NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2009

MATHEMATICS: PAPER II

Time: 3 hours 150 marks

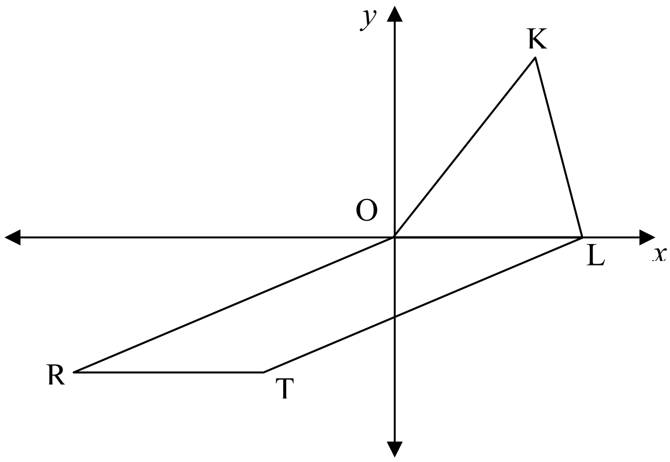
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages, and a green insert (pages i to iii) consisting of an Answer Sheet for Question 5 (c) and an Information Sheet. Please check that your paper is complete.
2. Write your examination number in the space provided in your Answer Book.
3. Answer ALL the questions. Answer Question 5 (c) on the Answer Sheet provided, and hand this in with your Answer Book.
4. Please note that diagrams are not necessarily drawn to scale.
5. All necessary working details must be shown.
6. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
7. Answers must be rounded off to one decimal digit, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) In the diagram below, an acute angled triangle, KOL, and a parallelogram, RTLO, are drawn.



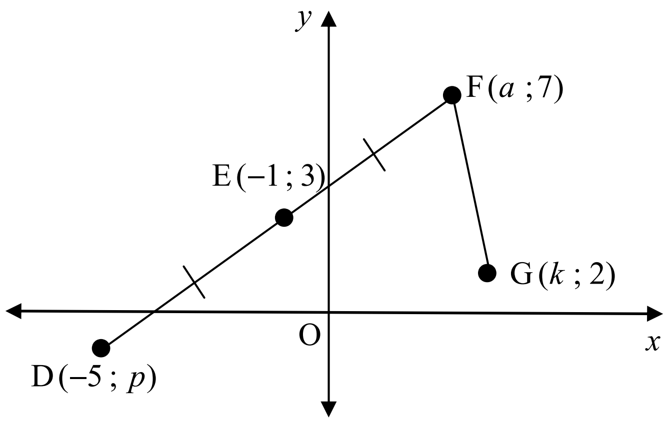
Write down using the given vertices:

(1) the line that has the same gradient as OR. (1)

(2) the line that has a negative gradient. (1)

(3) a line that has a gradient of zero. (1)

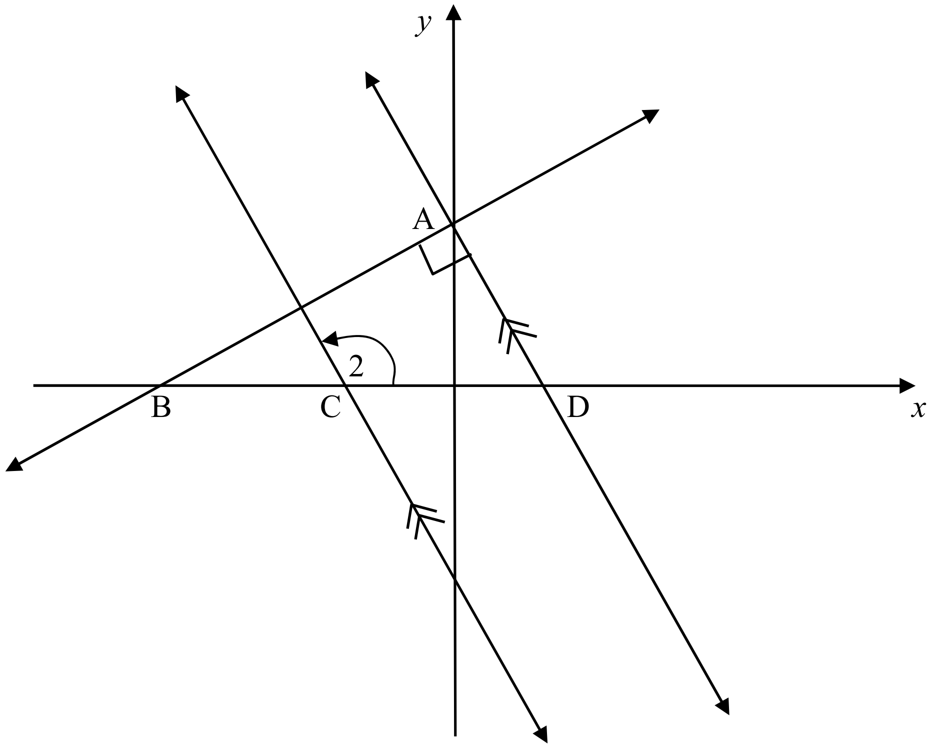
(b) In the diagram below, $E(-1\,;3)$ is the midpoint of line segment DF with $D(-5\,;p)$ and $F(a\,;7)$. $G(k\,;2)$ is a point in the first quadrant such that $EG = 5\sqrt{2}$ units.



(1) Determine the values of a and p. (4)

(2) Determine the value of k. (3)

(c) The equation of the line passing through A and D is given by $2y + 6x = 15$. The line passing through C is parallel to the line passing through A and D. B, C and D are points on the x-axis and A is a point on the y-axis.



Determine:

(1) the value of 2, the inclination of the line passing through C, rounded to one decimal digit. (4)

(2) the x-coordinate of B if $B\hat{A}D = 90°$. (4)

18 marks

QUESTION 2

(a) Calculate the value of M – T, rounded to three decimal digits, if:

$$\mathrm{M} = \sin\frac{336°}{2} \quad \text{and} \quad \mathrm{T} = \frac{\sin 336°}{2}$$ (3)

(b) Simplify to a single trigonometric ratio of A if $A \in (0° \,;\, 90°)$.

$$\sqrt{1-\sin \mathrm{A}\,.\cos \mathrm{A}\,.\tan \mathrm{A}}$$ (3)

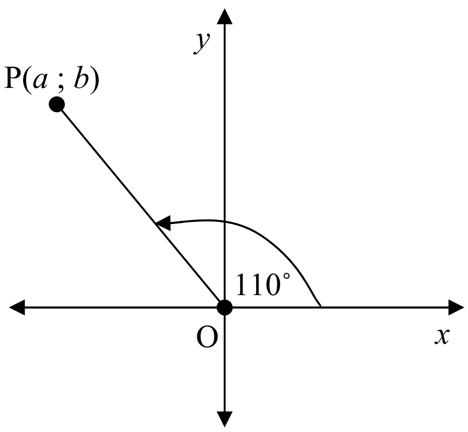
(c) Solve for θ, rounded to one decimal digit, if:

$$\cos\theta = \sqrt{3}-2 \;\text{ and }\; \theta \in [180° \,;\, 360°]$$ (3)

(d) Simplify as far as possible:

$$\frac{\sin(180° - 2\beta)}{2\cos(90° - \beta)}$$ (4)

(e) In the diagram below, OP makes an angle of 110° with the positive x-axis.



Determine, rounded to two decimal digits, the value of

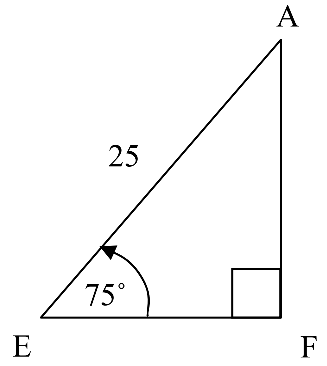
(1) $\dfrac{b}{a}$ (2)

(2) $\dfrac{b}{\sqrt{a^2+b^2}}$ (2)

17 marks

QUESTION 3

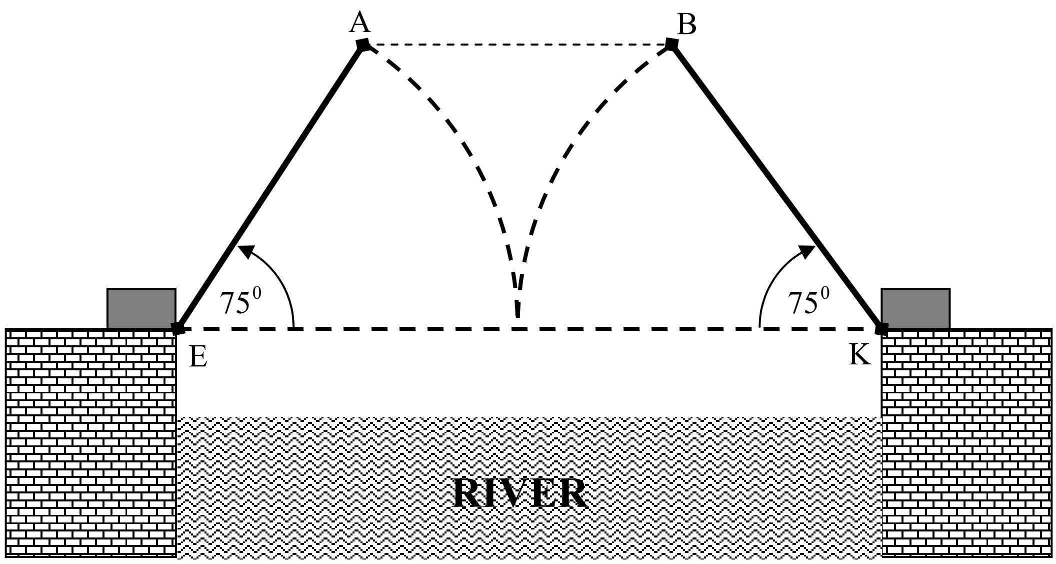
(a) (1) In the diagram below, ΔAEF is right angled at F.



Given: AE = 25 units and $A\hat{E}F = 75°$.

Determine the length of EF rounded to two decimal digits. (2)

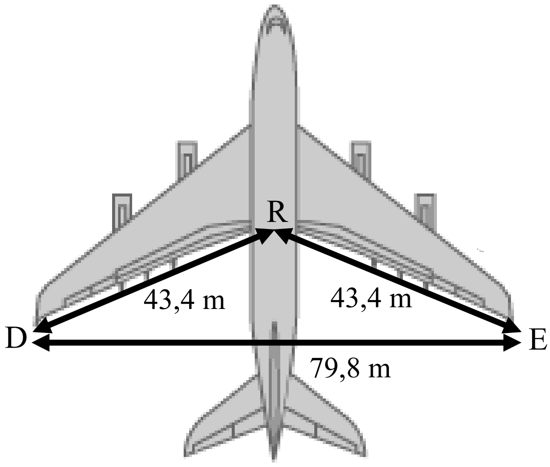
(2) The diagram below is an illustration of a vertical lift bridge positioned over a river. The bridge is made up of two arms each 25 metres in length, i.e. AE = BK = 25 metres. To allow ships to pass through, the arms AE and BK rotate upwards about E and K respectively, until they make an angle of 75° with the horizontal.

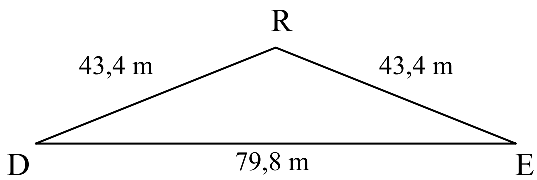


Determine the length of AB, the straight line distance between the tips of the bridge arms EA and KB. Give your answer rounded to 2 decimal digits. [Hint: Try to use your answer in (a) (1)] (4)

(b) The Airbus A380 is the largest passenger airplane in the world.

(1) In the diagram below, the top view of the plane is shown. The wing span of the plane is 79,8 metres and is shown by line DE.



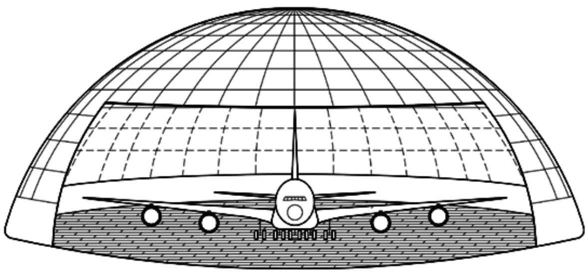


If it is given that DR = ER = 43,4 metres and DE = 79,8 metres, determine, rounded to one decimal digit,

(i) the size of angle $D\hat{R}E$. (4)

(ii) the area of ΔDRE. (3)

(2) The diagram below illustrates how an A380 Airbus might fit in a hangar constructed to form a dome of height of 45,72 metres. The dome is cut from a sphere of diameter 109,728 metres.



Determine, rounded to two decimal digits,

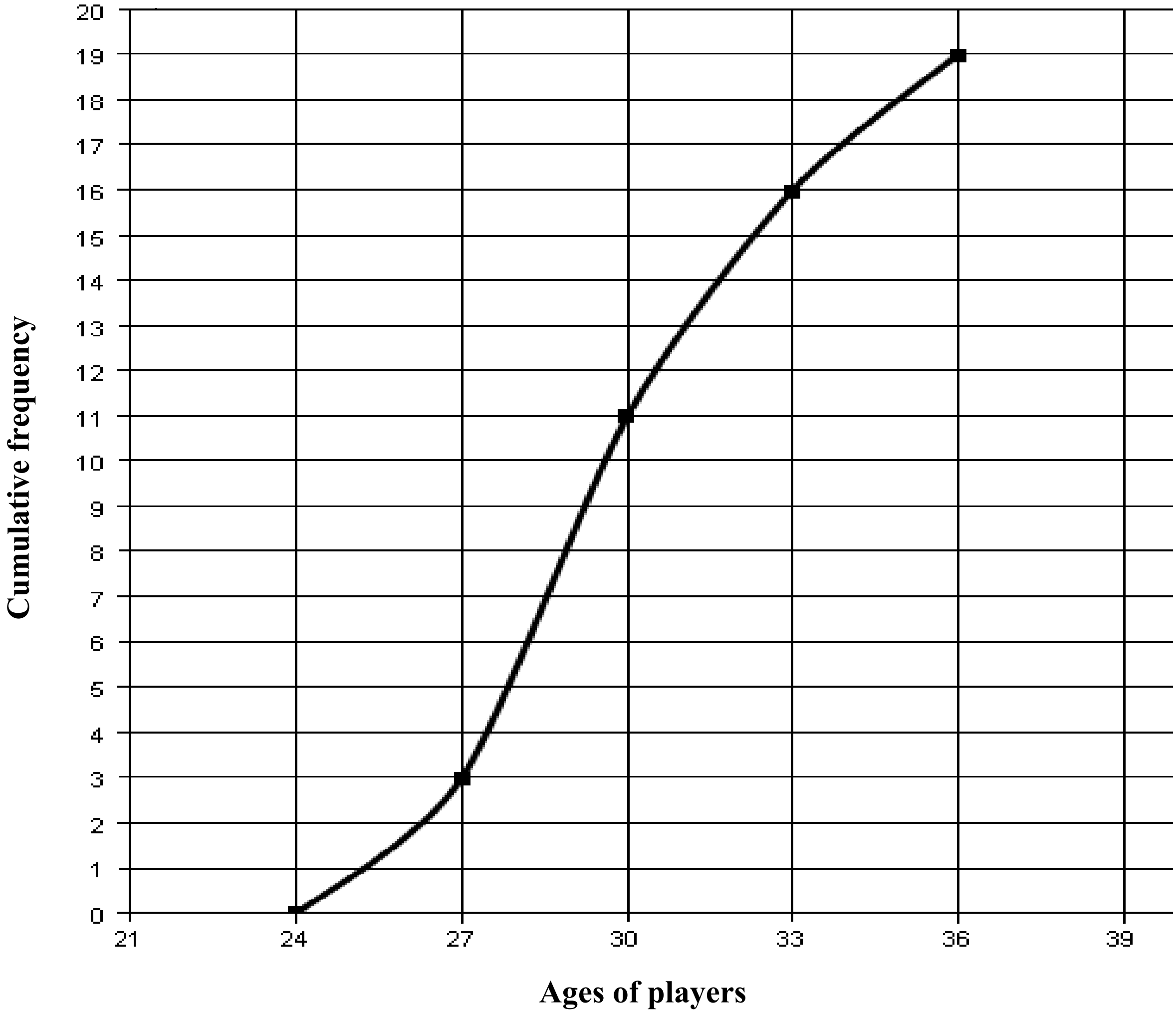
(i) the surface area of the dome using the formula $S = 2\pi rh$ where r is the radius of the sphere and h is the height of the dome. (2)

(ii) the cost per square metre of building the dome if it is estimated that the total cost of the dome will be approximately R160 million rands. (2)

17 marks

QUESTION 4

(a) The ages of the South African National Football Squad, known as BAFANA BAFANA, are summarised by the cumulative frequency curve below.



(1) How many players does the squad consist of? (1)

(2) Estimate, how many players are 30 or younger. (1)

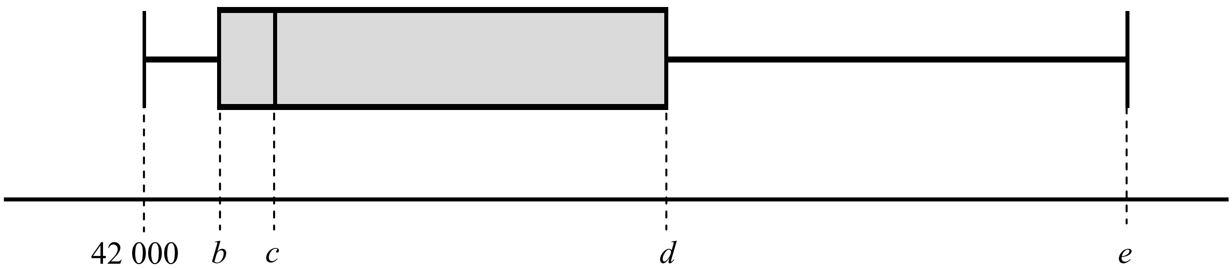
(3) Estimate, how many players are older than 27 but not older than 30. (1)

(4) Estimate, how many players are older than 30. (1)

(b) The stadiums for the FIFA WORLD CUP together with their capacities are given in the table below.

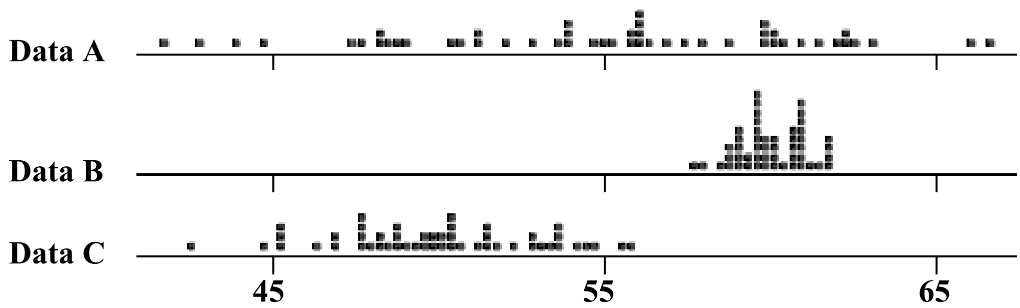
Stadium	**Capacity (Number of people)**
Green Point Cape Town	70 000
Durban	70 000
Ellis Park Johannesburg	61 000
Soccer City Johannesburg	94 700
Free State Bloemfontein	48 000
Port Elizabeth	48 000
Mbombela Nelspruit	46 000
Peter Mokaba Polokwane	46 000
Royal Bafokeng Rustenburg	42 000
Loftus Versveld Pretoria	50 000

The data in the table has been summarised by the box and whisker plot drawn below:



Determine the values of b, c, d and e. (4)

(c) The graph below shows the dotplots for three different sets of 50 numbers. The sets of numbers have different means and different standard deviations.



(1) Which of the data sets (A, B or C) has the smallest variance? (1)

(2) Which of the data sets (A, B or C) has the largest variance? (1)

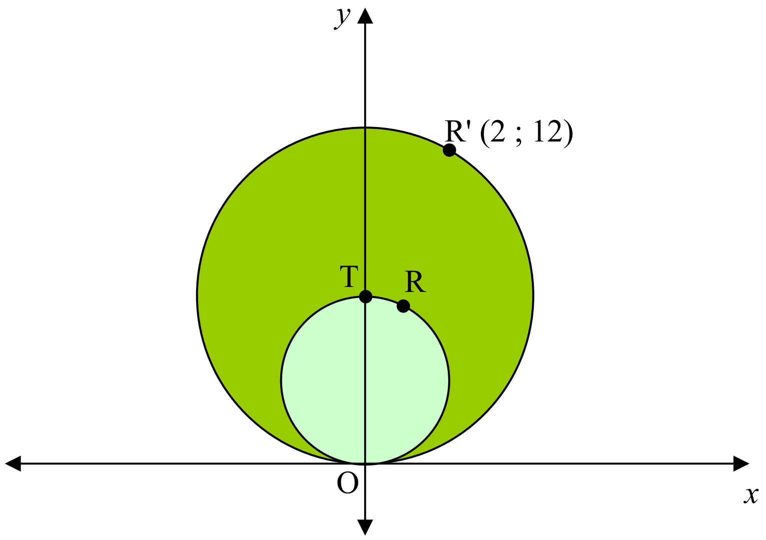
(3) Which of the data sets (A, B or C) has the smallest mean? (1)

(4) Which of the data sets (A, B or C) has the largest mean ? (1)

12 marks

QUESTION 5

(a) In the diagram below, two circles that touch internally at the origin are drawn. The larger circle has its centre at T, a point on the y-axis, and is the image of the smaller circle after an enlargement through the origin by a factor of k. R' (2 ; 12) is the image point of R, a point on the smaller circle.



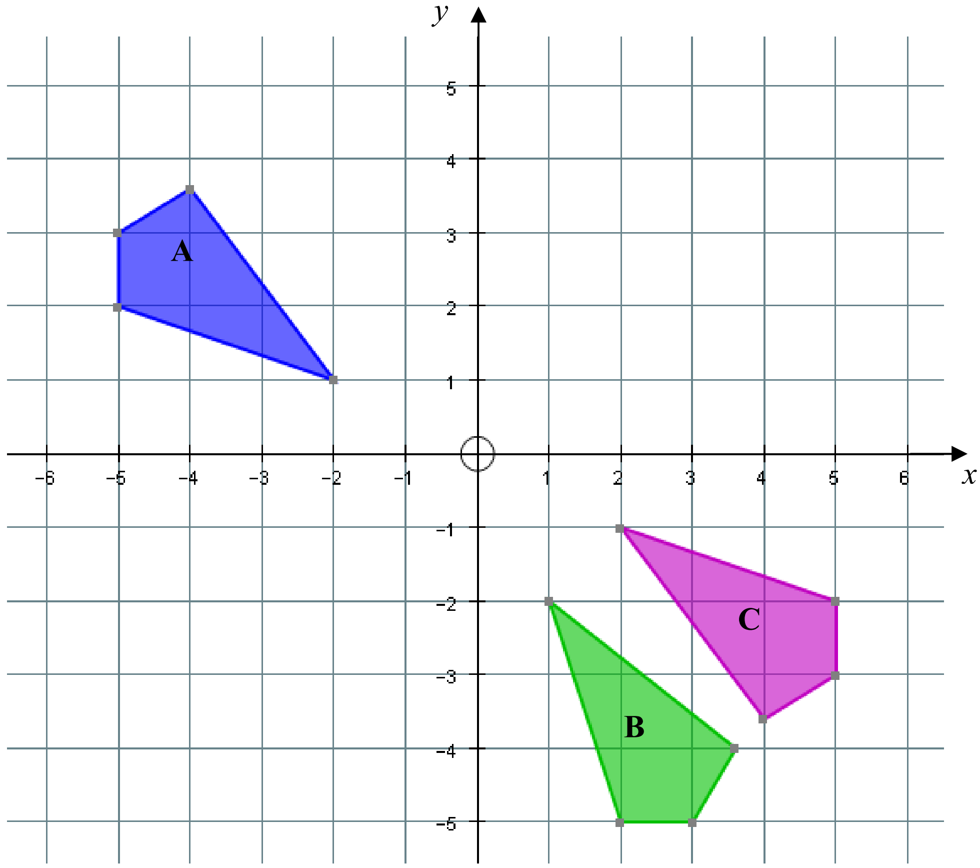
Determine,

(1) the value of k. (1)

(2) the coordinates of R. (2)

(3) the value of $\dfrac{\text{Area of smaller circle}}{\text{Area of larger circle}}$. (1)

(b) In the diagram below, quadrilateral A and its images B and C for different transformations are drawn.

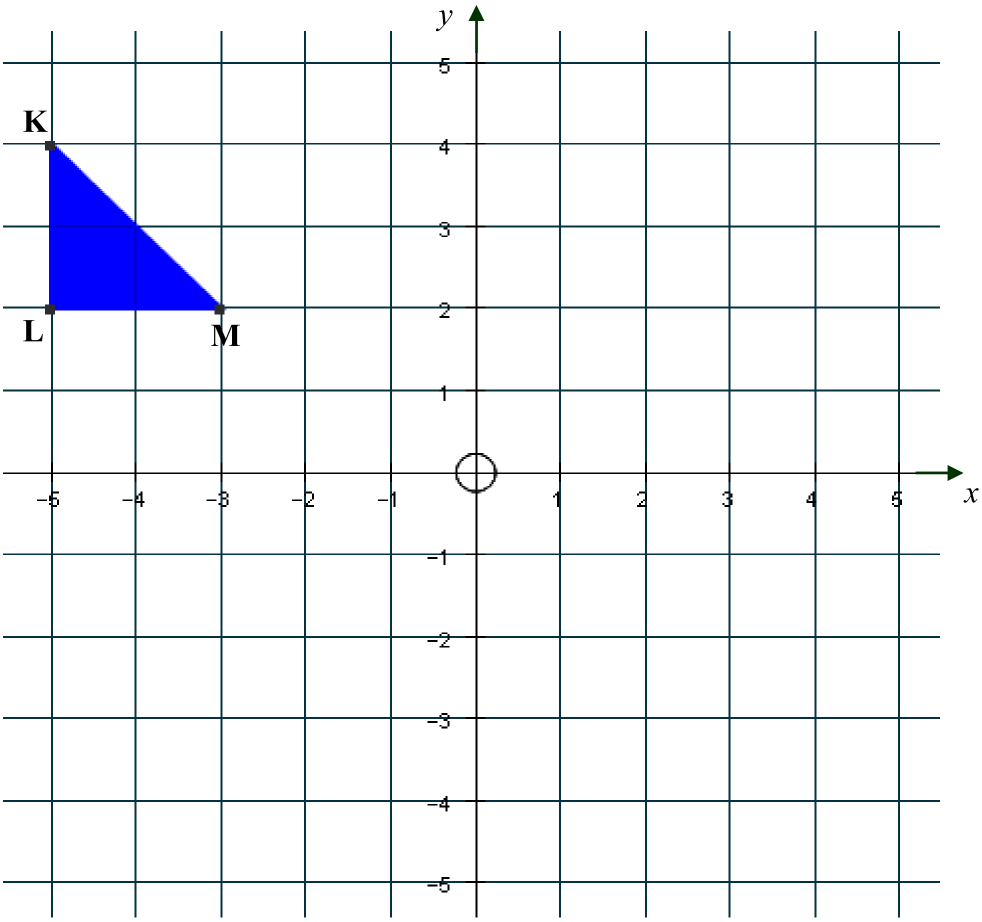


Describe FULLY a single transformation that takes:

(1) Shape A to Shape B. (2)

(2) Shape A to Shape C. (2)

(c) In the grid below, ΔKLM is given.
The image $\Delta K'L'M'$ is obtained by rotating ΔKLM about the origin in a clockwise direction through $90°$.



On the grid provided on the Answer Sheet, draw and clearly label $\Delta K'L'M'$. (3)

11 marks

Total for Section A: 75 marks

SECTION B

QUESTION 6

(a) Prove that:

$$\frac{\sin(7D).\cos(3D) - \cos(7D).\sin(3D)}{\tan(2D)} - 1 = \cos 4D \qquad (4)$$

(b) Determine the general solution to the following equation:

$$\tan^2 \theta = \tan^2 150° \qquad (5)$$

(c) (1) Prove that:

$$\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B \qquad (5)$$

(2) Hence, determine, without using a calculator, the value of:

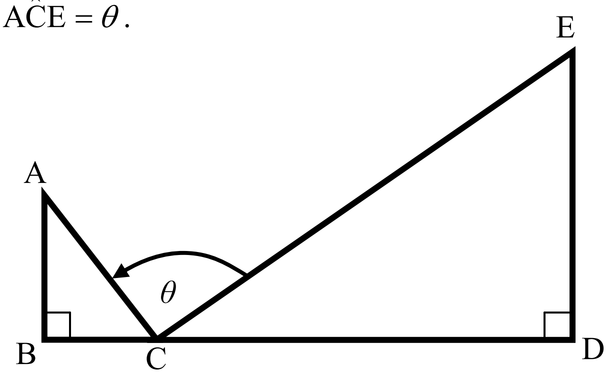
$$\cos^2 37{,}5° - \sin^2 7{,}5°$$

Leave your answer in surd form. (3)

17 marks

QUESTION 7

(a) In the diagram, BCD is a straight line with right angled triangles ABC and EDC meeting at C. $A\hat{C}E = \theta$.

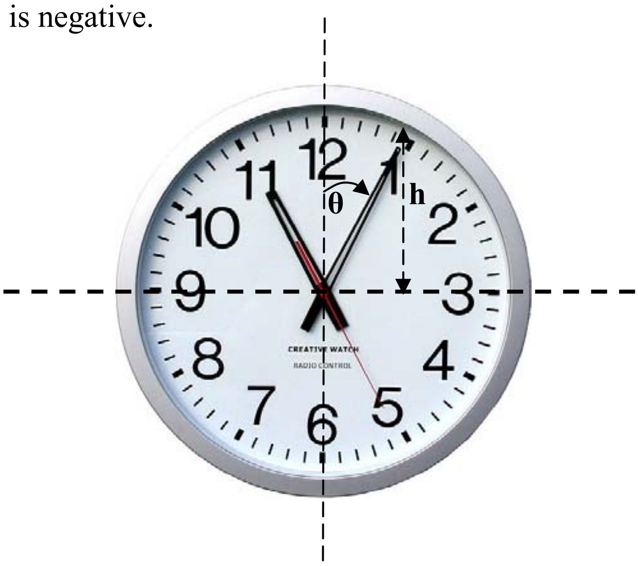


(1) The sides of triangle ABC are such that AC : BC = 2 : 1.
Write down the size of $B\hat{C}A$. (2)

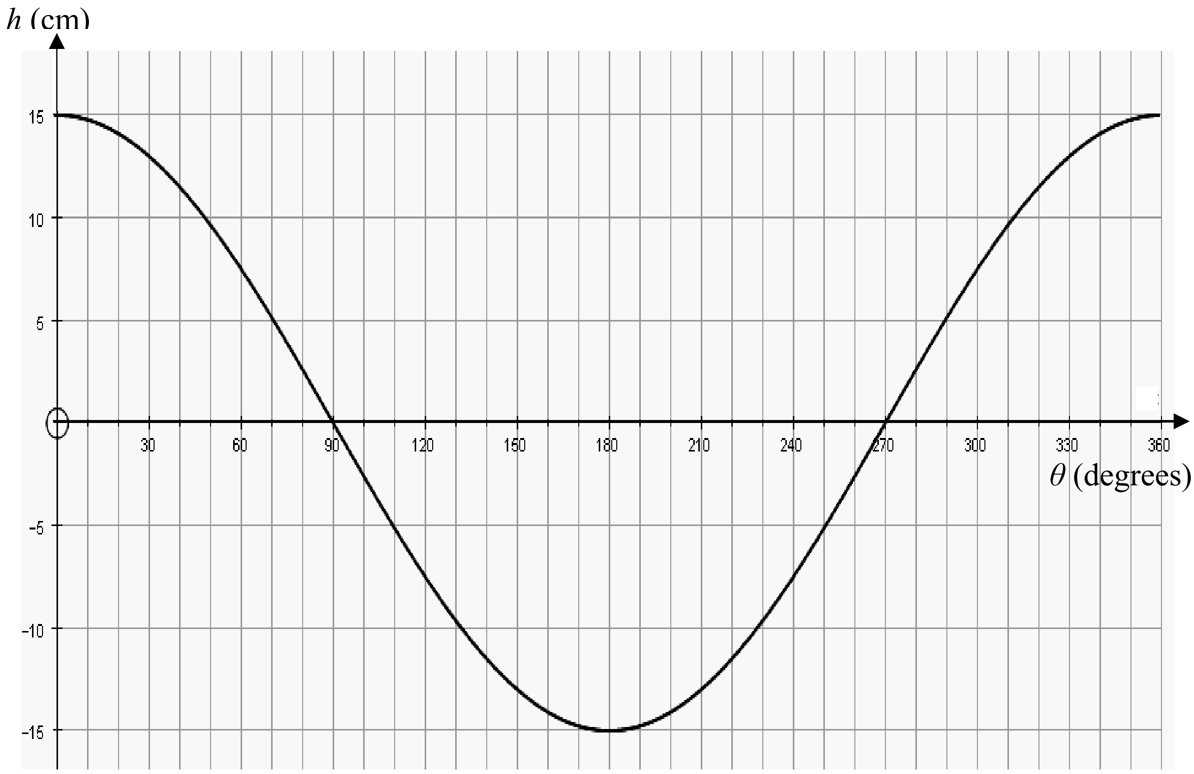
(2) The sides of triangle CDE are such that $\frac{DE}{BC} = \frac{CD}{AB} = \frac{CE}{AC} = 2$.
Write down the size of $E\hat{C}D$. (2)

(3) Hence, determine the value of $\tan\frac{\theta}{2}$. (2)

(b) In the diagram below, a wall clock is depicted.
Suppose h is the height of the tip of the minute hand above the horizontal line going through the 3 and the 9 on the clock. If the minute hand is below the horizontal line then h is negative.



The graph of h (in centimetres) versus θ (in degrees) where θ is the angle the minute hand makes with the vertical after 12:00, is drawn below.



Use the graph to answer the following questions:

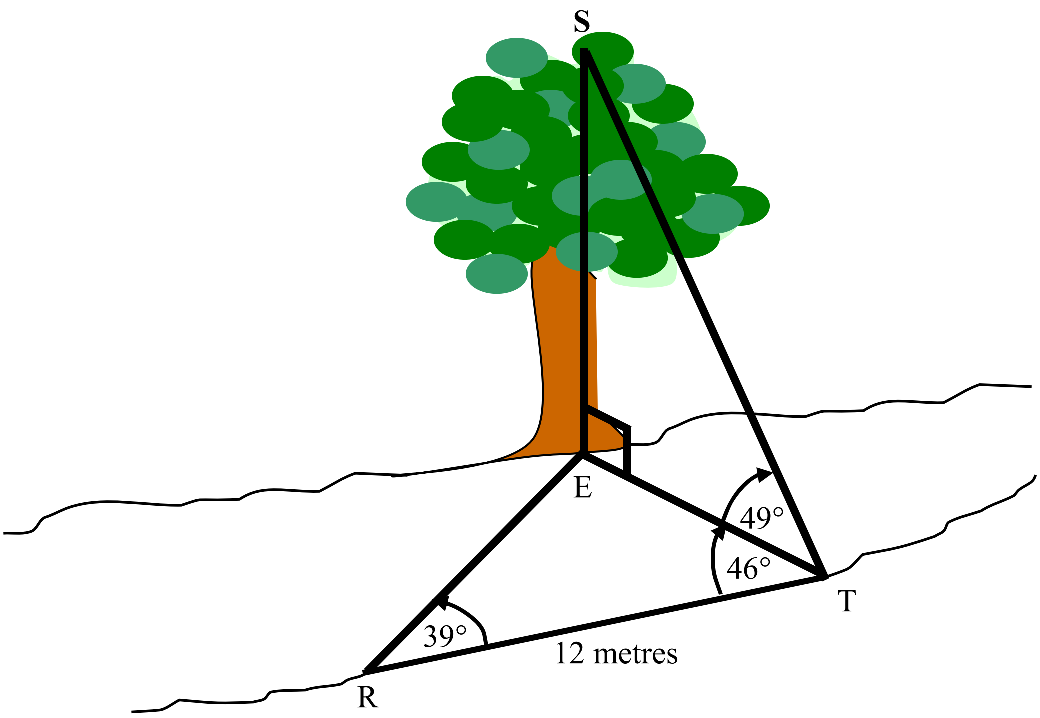
(1) How long, in centimetres, is the minute hand? (1)

(2) Write down an equation that will model the value of h, i.e. the equation of the curve shown. (1)

(3) Hence, determine the value of h when the time is exactly 12 minutes past the hour. (2)

(c) In the diagram below, a tree is situated on the opposite side of a river to R and T. The angle of elevation of S, the top of the tree, from T is 49˚. E is a point vertically below S and in the same horizontal plane as R and T.

$RT = 12$ metres, $R\hat{T}E = 46°$ and $E\hat{R}T = 39°$.



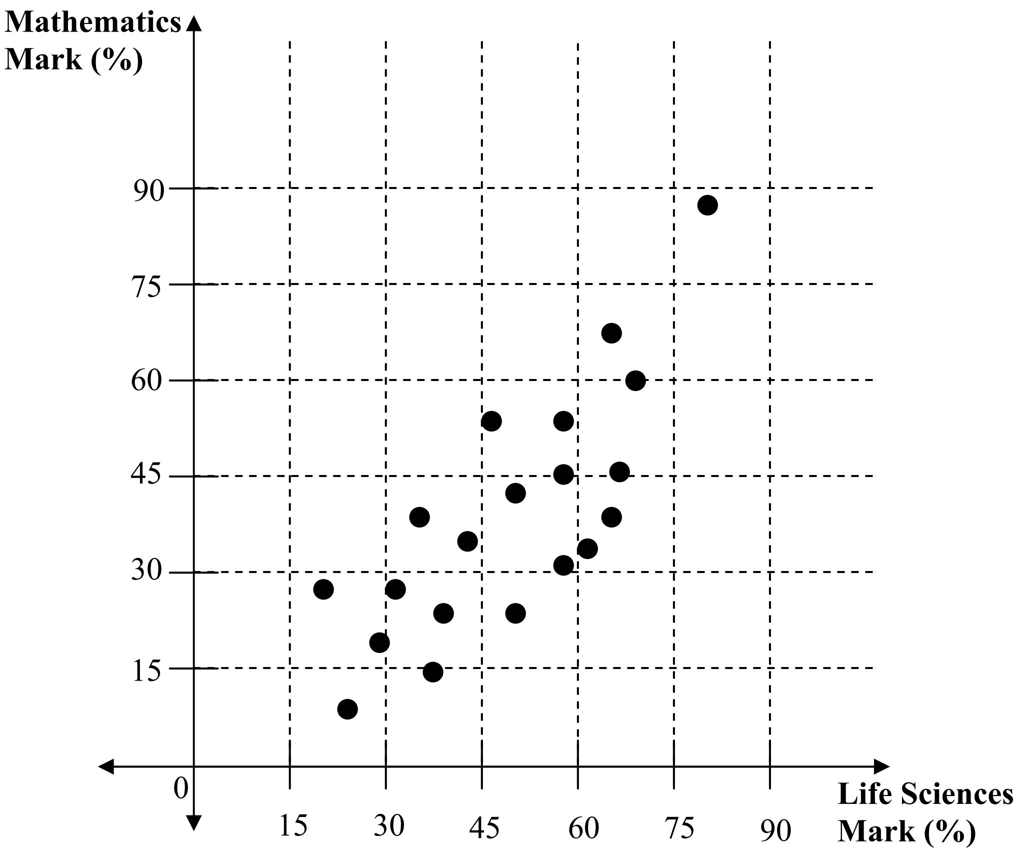
(1) Determine, ES, the height of the tree, rounded to one decimal digit. (4)

(2) Determine the width of the river, rounded to one decimal digit. (Assume that R, E and T are points on the banks of the river and that the width is constant). (3)

17 marks

QUESTION 8

(a) The scatter plot given below shows the marks for a group of students in Mathematics and Life Sciences.



(1) How many students are in the group? (1)

(2) State TRUE OR FALSE for each of the following:

(i) In general, the higher the mark for Life Sciences, the higher the mark for Mathematics. (1)

(ii) The student that got the lowest mark for Mathematics also got the second lowest mark for Life Sciences. (1)

(iii) The median mark for Mathematics is lower than the median mark for Life Sciences. (1)

(iv) The range for the Mathematics mark is smaller than the range for the Life Sciences Mark. (1)

(b) A group of students wrote a statistics test and the following data was obtained:

Standard Deviation	**Mean**
8	72

The teacher decided to increase the marks by adding 5 marks to each person's mark.
For the new set of marks, write down

(1) the mean. (1)

(2) the standard deviation. (1)

(c) Given: $\{m-4\ ;\ m\ ;\ m+1\ ;\ m+3\ ;\ m+5\}$

Determine the standard deviation for the given set of 5 numbers, using:

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(\bar{x}-x_i\right)^2}{n}$$

Show all working.
Give your answer rounded to two decimal digits. (6)

13 marks

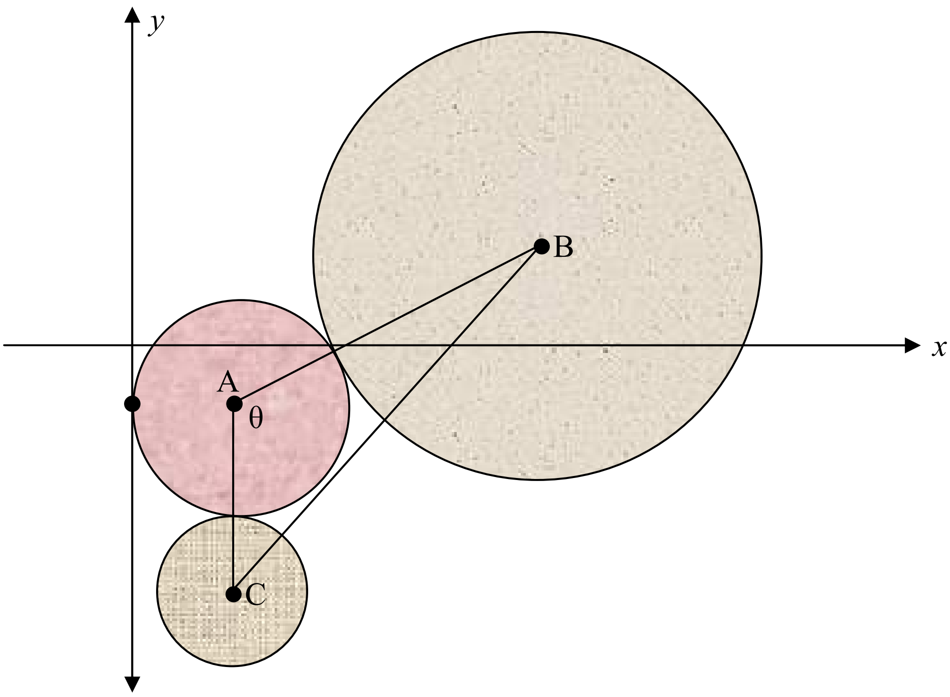
QUESTION 9

(a) Determine the equation of the tangent to the circle with equation.

$$x^2 + y^2 + 8y = 9$$

at the point $D(-4; -1)$. (6)

(b) In the diagram below, circle centre C touches circle centre A with C on the same vertical line as A.
Circle centre B touches circle centre A and $C\hat{A}B = \theta$.



AC = 3 units and AB = 6 units.

The equation of the circle centre A is given by $(x-2)^2 + (y+1)^2 = 4$.

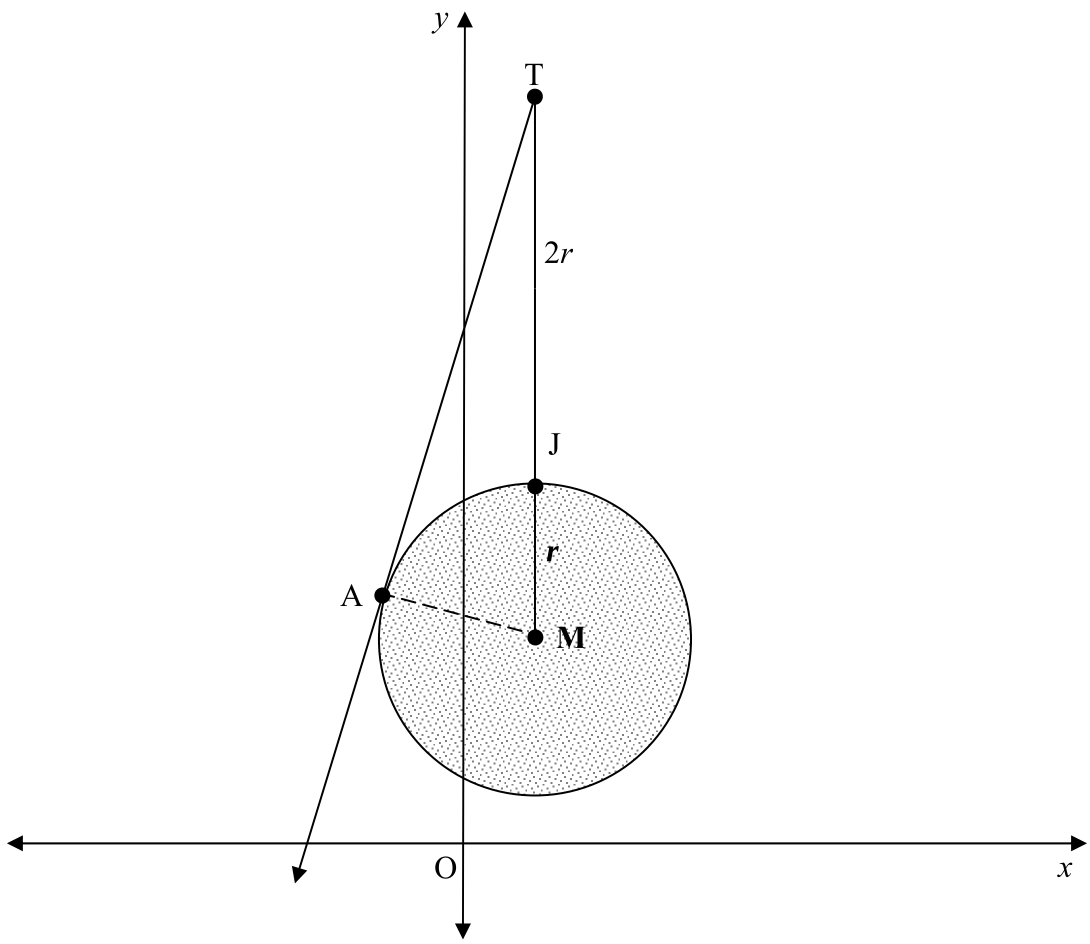
(1) Write down the equation of circle centre C, if AC is parallel to the y-axis. (3)

(2) Write down an expression in terms of θ for the area of ΔABC. (1)

(3) What value of $\theta \in [0°; 180°]$ will maximise the area of ΔABC? (1)

(4) Write down the equation of circle B when the area of ΔABC is a maximum. (3)

(c) The diagram below shows a circle centre M with radius r. T and J are points vertically above M with J on the circle and JT = 2MJ = $2r$. The tangent to the circle at A is drawn from T.



(1) Express the length of TA in terms of r, the radius of the circle. (2)

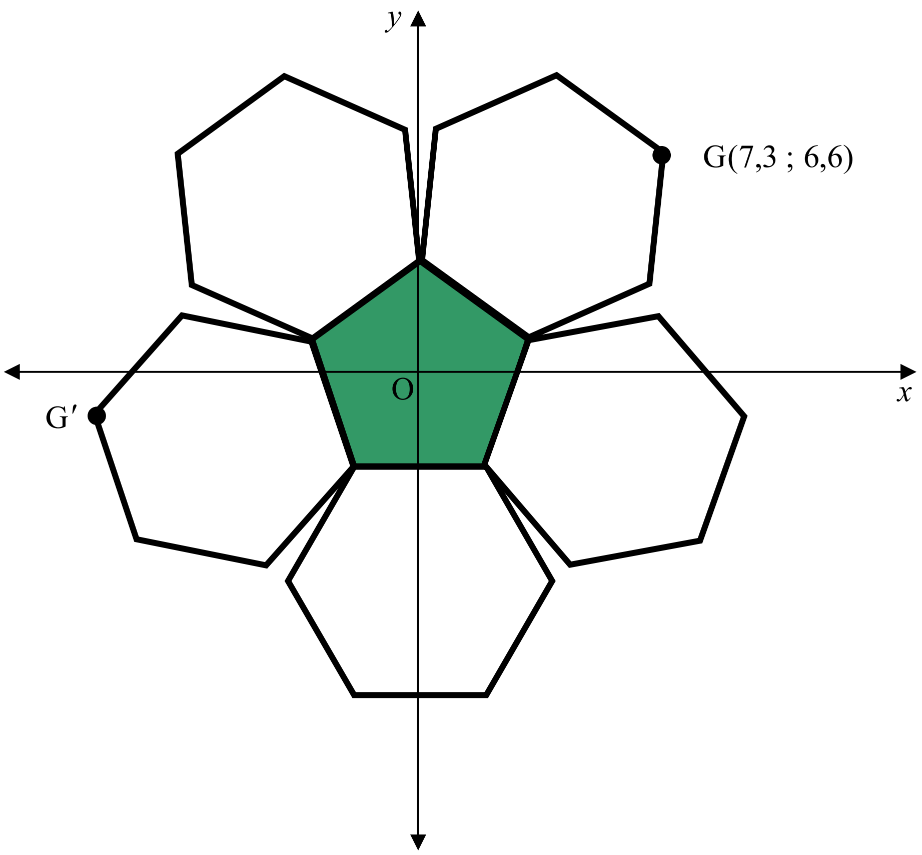
(2) Determine the gradient of the tangent TA, rounded to one decimal digit. (4)

20 marks

QUESTION 10

The photograph of the soccer ball shows that each side of a regular pentagon is a common side to a regular hexagon.

In the diagram below, the pentagon is drawn so that its centre is at the origin. The five hexagons are drawn so that they each have one side in common with a side of the pentagon. The curved surface of the ball is flattened to give the diagram below.



Use the transformation formula for rotation, or otherwise, to determine, rounded to one decimal digit, the coordinates of G'.

8 marks

Total for Section B: 75 marks

Total: 150 marks