education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2009

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, 1 information sheet and 2 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. TWO diagram sheets for answering QUESTION 5.4, QUESTION 7.4, QUESTION 12.2 and QUESTION 12.3 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-4)=5$ (3)

1.1.2 $4x^2-20x+1=0$ (round off your answer correctly to 2 decimal places) (4)

1.1.3 Solve simultaneously for x and y:

$$y-x+3=0$$
$$x^2-x=6+y$$ (6)

1.2 If m and n are rational numbers such that $\sqrt{m}+\sqrt{n}=\sqrt{7+\sqrt{48}}$, calculate a possible value of m^2+n^2. (5)
[18]

QUESTION 2

A cyclist, preparing for an ultra cycling race, cycled 20 km on the first day of training. She increases her distance by 4 km every day.

2.1 On which day does she cycle 100 km? (3)

2.2 Determine the total distance she would have cycled from day 1 to day 14. (3)

2.3 Would she be able to keep up this daily rate of increase in distance covered indefinitely? Substantiate your answer. (2)
[8]

QUESTION 3

Consider the following sequence: 5; 12; 21; 32; ...

3.1 Write down the next term of the sequence. (1)

3.2 Determine a formula for the n^{th} term of this sequence. (5)
[6]

QUESTION 4

4.1 Prove that $a + ar + ar^2 + ...$ (to n terms) $= \dfrac{a(1-r^n)}{1-r}$ for $r \neq 1$ (6)

4.2 Given the geometric series $15 + 5 + \dfrac{5}{3} + ...$

4.2.1 Explain why the series converges. (2)

4.2.2 Evaluate $\sum_{n=1}^{\infty} 5(3^{2-n})$ (3)

4.3 The sum of the first n terms of a sequence is given by $S_n = 2^{n+2} - 4$

4.3.1 Determine the sum of the first 24 terms. (1)

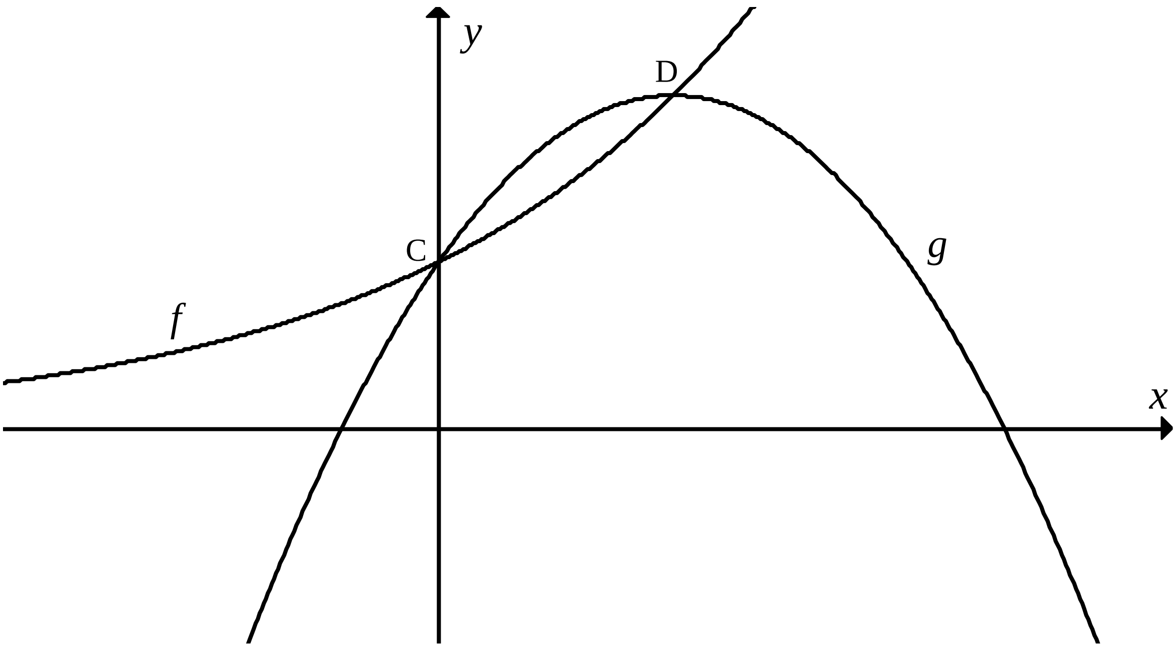
4.3.2 Determine the 24th term. (2)

4.3.3 Prove that the n^{th} term of the sequence is 2^{n+1}. (3)

[17]

QUESTION 5

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + b$, where b is a constant. The graphs of f and g intersect the y-axis at C. D is the turning point of g.

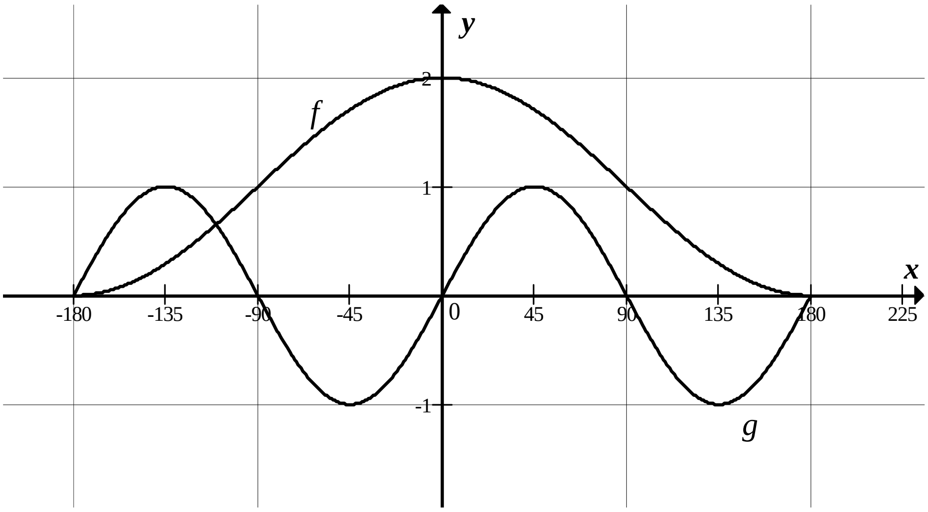


5.1	Show that $b = 2$.	(2)
5.2	Write down the coordinates of the turning point of g.	(2)
5.3	Write down the equation of $f^{-1}(x)$ in the form $y = ...$	(1)
5.4	Sketch the graph of f^{-1} on the system of axes on DIAGRAM SHEET 1 (attached). Indicate the x-intercept and the coordinates of one other point on your graph.	(3)
5.5	Write down the equation of h if $h(x) = g(x+1) - 2$.	(2)
5.6	How can the domain of h be restricted so that h^{-1} will be a function?	(1)
5.7	Determine the maximum value of $2^{2-(x-1)^2}$.	(2)

[13]

QUESTION 6

The sketch below shows the graphs of $f(x) = q + \cos x$ and $g(x) = \sin px$ for $x \in [-180° ; 180°]$.



6.1 Write down the values of *p* and *q*. (2)

6.2 Write down the range of *f*. (1)

6.3 Use the graphs to answer the following:

6.3.1 Explain how you would solve the equation $(2\sin x - 1)\cos x = 1$. (2)

6.3.2 Give ONE solution to the equation in QUESTION 6.3.1. (1)
[6]

QUESTION 7

Given: $f(x) = \dfrac{x+3}{x+1}$

7.1 Calculate the *x*- and *y*-intercepts of *f*. (4)

7.2 Show that $f(x) = \dfrac{2}{x+1} + 1$ (2)

7.3 Write down the equations of the vertical and horizontal asymptotes of *f*. (2)

7.4 Draw a sketch graph of $f(x)$ showing clearly the intercepts and asymptotes on the axes provided on DIAGRAM SHEET 1 (attached). (4)

7.5 Use your graph to solve: $\dfrac{2}{x+1} \geq -1$ (3)
[15]

QUESTION 8

8.1 A new cellphone was purchased for R7 200. Determine the depreciation value after 3 years if the cellphone depreciates at 25% p.a. on the reducing-balance method. (3)

8.2 Jill negotiates a loan of R300 000 with a bank which has to be repaid by means of monthly payments of R5 000 and a final payment which is less than R5 000. The repayments start one month after the granting of the loan. Interest is fixed at 18% per annum, compounded monthly.

8.2.1 Determine the number of payments required to settle the loan. (6)

8.2.2 Calculate the balance outstanding after Jill has paid the last R5 000. (5)

8.2.3 Calculate the value of the final payment made by Jill to settle the loan. (2)

8.2.4 Calculate the total amount that Jill repaid to the bank. (1)
[17]

QUESTION 9

9.1 Differentiate f from first principles if it is given that $f(x) = -5x^2 + 2$ (5)

9.2 Use the rules of differentiation to determine the following:

$D_x\left[(x-2)(x+3)\right]$ (3)

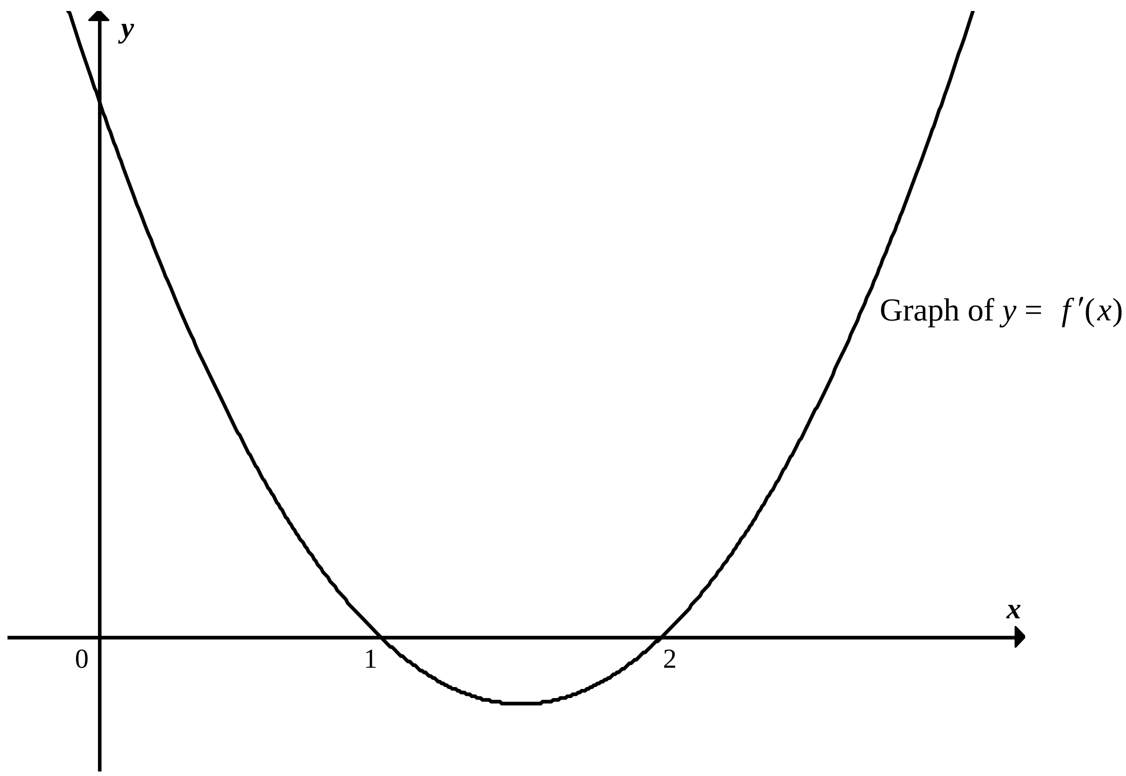
9.3 The depth h of petrol in a large tank, t days after the tank was refilled, is given by $h(t) = 12 - \dfrac{t}{4} - \dfrac{t^3}{6}$ metres for $0 \le t \le 4$.

9.3.1 What is the depth after 3 days? (1)

9.3.2 What is the rate of decrease in the depth after 2 days? (Give your answer in the correct units.) (5)
[14]

QUESTION 10

In the sketch below, the graph $y = ax^2 + bx + c$ represents the derivative, f', of f where f is a cubic function.



10.1 Write down the x-coordinates of the stationary points of f. (2)

10.2 State whether each stationary point in QUESTION 10.1 is a local minimum or a local maximum. Substantiate your answer. (4)

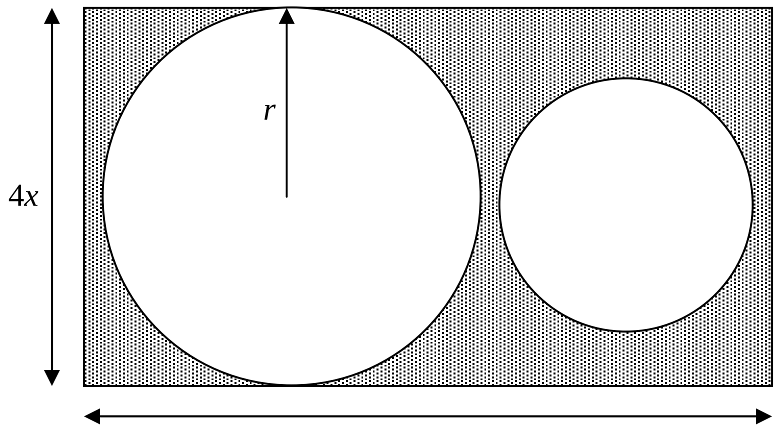
10.3 Determine the x-coordinate of the point of inflection of f. (1)

10.4 Hence, or otherwise, draw a sketch graph of f. (2)
[9]

QUESTION 11

Devan wants to cut two circles out of a rectangular piece of cardboard of 2 metres long and $4x$ metres wide. The radius of the larger circle is half the width of the cardboard and the smaller circle has a radius that is $\frac{2}{3}$ the radius of the bigger circle.

$A = lb$	$A = \pi r^2$	$P = 2(l + b)$	$C = 2\pi r$



11.1 Show that the area of the shaded region is $A(x) = 8x - \frac{52\pi}{9}x^2$. (5)

11.2 Determine the value of x, such that the area of the shaded region is a maximum. (3)

11.3 Calculate the total area of the circles, if the area of the shaded region is to be a maximum. (2)

[10]

QUESTION 12

Mr Lolwana has land available to sow mealies and plant sweet potatoes.

- He must use at least 7 hectares for the sowing of mealies and the planting of sweet potatoes altogether.
- He has at most R72 000 to spend and each hectare of mealies costs R12 000 to sow, whilst each hectare of sweet potatoes costs R6 000 to plant.
- He has at most 24 hours to sow and plant. It takes 2 hours to sow a hectare of mealies and 4 hours to plant a hectare of sweet potatoes.

Let x be the number of hectares of mealies sown.
Let y be the number of hectares of sweet potatoes planted.

12.1 Write down the constraints in terms of x and y to represent the above information. (You may assume that $x \geq 0$, $y \geq 0$.) (6)

12.2 Use the attached graph paper on DIAGRAM SHEET 2 to represent the constraints graphically. (3)

12.3 Clearly indicate the feasible region by shading it. (1)

12.4 If the profit is R30 000 per hectare of mealies and R18 000 per hectare of sweet potatoes, write down the function which gives the profit in terms of x and y. (2)

12.5 Determine the number of hectares of sweet potatoes that Mr Lolwana needs to plant and the number of hectares of mealies that he needs to sow in order to maximise his profit. (2)

12.6 Due to the global economic market, the profit per hectare of sweet potatoes has decreased to $\frac{2}{3}$ of the original profit. The profit per hectare of mealies has remained unchanged. Calculate the maximum profit that Mr Lolwana can now earn. (3)
[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

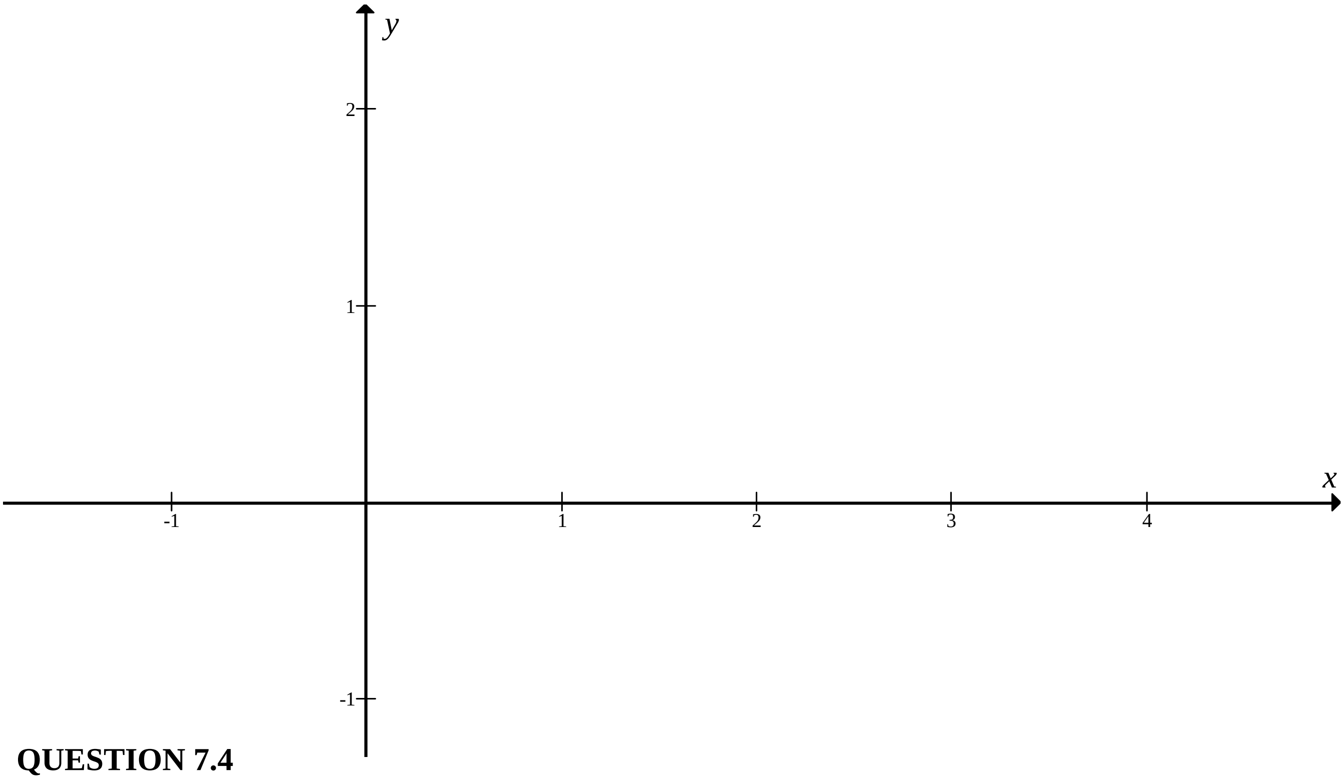
$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

CENTRE NUMBER:

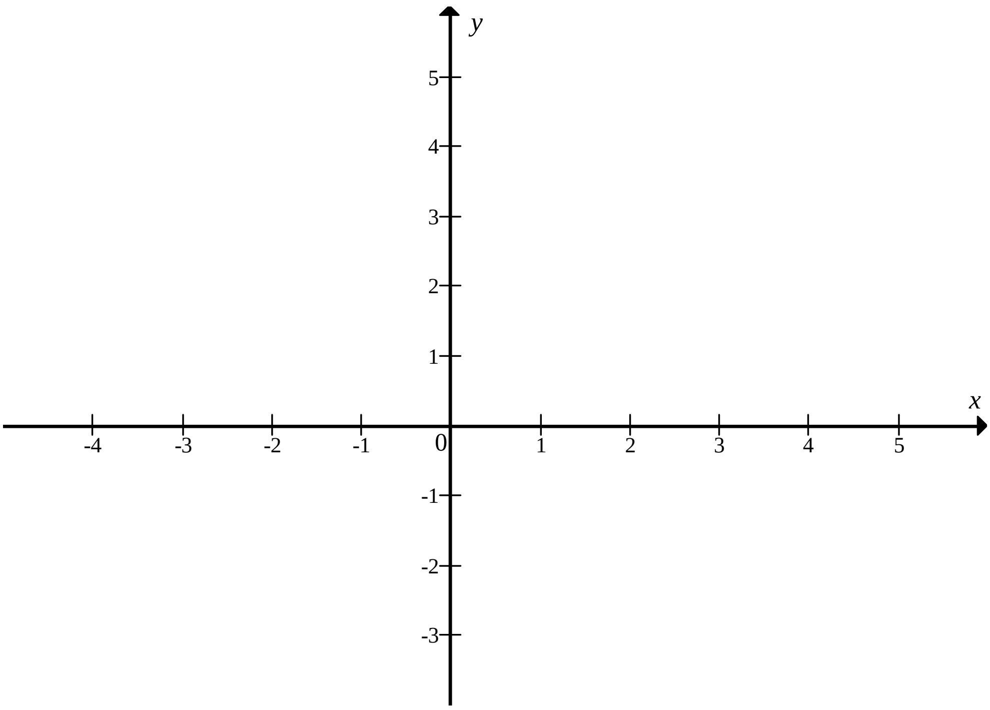
EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 5.4



QUESTION 7.4



CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 12.2 AND 12.3