HILTON COLLEGE

TRIAL EXAMINATION
AUGUST 2012

MATHEMATICS PAPER 3

Time: 2 hours   100 marks

GENERAL INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1.   This question paper consists of 13 pages and a separate formula sheet. Please check that your paper is complete.

2.   Read the questions carefully.

3.   This question paper consists of 11 questions. Answer all questions.

4.   Number your answers exactly as the questions are numbered.

5.   You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.

6.   Round off your answers to one decimal digit where necessary, unless otherwise stated.

7.   All necessary working details must be shown.

8.   It is in your own interest to write legibly and to present your work neatly.

9.   Please note that the diagrams are **NOT** necessarily drawn to scale.

Please do not turn over this page until you are asked to do so.

Laundry number: MEMORANDUM

QUESTION 1

Consider the sequence   3 ; 8 ; 13 ; 18 ; 23 ; …………

(a)   If the sequence continues to behave in the same manner then write
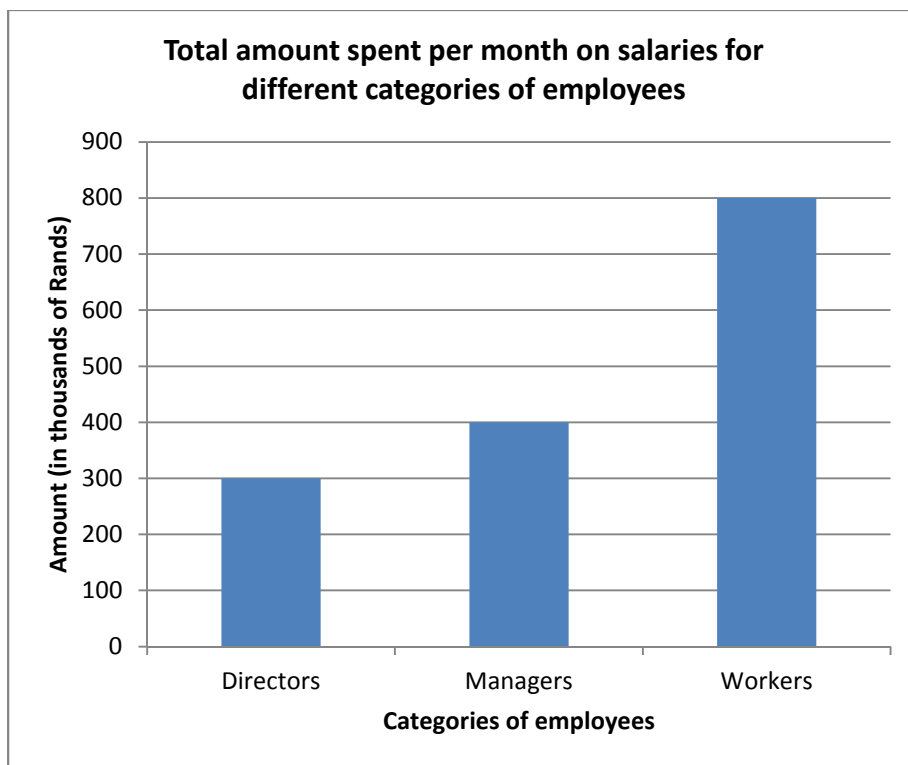down the next two terms.   (2)

28 ; 33

(b)   Write down a recursive formula that will determine the nth term of the sequence.   (3)

$T_{n+1} = T_n + 5$ *and* $T_1 = 3$

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QUESTION 2

In a company there are 2 directors, 16 managers and 200 workers. At salary
negotiations the directors presented the following bar chart to show the total
amount spent on salaries for the different categories of employees.

(a)   The directors stated that the total salaries earned by workers are twice
        as much earned by Managers. Do you agree with this statement?
        Motivate your answer.   (3)

        This is correct since the workers earned R800 000 while the Managers earned R400 000

(b)   The directors suggest that the percentage increase in the salaries for
        workers should be the lowest since the workers wage bill is the largest.
        Do you think this is fair? Explain.   (3)

        No, this would not be fair. The individual salaries for workers are much lower than for
        managers. It is just that there are far more workers that makes the total salary bill higher

(c)   Calculate the mean monthly salary for the workers.   (2)

        $\dfrac{800\,000}{200}$

        mean   worker's   salary   is   R4000

(d)   When discussing the salary increases with the workers the directors use the following
        graph to highlight the big difference between the amounts spent on workers compared
        with the other employees:
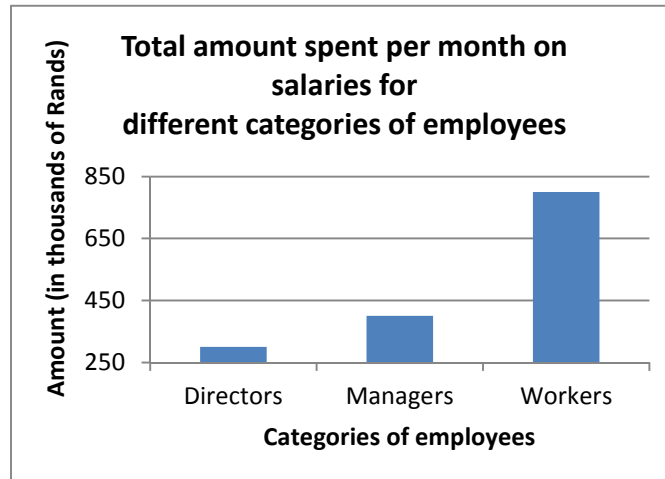

        Explain why this graph could be misleading.   (2)

        The vertical axis does not start at zero, thereby making the gap between the totals look
        even larger than it is.

        | 10 |

QUESTION 3

The duration of telephone calls made by a receptionist was monitored for a week.
The data obtained is represented by the normal distribution curve below.
The mean time was 180 seconds with a standard deviation of 30 seconds.
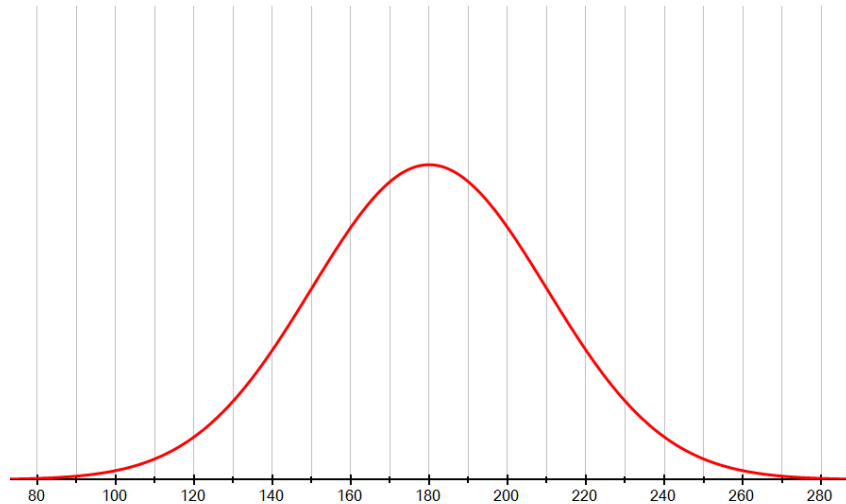
It is also known that:

> Approximately 68% of the duration of the telephone calls recorded is within one standard deviation of the mean.
>
> Approximately 96% of the duration of the telephone calls recorded is within two standard deviations of the mean.
>
> Approximately 100% of the duration of the telephone calls recorded is within three standard deviations of the mean.

(a)   What percentage of calls made was between 150 seconds and 240 seconds in duration.   (3)

34% of the calls will fall one standard deviation below the mean and 48% will fall two standard deviations above the mean. So, a total of 82% of the calls will lie between 150 and 240 seconds in duration.

(b)   What % of the calls made were in excess of 150 seconds?   (3)

34% of the calls will fall two standard deviations below the mean, so 84% of the calls were in excess of 150 seconds.

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QUESTION 4

During the month of July a number of patients visited a local hospital suffering from bronchitis. The table shows the cumulative number of patients treated as per the dates given.

Dates in the month of July	3	5	8	12	15	19	22	26
Number of patients treated	270	275	376	420	602	684	800	820

(a)   Draw a scatter plot of the above data on the axes provided below.   (3)
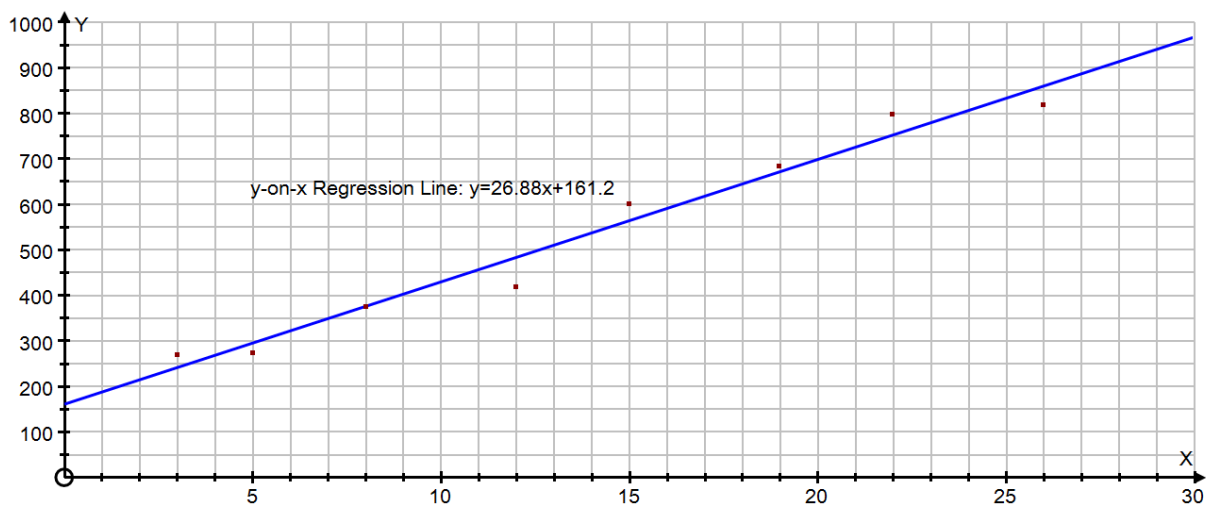
(b)   Determine the equation of the least squares regression line of
         patients treated (y) on date (x)   (4)

$$y = 26.88x + 161.2$$

(c)   Draw the least squares regression line on the above grid.   (2)

         See above

(e)   Estimate how many patients had been treated by 24 July.   (2)

$$y = 26.88(24) + 161.2$$
$$= 806$$

(f)   Determine the correlation coefficient of the data. Interpret this result.   (3)

$$r = 0.985$$

There is a very strong positive, linear correlation. As one increases so does the other.

(g)   The average grass length in a field is correlated against the number of cows. The
        correlation coefficient comes out to -0.7

        (1)   Explain what this means   (2)

                There is a reasonably strong indirect correlation. As the number of cows
                increases the average grass length decreases.

        (2)   Draw a possible scatter plot for the data on the axes below. Your scatter plot
                should have a minimum of 10 dots.   (2)

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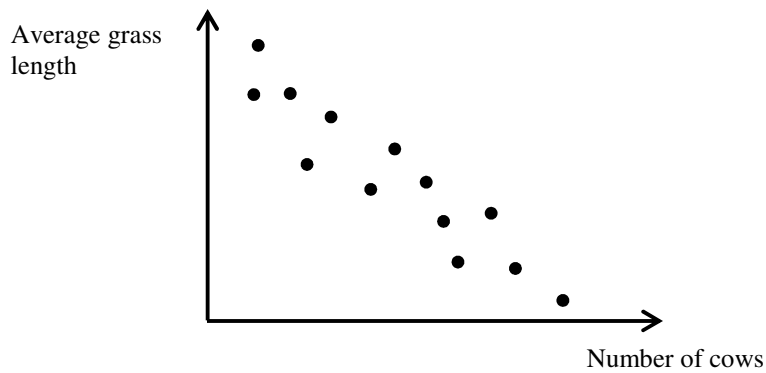
QUESTION 5

All answers containing factorials must be calculated e.g. 5! = 120

(a)   Three cards are selected at random (without replacement) from a standard
pack of 52 cards (jokers are excluded).
Find the probability (as a percentage to 1 decimal place) that the cards
are all the same colour.   (5)

$$P(all \quad 3 \quad same \quad colour) = P(all \quad red) + P(all \quad black)$$
$$= \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} + \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50}$$
$$= 23.5\%$$

(b)   If $P(A) = \frac{3}{8}$ *and* $P(B) = \frac{1}{4}$ then find

(1) $P(A \cup B)$ if A and B are mutually exclusive events.   (2)

$$P(A \cup B) = P(A) + P(B)$$
$$= \frac{3}{8} + \frac{1}{4}$$
$$= \frac{5}{8}$$

(2) $P(A \cup B)$ if A and B are independent events.   (5)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{3}{8} + \frac{1}{4} - \frac{3}{8} \times \frac{1}{4}$$
$$= \frac{17}{32} \approx 53.1\%$$

(c)   Three married couples: Mr and Mrs Smith; Mr and Mrs Botha;
Mr and Mrs Ndlovu are to be seated on a bench.

(1)   How many different arrangements are possible if there are no
restrictions as to where people sit?   (2)

$$6! = 720 \quad possible \quad arrangements$$

(2) Find the probability that Mr and Mrs Ndlovu land up
     sitting next to each other. (4)

Number of ways they sit next to one another is
$5 \times 2! = 48$

So, the probability that they sit next to

one another is $\dfrac{240}{720} = 33.3\%$

(d) Using the letters in the word MILLENIUM determine:

(1) The number of 9 letter words that can be formed. (2)

$\dfrac{9!}{2!2!2!} = 45360$

(2) The probability that the new word will start and end on the letter 'L' (2)

number starting and ending with $L = \dfrac{7!}{2!2!} = 1260$

so, $P(start \ and \ end \ with \ L) = \dfrac{9!}{2!2!2!} = \dfrac{1260}{45360} = 2.8\%$
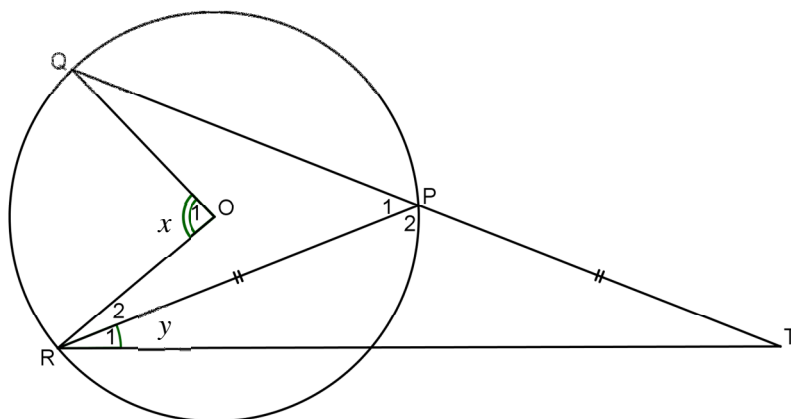
$\boxed{22}$

QUESTION 6

(a) Complete : "The angle subtended by a chord or arc at the centre of the circle
     is…. (1)

     equal to twice the size of the angle it subtends at the circumference

(b) In the figure below O is the centre of the circle and PT = PR.
     Let $\hat{R}_1 = y$ *and* $\hat{O}_1 = x$

NO REASONS REQUIRED

(a)   Express $\hat{P_1}$ in terms of x (1)

$\hat{P_1} = 2y$

(b)   Express x in terms y (2)

$x = 2\hat{P_1}$
$\therefore x = 4y$

(c)   If TQ = TR and $x = 120°$ calculate the measures of

   (1) y (2)
   $$y = \frac{120°}{4} = 30°$$

   (2) $\hat{R_2}$ (3)
   $\hat{T} = 30°$
   $$\therefore T\hat{R}Q = \frac{150°}{2} = 75°$$
   $but \quad ORQ = 30°$
   $\therefore R_2 = 75° - 30° - 30° = 15°$

$$\boxed{9}$$

QUESTION 7

In the figure below TP and TS are tangents to the circle. R is a point on the circumference. Q is a point on PR such that $\hat{Q}_1 = \hat{P}_1$. SQ is drawn. Let $\hat{P}_1 = x$
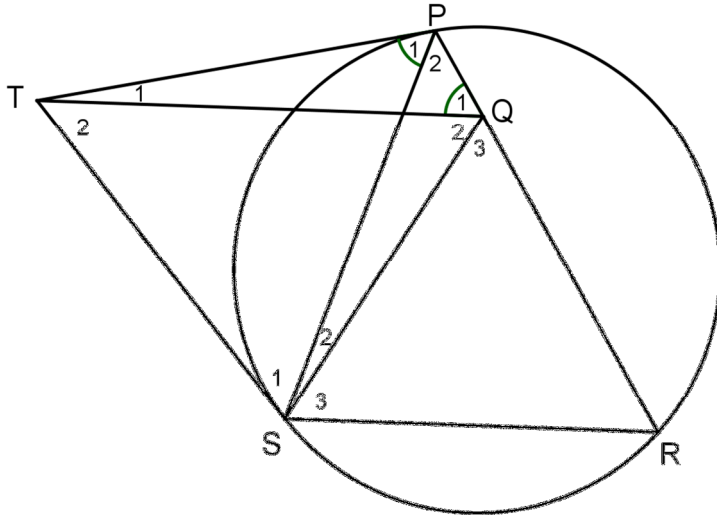
(a)   Prove TQ ∥ SR   (4)

$\hat{P}_1 = \hat{R}$ $(tan\text{-}chord)$

$but \quad \hat{P}_1 = \hat{Q}_1$ $(given)$

$\therefore \hat{Q}_1 = \hat{R}$

$\therefore TQ \parallel SR$ $(corresponding \ \angle's \ equal)$

(b)   Prove QPTS is a cyclic quadrilateral.   (4)

$\hat{S}_1 = x$ $(tangents \ from \ same \ point)$

$\therefore \hat{S}_1 = \hat{Q}_1$

$\therefore QPTS \ is \ cyclic \ (converse \ \angle's \ in \ same \ segment)$

(c)   Prove TQ bisects $S\hat{Q}P$ (3)

$\hat{Q}_2 = \hat{P}_1 = x$ $(\angle's \ in \ same \ segment)$

$but \quad \hat{Q}_1 = x \quad proved$

$\therefore \hat{Q}_1 = \hat{Q}_2$

$\therefore TQ \ bisects \ S\hat{Q}P$

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QUESTION 8

In the figure $AQ \parallel RT$. $\dfrac{BQ}{QC} = \dfrac{3}{5}$ *and* $\dfrac{BR}{RA} = \dfrac{1}{2}$.
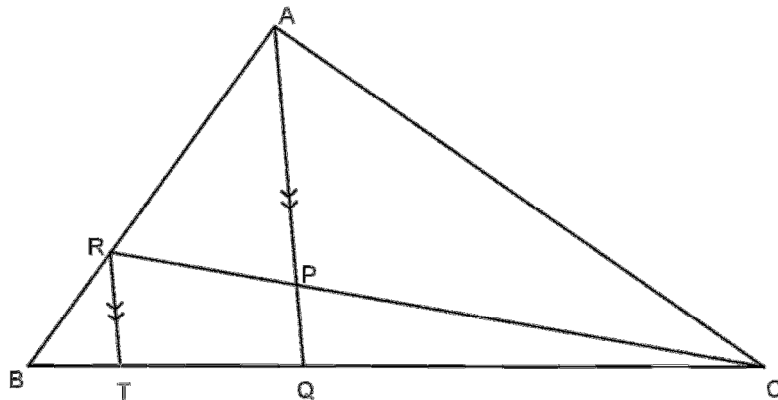
(a)   If $BT = k$ calculate TQ in terms of k.   (3)

$$\dfrac{BT}{TQ} = \dfrac{BR}{RA} = \dfrac{1}{2} \quad (prop. \ \ int. \ \ theorem)$$

$$\therefore if \quad BT = k \quad then \quad TQ = 2k$$

(b)   Hence, or otherwise, calculate the numerical value of

(1) $\dfrac{CP}{PR}$ (3)

$$\dfrac{CP}{PR} = \dfrac{5}{2}$$

(2) $\dfrac{area \quad \Delta CPQ}{area \quad \Delta CRT}$, giving reasons.   (4)

$$R\hat{C}T \quad is \quad common$$

$$C\hat{Q}P = C\hat{T}R \quad (corresp. \ \ \angle's \ \ on \ \ \parallel \ \ lines)$$

$$C\hat{P}Q = C\hat{R}T \quad (corresp. \ \ \angle's \ \ on \ \ \parallel \ \ lines)$$

$$\therefore \Delta CPQ = \Delta CRT \quad (AAA)$$

$$\therefore \dfrac{\Delta CPQ}{\Delta CRT} = \left(\dfrac{CP}{CR}\right)^2 = \left(\dfrac{5}{7}\right)^2 = \dfrac{25}{49} \quad (similar \ \ figures)$$

QUESTION 9

AB is the diameter of semi-circle ADCB. Chords AC and BD intersect at E.
EP is perpendicular to AB

$\boxed{10}$
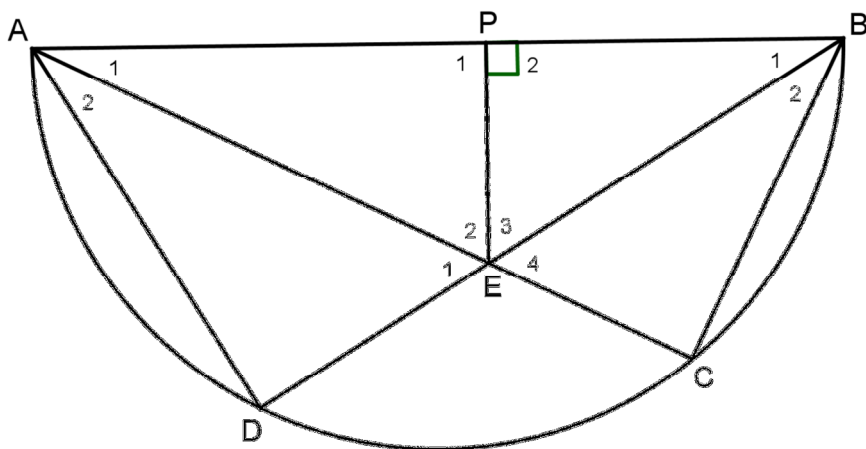
(a)   Prove that $\Delta BPE \;|||\; \Delta BDA$ (3)

$\hat{D} = \hat{P_2} = 90° \quad (\angle \quad in \quad semi - \odot \quad and \quad given)$

$\hat{B_1} \quad is \quad common$

$\therefore B\hat{A}D = \hat{P_3} \quad (3^{rd} \quad \angle \quad of \quad \Delta)$

$\therefore \Delta BPE \;|||\; \Delta BDA \quad (AAA)$

(b)   Hence show that $\dfrac{BP}{BD} = \dfrac{PE}{AD}$ (2)

$\dfrac{BP}{BD} = \dfrac{PE}{AD} \quad (||| \quad \Delta's)$

(c)   Prove that $AB^2 = BD^2 + \dfrac{BD^2 \times PE^2}{BP^2}$ (4)

$AB^2 = BD^2 + AD^2 \quad (Pythagoras)$

$but \quad AD = \dfrac{BD \times PE}{BP} \quad (from \quad (b))$

$\therefore AD^2 = \dfrac{BD^2 \times PE^2}{BP^2}$

$\therefore AB^2 = BD^2 + \dfrac{BD^2 \times PE^2}{BP^2}$

$\boxed{9}$