}{\sin 126{,}4°}$ ✓✓

$OA = \frac{30 \cdot \sin 126{,}4°}{\sin 15{,}6°}$

$= 89{,}79187129$ ✓

$\therefore r = OA \sin 17°$ ✓

$= 26{,}252602$

$= 26{,}25$ ✓ (5)

→

(2) $5 = 4\pi (26{,}25)^2$ ✓ $= 8659\ m^2$ ✓ (2)

(3) (i) $V = \frac{2}{3}\pi r^3$ ✓ $+ \pi r^2 (18) - \pi\left(\frac{18^3}{3}\right)$ ✓

$= \frac{2}{3}\pi r^3 + 18\pi r^2 - 1944\pi$ ✓ (3)

(ii) $S = \frac{dV}{dr} = 2\pi r^2$ ✓ $+ 36\pi r$ ✓

$= 2\pi (26{,}25)^2 + 36\pi (26{,}25)$ ✓

$= 2323{,}1\ m^2$ ✓ (4)

(d) (1) 3 ✓ (1)

(2) $\theta = 0° + k\,180°$ ✓ $k \in Z$

32 marks

QUESTION 8

(a) (1) $x^2 - 12x + y^2 = 64$

$(x-6)^2 - 36 + y^2 = 64$

$(x-6)^2 + y^2 = 100$ ✓✓

$r = 10$ ✓

centre (6 ; 0)

$(x-20)^2 + y^2 = 100$

centre (20 ; 0) (3)

(2) $(x+8)^2 + y^2 = 100$ ✓✓ (2)

(3) BC = 6 ✓✓ (2)

(b) (1) (i) $y = 2x$

$\tan\theta = 2$ ✓

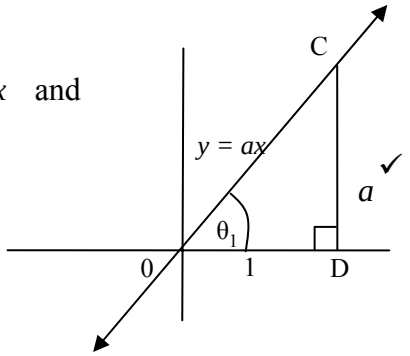
$\theta = 63{,}43°$ ✓ (2)

(ii) $y = \frac{1}{2}x$

$\theta = 26{,}57°$ ✓ (1)

(2) add up to 90° ✓

(3) Let θ_1 and θ_2 be the inclinations of $y = ax$ and $y = \frac{1}{a}x$ respectively.



The $\tan\theta_1 = a$

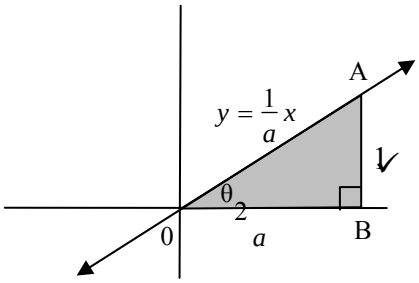
$\tan\theta_2 = \frac{1}{a}$ ✓

$\Delta ODC \equiv \Delta ABO$ SAS ✓

$\therefore \hat{A} = \theta_1$

$\therefore \theta_1 + \theta_2 = 90°$

or



$\tan\theta_1 = a$ and $\tan\theta_2 = \frac{1}{a}$ ✓

$\therefore \tan\theta_1 = \frac{1}{\tan\theta_2}$ $\therefore \tan\theta_1 . \tan\theta_2 = 1$ ✓

$\therefore \frac{\sin\theta_1}{\cos\theta_1} \times \frac{\sin\theta_2}{\cos\theta_2} = 1$

$\therefore \sin\theta_1 . \sin\theta_2 = \cos\theta_1 . \cos\theta_2$ ✓

$\therefore \cos(\theta_1 + \theta_2) = 0$

$\therefore a + \theta = 90°$ ✓ (5)

17 marks

Total: 150 marks