education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2009

MEMORANDUM

MARKS: 150

This memorandum consists of 17 pages.

QUESTION 1

1.1.1	$x(x-4)=5$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$	✓ standard form ✓ factors ✓ both answers (3)
1.1.2	$4x^2-20x+1=0$ $x=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(4)(1)}}{2(4)}$ $x=\dfrac{20\pm\sqrt{384}}{8}$ $x=4{,}95$ or $x=0{,}05$	✓ substitution into formula ✓ 384 ✓ ✓ answers (4)
1.1.3	$y=x-3$ $x^2-x=6+(x-3)$ $x^2-2x-2=0$ $(x-3)(x+1)=0$ $x=3$ or $x=-1$ $y=0$ or $y=-4$ Solutions are $(x\,;y)=(3\,;0)$ or $(-1\,;-4)$ **OR** $x=y+3$ $(y+3)^2-(y+3)=6+y$ $y^2+6y+9-y-3=6+y$ $y^2+4y=0$ $y(y+4)=0$ $y=0$ or $y=-4$ $x=3$ or $x=-1$ Solutions are $(x\,;y)=(3\,;0)$ or $(-1\,;-4)$ **OR** $y=x-3$ $y=x^2-x-6$ $y=(x-3)(x+2)$ $y=y(x+2)$ $x+2=1$ or $y=0$ $x=-1$ or $x=3$ $y=-4$ Solutions are $(x\,;y)=(3\,;0)$ or $(-1\,;-4)$	✓ $y=x-3$ ✓ substitution ✓ factors ✓✓ answers (6) **OR** ✓ $x=y+3$ ✓ substitution ✓ simplify ✓factors ✓ ✓answers (6) **OR** ✓ $y=x-3$ ✓ factorise ✓ substitution ✓equal ✓ ✓answers (6)

1.2	$\sqrt{m}+\sqrt{n}=\sqrt{7+\sqrt{48}}$ $m+2\sqrt{mn}+n=7+\sqrt{48}$ $m+2\sqrt{mn}+n=7+2\sqrt{12}$ $m+n=7$ $mn=12$ $(m+n)^2=7^2$ $m^2+2mn+n^2=49$ $m^2+n^2=49-2mn$ $=49-2(12)$ $=25$	✓ squaring both sides ✓ equating numbers and surds ✓✓ squaring both sides in $m + n = 7$ ✓ answer (5)
	OR	**OR**
	$\sqrt{m}+\sqrt{n}=\sqrt{7+\sqrt{48}}$ $m+2\sqrt{mn}+n=7+\sqrt{48}$ $m+2\sqrt{mn}+n=7+2\sqrt{12}$ $m+n=7$ and $mn=12$ are possible solutions $m=7-n$; $n(7-n)=12$; $n^2-7n+12=0$; $(n-4)(n-3)=0$; $n=4$ or $n=3$; $m=3$ or $m=4$ or $n=7-m$; $m(7-m)=12$; $m^2-7m+12=0$; $(m-4)(m-3)=0$; $m=4$ or $m=3$; $n=3$ or $n=4$ $\therefore m^2+n^2=3^2+4^2$ $=25$	✓ squaring both sides of $m + n = 7$ ✓ equating numbers and surds ✓✓ solving simultaneously for m and n ✓ answer (5)
	OR	**OR**
	$\sqrt{m}+\sqrt{n}=\sqrt{7+\sqrt{48}}$ $m+2\sqrt{mn}+n=7+\sqrt{48}$ $m+2\sqrt{mn}+n=7+2\sqrt{12}$ By inspection $(m ; n) = (4 ; 3)$ or $(3 ; 4)$ $\therefore m^2+n^2=3^2+4^2$ $=25$	✓✓ squaring both sides of $m + n = 7$ ✓✓ answers for m and n ✓ answer (5) **[18]**

QUESTION 2

2.1	20 ; 24 ; 28 ; 32 ; … 4 4 4 $T_n = 20 + (n - 1)\,4$ $100 = 20 + 4n - 4$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. **OR** $T_n = 4n + 16$ $100 = 4n + 16$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. **OR** $100 = 20 + 80$ $= 20 + 4(21 - 1)$ $\therefore n = 21$	✓ sequence ✓ T_0 ✓ 21 days (3) **OR** (Answer only – full marks)
2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{14} = \frac{14}{2}[2(20) + (14-1)4]$ $= 644$ km	✓ formula ✓ substitution ✓ answer (3)
2.3	No. It will not be humanly possible to just keep on increasing the distance covered indefinitely. For example: $T_{1\,000} = 4\,016$ km in one day.	✓ no ✓ reason (2) **[8]**

QUESTION 3

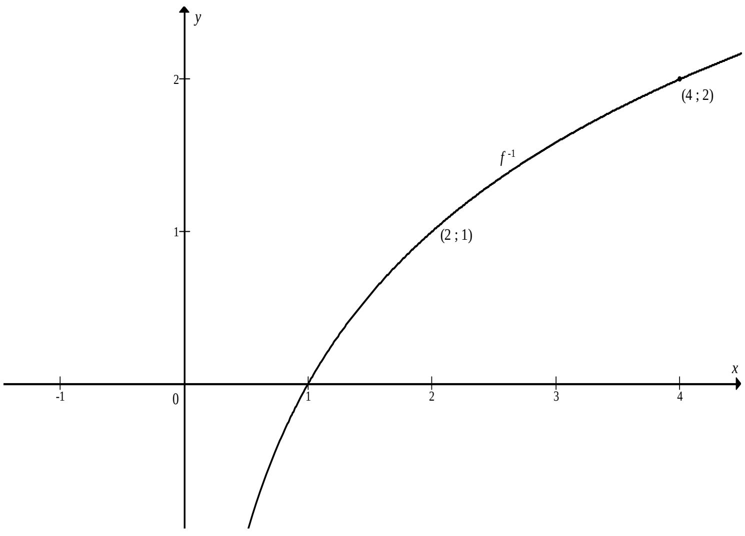
3.1	45	✓ answer (1)
3.2	$T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $3a + b = 7$ $3 + b = 7$ $b = 4$ $a + b + c = 5$ $1 + 4 + c = 5$ $c = 0$ $T_n = n^2 + 4n$	✓ value of a ✓ substitution ✓ value of b ✓ substitution ✓ value of c (5)
	OR	**OR**
	$T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $T_0 = 0 = c$ $T_n = n^2 + bn + 0$ $5 = (1)^2 + (1)b$ $b = 4$ $T_n = n^2 + 4n$	✓ value of a ✓✓ value of c ✓ substitution ✓ value of b (5)
	OR	**OR**
	If $T_n = an^2 + bn + c$ $5 = T_1 = a + b + c$ $12 = T_2 = 4a + 2b + c$ $21 = T_3 = 9a + 3b + c$ $\Rightarrow 3a + b = 7$ $\Rightarrow 5a + b = 9$ $\Rightarrow$ $a = 1$, $b = 4$, $c = 0$	✓ setting up equations ✓ simultaneous equations ✓✓✓answers (5) **[6]**

QUESTION 4

4.1	$S = a + ar + ar^2 + ... + ar^{n-1}$ $rS = ar + ar^2 + ... + ar^{n-1} + ar^n$ $S - rS = a - ar^n$ $S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$	✓ setting up of S ✓ *r*S ✓ subtraction ✓✓ common factors ✓ division (6)
4.2.1	$15 ; 5 ; \frac{5}{3} ; \ldots$ $r = \frac{5}{15} = \frac{1}{3}$ The series converges because – 1< r < 1	✓ value of *r* ✓ explanation (2)
4.2.2	$S_\infty = \frac{15}{1-\frac{1}{3}}$ $= \frac{45}{2}$	✓ *a* = 15 ✓ substitution into correct formula ✓answer (3)
4.3.1	$S_{24} = 2^{24+2} - 4$ $= 67108860$	✓ answer (1)
4.3.2	$S_{24} = 2^{24+2} - 4 = 67108860$ $S_{23} = 2^{23+2} - 4 = 33554428$ $T_{24} = 33554432$ **OR** $T_{24} = S_{24} - S_{23}$ $= 2^{26} - 2^{25}$ $= 2 \times 2^{25} - 2^{25}$ $= 2^{25}$	✓ both sums ✓ answer (2) **OR** ✓ both sums ✓ answer (2)

4.3.3	$a = S_1 = 2^{1+2} - 4 = 4$ $T_2 = S_2 - S_1 = 2^{2+2} - 4 - 4 = 8$ $r = \frac{8}{4} = 2$ $\therefore T_n = 4(2)^{n-1}$	✓ $a = 4$ ✓ second term ✓ answer (3)
	OR	**OR**
	$T_1 = S_1 = 2^{1+2} - 4 = 4 = 2^2$ $T_2 = 8 = 2^3$ $T_3 = 16 = 2^4$ $T_n = 2^{n+1}$	✓ $a = 4$ ✓ develop pattern ✓ answer (3)
	OR	**OR**
	$T_n = S_{n+1} - S_n$ $= 2^{n+2} - 2^{n+1}$ $= 2 \times 2^{n+1} - 2^{n+1}$ $= 2^{n+1}$	✓ formula ✓ substitution ✓ simplification (3) **[17]**
	NOTE: If $T_n = 2^{n+1}$ then $a = T_1 = 4$ $S_n = \frac{4(2^n - 1)}{2 - 1}$ $= 4(2^n - 1)$ maximum of $\frac{2}{3}$ marks $= 2^{n+2} - 1$	

QUESTION 5

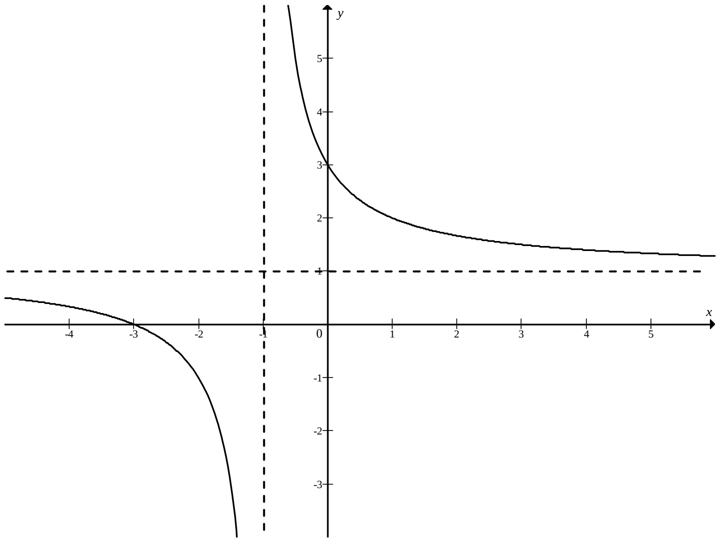
5.1	$1 = -(0-1)^2 + b$ $1 = -1 + b$ $b = 2$	✓ substitution (0 ; 1) ✓ simplification (2)
5.2	$g(x) = -(x-1)^2 + 2$ Turning point of *g*: (1 ; 2) $f(1) = 2^1 = 2$ (1 ; 2) lies on *f*. (1 ; 2) lies on both *f* and *g* D(1 ; 2)	✓ (1 ; 2) TP ✓ substitution into *f* ✓ (1 ; 2) lies on both *f* and *g*. (2)
5.3	$y = \log_2 x$	✓ answer (1)
5.4		✓ *y*-intercept ✓ one other point ✓ shape – increasing (3)
5.5	$h(x) = -(x-1+1)^2 + 2 - 2$ $= -x^2$ **OR** Shift one unit to the left and shift two units down to give $y = -x^2$	✓translation ✓ answer (2)
5.6	$x \leq 0$ **OR** $x \geq 0$	✓ answer (1)
5.7	Max. value of $2^{2-(x-1)^2}$ occurs at max. value of $2-(x-1)^2$ $= 2^2$ $= 4$	✓ at (1 ; 2) ✓ answer (2) **[13]**

QUESTION 6

6.1	$p = 2$ $q = 1$	✓ answer ✓ answer (2)
6.2	$y \in [0\ ;\ 2]$ **OR** $0 \leq y \leq 2$	✓ answer (1)
6.3.1	$\cos x(2\sin x - 1) = 1$ $2\sin x\cos x - \cos x = 1$ $\sin 2x = \cos x + 1$ This would be solved by finding the *x*-values of the points of intersection of the graphs of *f* and *g*.	✓ manipulation ✓ answer (2)
6.3.2	180° **OR** – 180° **OR** about – 112,5°	✓ answer (1) **OR** ✓ answer (1) **OR** ✓ answer (1) **[6]**

QUESTION 7

7.1	$f(0) = \frac{0+3}{0+1}$ $f(0) = 3$ *y*-intercept (0 ; 3) *x*-intercepts $0 = \frac{x+3}{x+1}$ …… $(x \neq -1)$ $x = -3$ *x*-intercept (– 3 ; 0)	✓ substitution $x = 0$ ✓ answer ✓ substitution $y = 0$ ✓ answer (4)

7.2	$\frac{2}{x+1}+1$ $=\frac{2+x+1}{x+1}$ $=\frac{x+3}{x+1}$ **OR** $\frac{x+3}{x+1}$ $=\frac{(x+1)+2}{x+1}$ $=\frac{x+1}{x+1}+\frac{2}{x+1}$ $=\frac{2}{x+1}+1$	✓ LCD ✓ simplification (2) **OR** ✓ split the fraction ✓ simplification (2)
7.3	Vertical asymptote: $x = -1$ Horizontal asymptote: $y = 1$	✓ answer ✓ answer (2)
7.4		✓✓ asymptotes ✓ shape ✓ intercepts (4) **NOTE:** If the graph does not represent a function, candidates do not get the mark for shape.
7.5	$\frac{2}{x+1} \geq -1$ $\frac{2}{x+1}+1 \geq 0$ $x \in (-\infty\ ;-3] \cup (-1\ ;\infty)$ **OR** $x \leq -3$ or $x > -1$	✓ manipulation ✓✓ answer (3) **NOTE:** 0 marks for $-1 < x \leq -3$ **[15]**

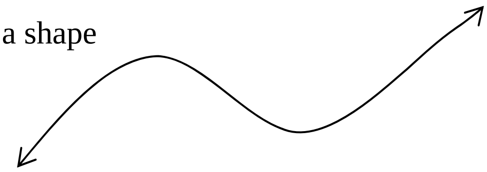
QUESTION 8

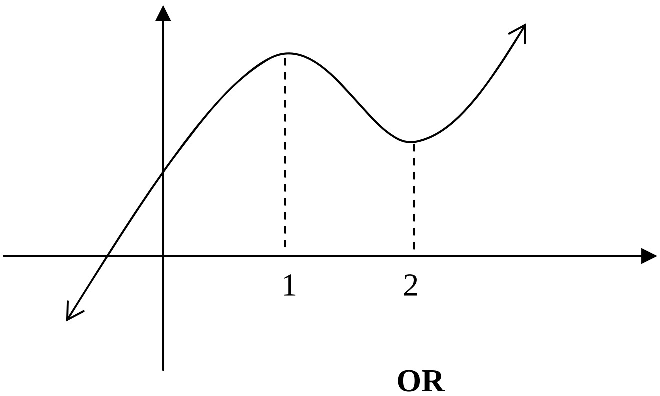
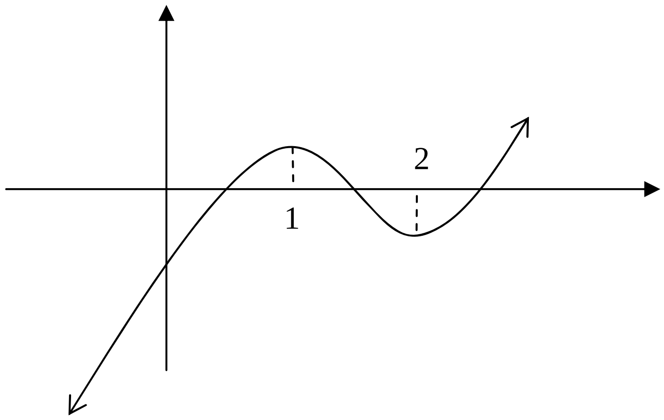
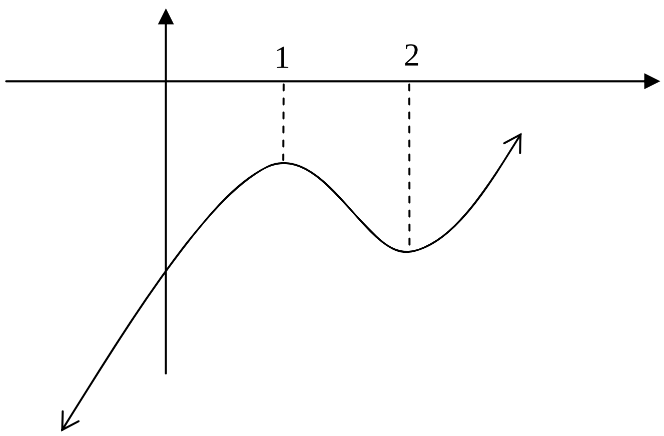
8.1	Depreciation value = $7\ 200(1-0,25)^3$ = R3 037,50	✓ formula ✓ substitution ✓ answer (3)
8.2.1	$300\ 000 = \frac{5\ 000\left[1-(1,015)^{-n}\right]}{0,015}$ $4\ 500 = 5\ 000 - 5\ 000(1,015)^{-n}$ $5\ 000(1,015)^{-n} = 500$ $(1,015)^{-n} = 0,1$ or $(1,015)^{n} = 10$ $-n = \frac{\log 0,1}{\log 1,015}$ $n = 154,65$ Number of payments = 155	✓ formula ✓ substitution ✓ use of logs ✓ use of logs ✓ answer ✓ answer (6)
8.2.2	Balance outstanding $= 300\ 000\left(1+\frac{0.18}{12}\right)^{154} - \frac{5\ 000\left[\left(1+\frac{0.18}{12}\right)^{154}-1\right]}{\frac{0.18}{12}}$ = R3 230,50	✓✓ setting up equation ✓✓ substitution ✓ answer (5)
8.2.3	Amount paid in last month $= 3\ 230,50\left(1+\frac{0.18}{12}\right)$ = R3 278,96	✓ substitution into correct formula ✓ answer (2)
8.2.4	Total repaid = (154 × 5 000) + 3 278,96 = R773 278,96	✓ answer (1) **[17]**

QUESTION 9

9.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-5(x+h)^2+2-(-5x^2+2)}{h}$ $=\lim_{h\to 0}\frac{-5(x^2+2xh+h^2)+2+5x^2-2}{h}$ $=\lim_{h\to 0}\frac{-10xh-5h^2}{h}$ $=\lim_{h\to 0}\frac{h(-10x-5h)}{h}$ $=\lim_{h\to 0}(-10x-5h)$ $=-10x$ **OR** $f(x+h)=-5(x+h)^2+2$ $=-5x^2-10xh-5h^2+2$ $\frac{f(x+h)-f(x)}{h}=\frac{-5x^2-10xh-5h^2+2+5x^2-2}{h}$ $=\frac{-10xh-5h^2}{h}$ $=-10x-5h$ $\therefore\ f'(x)=\lim_{h\to 0}(-10x-5h)=-10x$	✓ method ✓ substitution ✓ simplification ✓ factorising ✓ answer (5) **OR** ✓ substitution ✓ answer ✓ substitution ✓ answer ✓ answer (5)
9.2	$D_x[(x-2)(x+3)]$ $=D_x[x^2+x-8]$ $=2x+1$	✓ simplification ✓✓answer (3)
9.3.1	Depth after 3 days = $12-\frac{1}{4}(3)-\frac{1}{6}(3)^3=\frac{27}{4}$ = 6,75 m	✓ answer (1)
9.3.2	Rate of decrease in depth = $h'(t)=-\frac{1}{4}-\frac{1}{2}t^2$ $=-\frac{1}{4}-\frac{1}{2}(2)^2$ Rate of decrease in depth after 2 days $=-\frac{9}{4}$ = –2,25 metres/day Rate of decrease in depth = 2,25 metres per day	✓ $h'(t)$ ✓derivative ✓ substitution of $t=2$ ✓answer (2,25) ✓ units (metres per day) (5) **[14]**

QUESTION 10

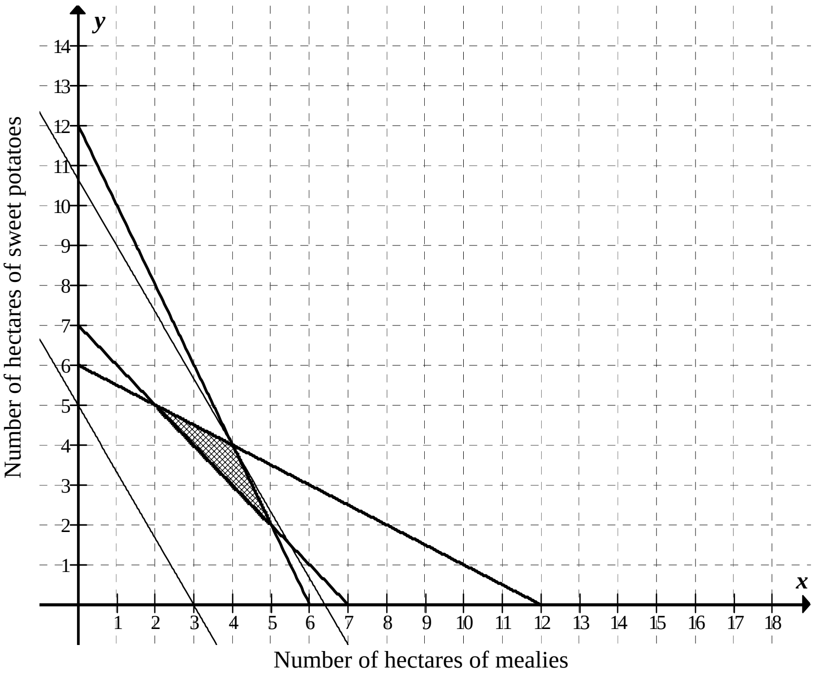
10.1	$x = 1$ and/or $x = 2$	✓✓ answer (2)
10.2	When $x < 1$, $f'(x) > 0$ and so f is increasing When $1 < x < 2$, $f'(x) < 0$ and so f is decreasing When $x > 2$, $f'(x) > 0$ and so f is increasing At x = 1: local maximum At x = 2: local minimum **OR** $f'(x) = ax^2 + bx + c$ is minimum-valued $\therefore a > 0$ $\therefore f$ has a shape  At x = 1: local maximum At x = 2: local minimum **OR** $f'(x)$: + 0 – 0 + x: 1 2 At x = 1: local maximum At x = 2: local minimum	✓ $f'(x) > 0$ ✓ $f'(x) < 0$ ✓answer ✓ answer (4) **OR** ✓ $f'(x)$ minimum-valued ✓ $a > 0$ ✓ answer ✓ answer (4) **OR** ✓✓ number line ✓ answer ✓ answer (4)
10.3	$x = \frac{1+2}{2} = 1{,}5$	✓ answer (1)

10.4	 **OR**  **OR** 	✓ shape ✓*x*-values of turning points correct (2) **[9]**

QUESTION 11

11.1	$r = 2x$ Area rectangle = $8x$ Radius small circle = $\frac{2}{3}r$ $A(x) = \text{area rectangle} - \text{area circles}$ $A(x) = 8x - \left[\pi r^2 + \pi\left(\frac{2}{3}r\right)^2\right]$ $A(x) = 8x - \pi(2x)^2 - \pi\left(\frac{2}{3}(2x)\right)^2$ $A(x) = 8x - 4\pi x^2 - \frac{16}{9}\pi x^2$ $A(x) = 8x - \frac{52\pi}{9}x^2$	✓ $r = 2x$ ✓ area rectangle = $8x$ ✓ radius small circle $= \frac{2}{3}r$ ✓ formula ✓ $\frac{16}{9}\pi x^2$ (5)
11.2	$A'(x) = 8 - \frac{104}{9}\pi x$ $0 = 8 - \frac{104}{9}\pi x$ $x = \frac{72}{104\pi}$ $x = \frac{9}{13\pi}$ $x = 0{,}2203683827...$ $x = 0{,}22$ m	✓ $A'(x) = 8 - \frac{104}{9}\pi x$ ✓ $A'(x) = 0$ ✓ answer (3)
11.3	Area of circles $= \frac{52\pi}{9}x^2$ $= \frac{52\pi}{9}(0{,}22)^2$ $= 0{,}88 \text{ m}^2$ **OR** Area of circles $= \frac{52\pi}{9}x^2$ $= \frac{52\pi}{9}\left(\frac{9}{13\pi}\right)^2$ $= \frac{36}{13\pi} \text{ m}^2$	✓ substitution ✓ answer (2) **[10]**

QUESTION 12

12.1	$x + y \geq 7$ $12\ 000x + 6\ 000y \leq 72\ 000 \quad \therefore 2x + y \leq 12$ $2x + 4y \leq 24 \ \therefore \ x + 2y \leq 12$ $x, y \geq 0$	✓✓ constraint ✓✓ constraint ✓✓ constraint (✓correct expression, ✓correct inequalities) (6)
OR 12.2 and 12.3		✓✓✓line graphs (3) ✓ feasible region (1)
12.4	P = 3 000x + 1 800y	✓✓ answer (2)
12.5	Maximum at (4 ; 4) 4 hectares of mealies and 4 hectares of sweet potatoes	✓ search line ✓ answer (2)
12.6	Profit per hectare sweet potatoes $= \frac{2}{3} \times 18\ 000$ $= 12\ 000$ P = 30 000x + 12 000y $y = -\frac{5}{2}x + \frac{P}{12\ 000}$ Maximises at (5 ; 2) $P = 30\ 000(5) + 12\ 000(2)$ Profit = R174 000 per hectare	✓ 12 000 ✓ gradient of $-\frac{5}{2}$ or P = 30 000x + 12 000y ✓ answer (3) **[17]**

TOTAL: 150

QUESTION 12.2 AND 12.3