education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2009

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 1 information sheet and 2 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

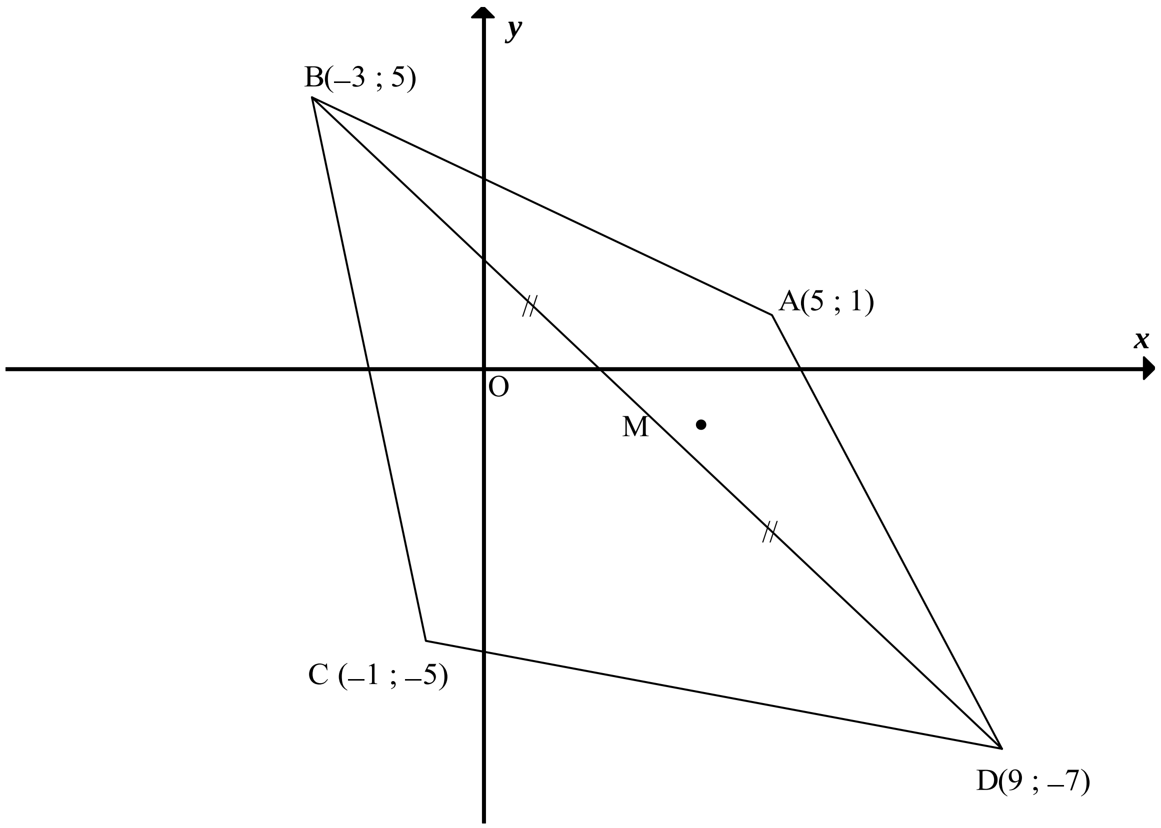
6. TWO diagram sheets for answering QUESTION 8.2, QUESTION 9.1 and QUESTION 11.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

ABCD is a quadrilateral with vertices A(5 ; 1), B(−3 ; 5), C(−1 ; −5) and D(9 ; −7).



1.1 Calculate the gradient of AC. (2)

1.2 Determine the equation of AC in the form $y = \ldots$ (3)

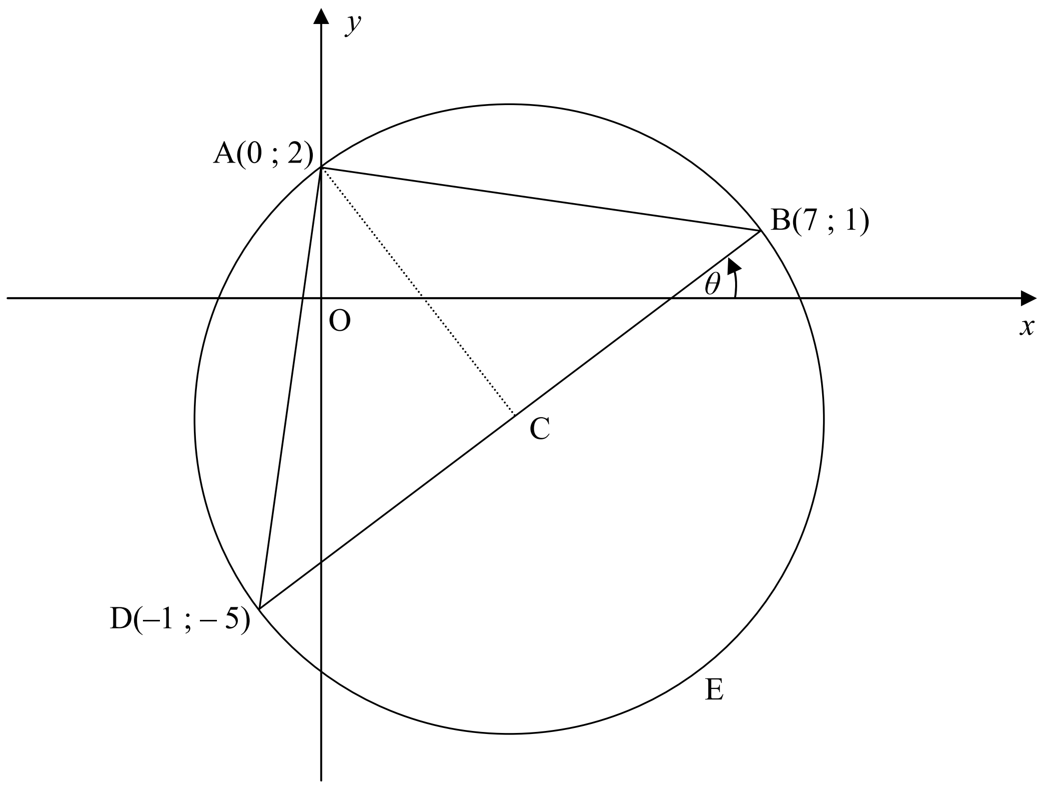
1.3 Hence, or otherwise, show that the midpoint M of BD lies on AC. (3)

1.4 Show that $\hat{AMB} = 90°$. (3)

1.5 Calculate the area of Δ ABC. (5)
[16]

QUESTION 2

2.1 The circle that passes through the points A(0 ; 2), B(7 ; 1) and D(−1 ; −5) is given below.



2.1.1 Calculate C, the coordinates of the midpoint of BD. (2)

2.1.2 Show that CA = CB. (3)

2.1.3 Hence, give the equation of the circle. (2)

2.1.4 Calculate the angle θ that BD makes with the positive x-axis. (3)

2.1.5 If AC is extended to meet the circle at E, calculate the coordinates of E. (2)

2.1.6 Explain why ABED is a rectangle. (3)

2.1.7 Determine the equation of the tangent to the circle at B in the form of $y = \ldots$ (3)

2.2 A circle is represented by the equation $x^2 + 2x + y^2 - 4y - 5 = 0$.

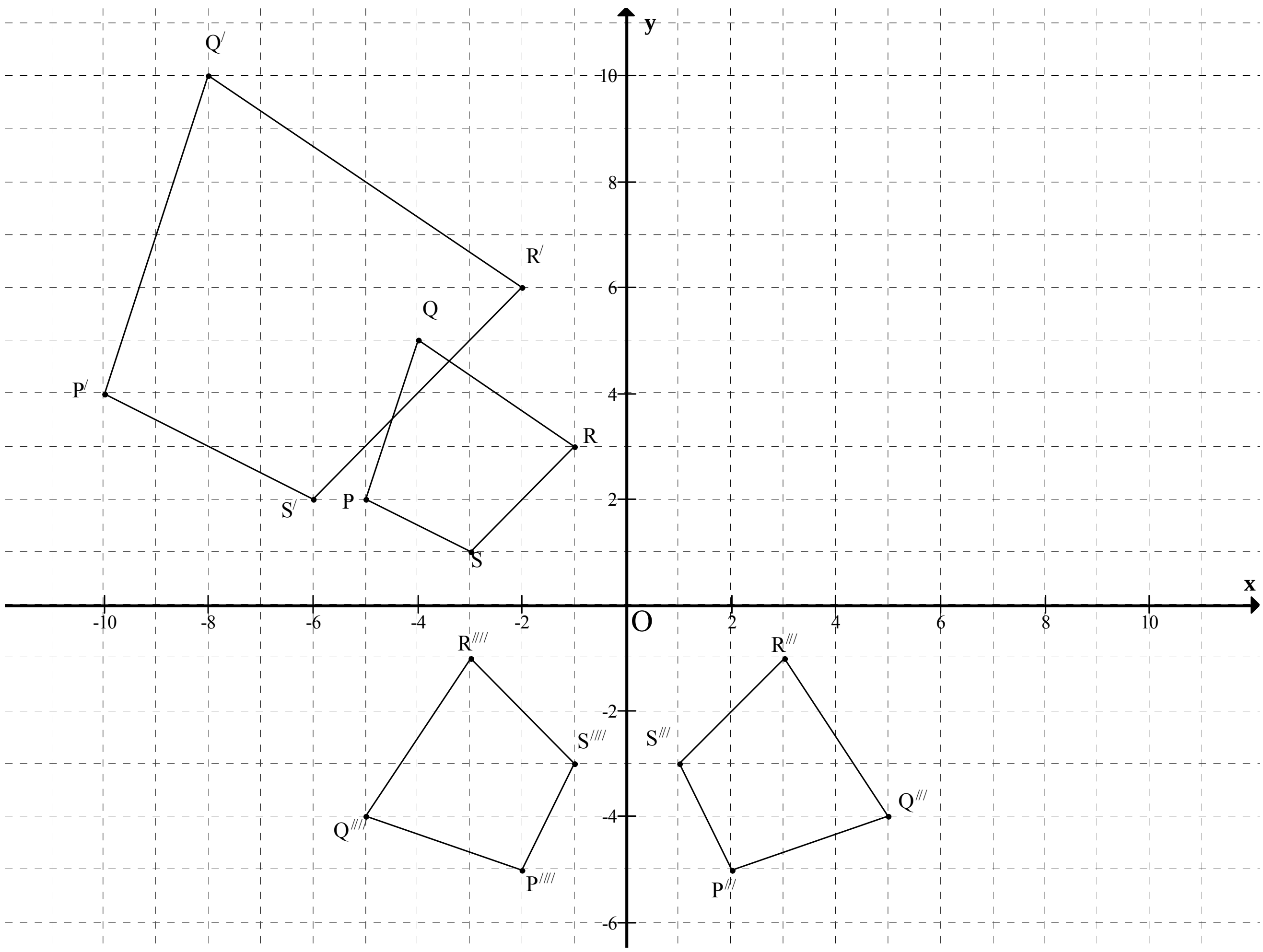
2.2.1 A transformation moves every point 2 units to the left and 4 units up. Determine the equation of the new circle after the transformation. (3)

2.2.2 Does the origin lie within the new circle? Give a reason for your answer. (2)

[23]

QUESTION 3

A quadrilateral with vertices P, Q, R and S was transformed into various images.



3.1 Describe the transformation of quadrilateral PQRS if it is transformed to image $P^{/}Q^{/}R^{/}S^{/}$. (2)

3.2 If quadrilateral $P^{//}Q^{//}R^{//}S^{//}$ is the image of quadrilateral PQRS after the transformation defined by $(x; y) \rightarrow (x+3; y+4)$, determine the coordinates of $R^{//}$. (2)

3.3 Write down the general rule for the transformation of quadrilateral PQRS to image $P^{///}Q^{///}R^{///}S^{///}$. (2)

3.4 Determine the size of the angle θ through which quadrilateral PQRS was rotated in respect to the origin, to obtain quadrilateral $P^{////}Q^{////}R^{////}S^{////}$. (4)
[10]

QUESTION 4

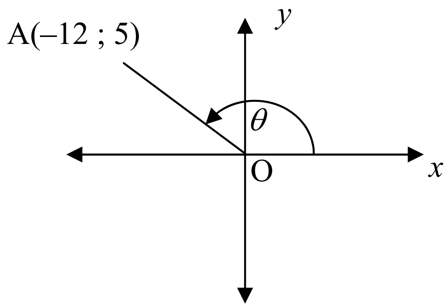
4.1 A straight line with equation $y = 2x$ is rotated anticlockwise about the origin through an acute angle θ. The equation of the new line is $y = -6x$. Determine θ. (4)

4.2 A point P(3 ; – 4) is rotated about the origin in a clockwise direction through an angle of 150° to obtain $P^{/}$. Its image is then reflected about the y-axis to obtain $P^{//}$. Calculate, **without the use of a calculator**, the coordinates of $P^{//}$. (8)

[12]

QUESTION 5

5.1 In the Cartesian plane below, the point $A(-12\ ;\ 5)$ and the angle θ are shown.



Determine, writing your answer as a single fraction:

5.1.1 $\tan\theta$ (1)

5.1.2 $\cos\theta\sin\theta$ (4)

5.2 Simplify the following to a single trigonometric ratio:

$$\frac{\sin(90^\circ - x)\tan(360^\circ - x)}{\cos(180^\circ - x)}$$ (4)

5.3 Simplify completely, **without the use of a calculator**:

$$\frac{\cos(-60^\circ) + \tan 135^\circ}{\tan 315^\circ + \cos 660^\circ}$$ (4)

5.4 Find the general solution of $\frac{1}{2}\sin x = -0{,}243$. (5)

5.5 Find a value for x if $\cos x\ ;\ \sin x\ ;\ \sqrt{3}\sin x$ is a geometric sequence. (3)

[21]

QUESTION 6

6.1 If $\sin 24° = p$, express the following in terms of p, **without the use of a calculator**:

6.1.1 $\cos 24°$ (2)

6.1.2 $\sin 12° \cos 12° - \sin(-66°)\tan 204°$ (6)

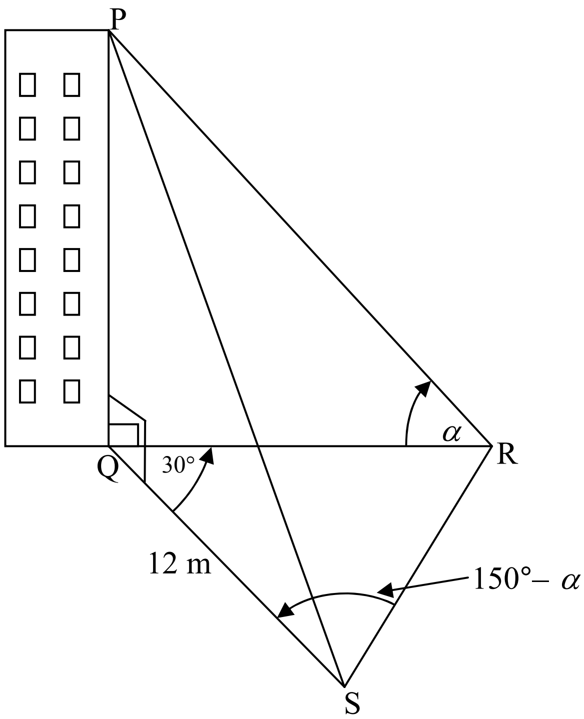
6.2 Prove that $\dfrac{2\sin x + \sin 2x}{4 + 3\cos x - \cos 2x} = \dfrac{2\sin x}{5 - 2\cos x}$ (4)

[12]

QUESTION 7

7.1 In the sketch below, PQ is a vertical building. Q, R and S are points on the same horizontal plane. The angle of elevation of P, the top of the building, from R is α.

$R\hat{Q}S = 30°$ and $Q\hat{S}R = 150° - \alpha$.
$QS = 12$ m

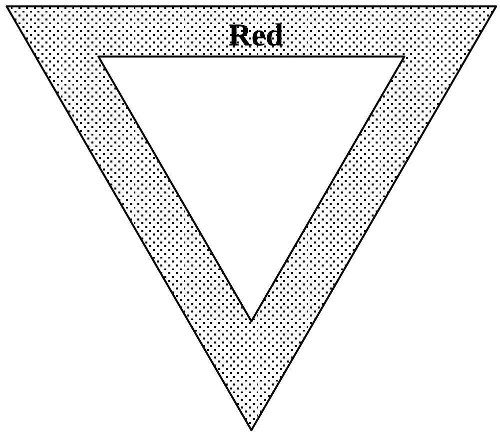


7.1.1 Determine QR in terms of $\sin\alpha$ and $\cos\alpha$. (Hint: Use the sine rule in ΔQRS.) (5)

7.1.2 Hence, or otherwise, show that the height PQ of the building is $PQ = 6 + 6\sqrt{3}\tan\alpha$. (5)

7.1.3 Hence, or otherwise, calculate α, the angle of elevation of P from R, if PQ = 23 m. (3)

7.2 A yield sign consists of two equilateral triangles. The length of the side of the inner triangle is 50 cm and the length of the side of the outer triangle is 80 cm.

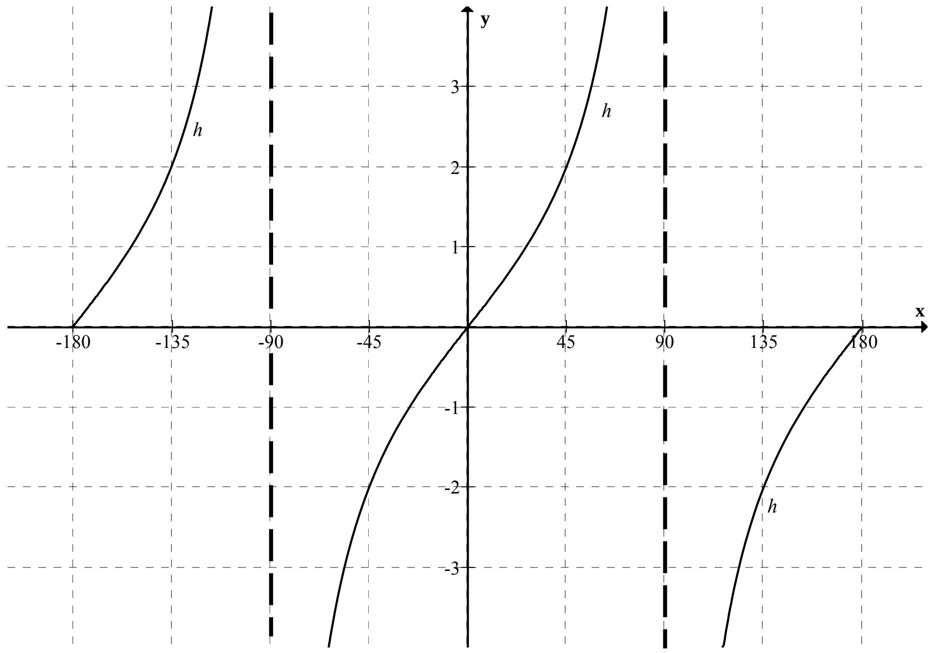


Calculate the area of the red part of the yield sign. (Indicated as the shaded region on the diagram). (4)

[17]

QUESTION 8

The graph of $h(x) = a\tan x$ for $x \in [-180°\,;180°]$, $x \neq -90°, x \neq 90°$, is sketched below.



8.1 Write down the value of a. (2)

8.2 If $f(x) = \cos(x + 45°)$, sketch the graph of f for $x \in [-180°\,;180°]$ on the same system of axes as h on DIAGRAM SHEET 1 (attached). (4)

8.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°\,;180°]$? (1)

8.4 Let θ be the smallest positive value of x at which h and f intersect. Is θ bigger than 14,5° or smaller than 14,5°? Give a reason for your answer. (3)

[10]

QUESTION 9

UNAIDS issued their 2007 report on 20 November 2008, providing an overview on the state of the AIDS epidemic. Using modern methods, UNAIDS was able to estimate how many new people throughout the world were infected each year.

YEAR	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
New infections (in millions)	3,2	3,1	3,0	2,9	2,8	2,7	2,5

[Source: www.data.org/issues/UNAIDS-2007 – AIDS EPIDEMIC UPDATE]

9.1 Draw a scatter plot for the data on the graph paper provided on DIAGRAM SHEET 2 (attached). (3)

9.2 Describe the trend that is observed in these estimates. (1)

9.3 Give TWO possible reasons for this trend. (2)
[6]

QUESTION 10

The following set of data shows the individual travelling times, in minutes, that 20 learners took to get to school one morning.

40	21	23	27	20	29	35	19	20	45
18	36	17	19	39	25	27	22	21	17

10.1 Calculate the mean time it took for a learner to get to school on this particular morning. (2)

10.2 Calculate the standard deviation of the above set of data. (2)

10.3 The educator says that all learners took about the same time to get to school. Do you agree with the educator? Give a reason for your answer. (2)
[6]

QUESTION 11

The price of 95-octane unleaded petrol in Gauteng for the period January 2007 to July 2008 is shown below. The price is in South African cents per litre.

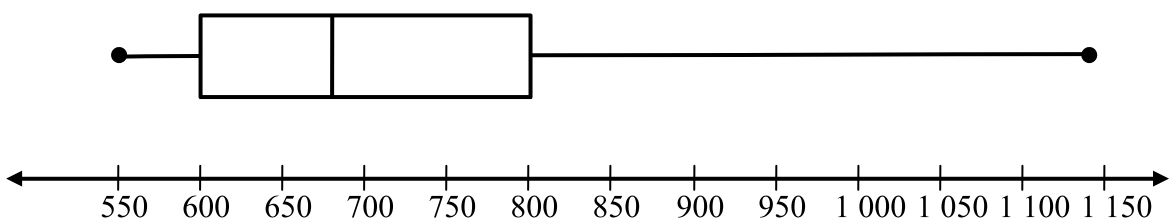
January 2007	598	February 2007	575	March 2007	599
April 2007	667	May 2007	701	June 2007	724
July 2007	716	August 2007	701	September 2007	691
October 2007	701	November 2007	704	December 2007	747
January 2008	747	February 2008	764	March 2008	825
April 2008	891	May 2008	946	June 2008	996
July 2008	1 070				

[Source: www.sasol.com]

11.1 Determine the median, lower quartile and upper quartile for the data. (4)

11.2 Draw a box and whisker diagram on DIAGRAM SHEET 2 (attached). (2)

11.3 The box and whisker diagram for the price of diesel for the same period as above is shown below. The lower quartile is 600 and the upper quartile is 800.

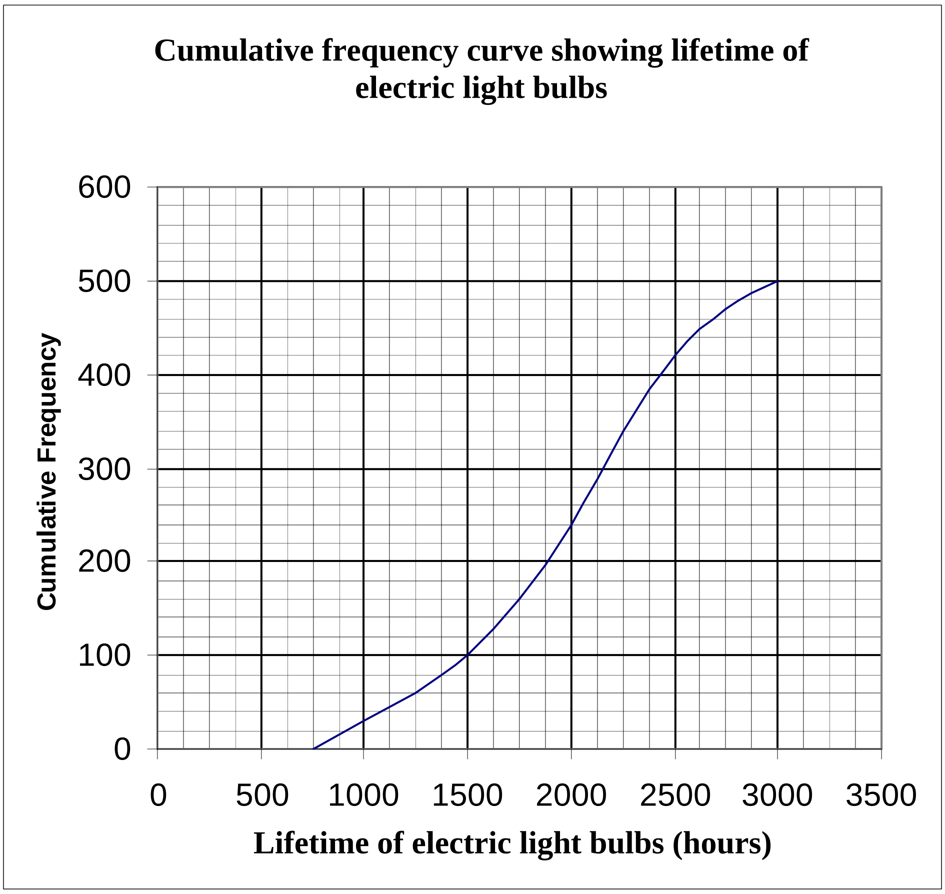


How many data points are there, strictly between 600 and 800? (2)

[8]

QUESTION 12

The lifetime of electric light bulbs was measured in a laboratory. The results are shown in the cumulative frequency diagram below.



12.1 Use the above cumulative frequency curve to determine the following:

12.1.1 How many light bulbs were tested (1)

12.1.2 The median lifetime of the electric light bulbs tested (2)

12.1.3 The interquartile range (2)

12.1.4 The number of electric light bulbs with a lifetime of between 1 750 and 2 000 hours (2)

12.2 If the cost of one light bulb is R5,00, determine the amount spent on purchasing the light bulbs that lasted longer than 2 500 hours. (2)
[9]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} \left(a + (i-1)d\right) = \frac{n}{2}\left(2a + (n-1)d\right)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

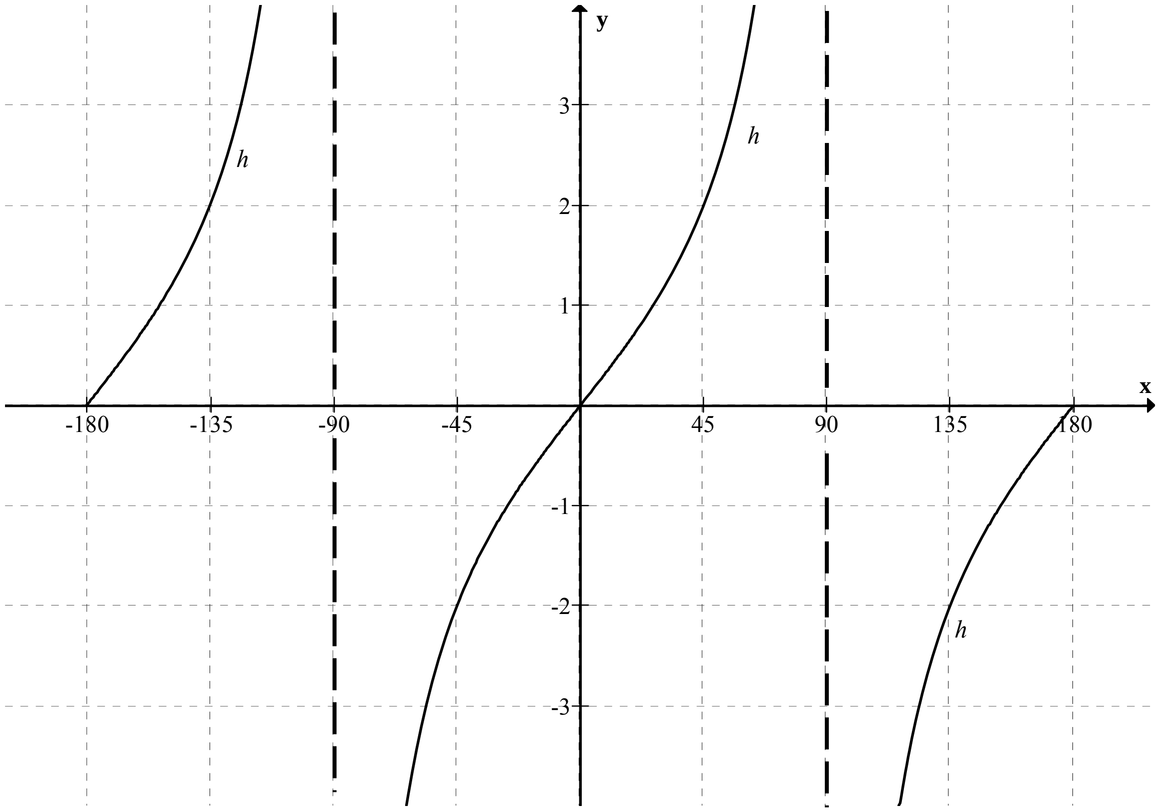
$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 8.2

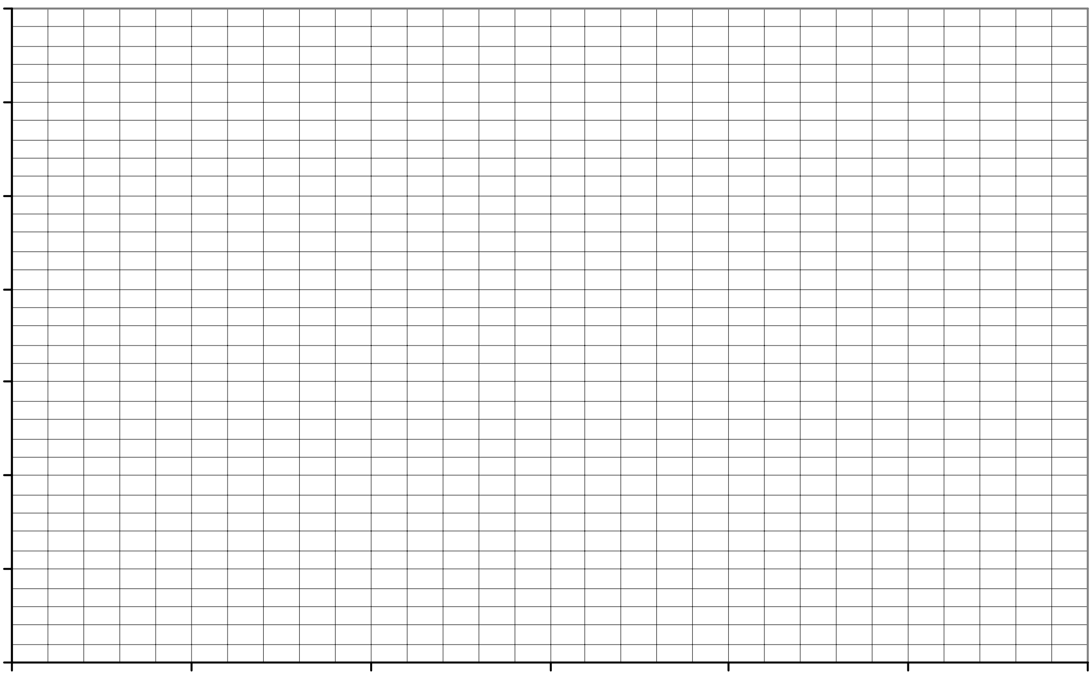


CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 9.1



QUESTION 11.2

Petrol