

GR 12

JULY 2010

MEMO (P1)

SECTION A

Question 1

a) $x^2 - 6x = 10 - 30x$

$\therefore x^2 + 24x - 10 = 0$

$\therefore x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{-24 \pm \sqrt{616}}{2}$

$= -12 \pm \sqrt{154}$

b) $x^2 - 3x + 2 \leq 6$

$\therefore x^2 - 3x - 4 \leq 0$

$\therefore (x-4)(x+1) \leq 0$

$-1 \leq x \leq 4$

c) $3(4^{2x}) = 14,2$

$\therefore 4^{2x} = \frac{14,2}{3}$

$\therefore \log_4 \left(\frac{14,2}{3}\right) = 2x$

$\therefore 0,56 = x$

[12]

Question 2

a) (1) $\log_2 4 + \log_2 \frac{1}{2}$

$= (2) + (-1)$

$= 1$

(2) $\frac{\log 3 - 2 \log 5}{4 \log 5 - 2 \log 3}$
 $= \frac{(\log 3 - 2 \log 5)}{\sqrt{-2}(\log 3 - 2 \log 5)}$

$= -\frac{1}{2}$

b) $\therefore \log \frac{(x+3)^2}{x} = 1$

$\therefore \frac{(x+3)^2}{x} = 10$

$\therefore x+3 = 10x$

$\therefore 3 = 9x$

$\therefore \frac{1}{3} = x$

[11]

Question 3

a) Hyperbola

b) $y = -1$; $x = 3$

c) X-intercept: Y-intercept:

$0 = \frac{5}{x-3} - 1$

$y = \frac{5}{0-3} - 1$

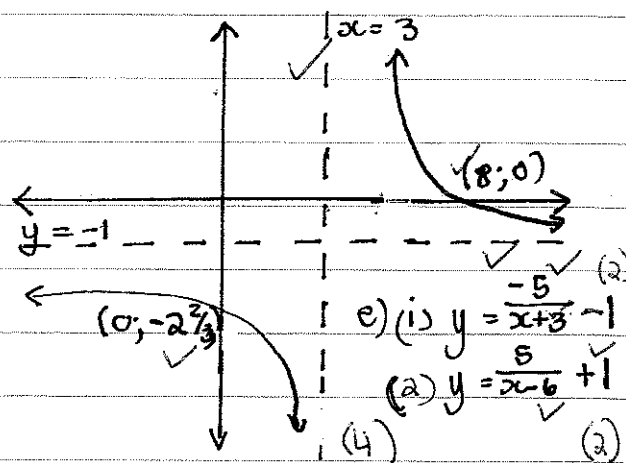
$\therefore 1 = \frac{5}{x-3}$

$= -2 \frac{2}{3}$

$\therefore x-3 = 5$

$\therefore x = 8$

d)



e) (i) $y = \frac{5}{x+3} - 1$

(a) $y = \frac{5}{x-6} + 1$

[14]

Question 6

ANSWER SHEET

Question 7

a) CD = 4 ✓

X-intercept: (y=0)

$$-(x+1)^2 + 4 = 0 \quad \checkmark$$

$$\therefore -(x^2 + 2x + 1) + 4 = 0$$

$$\therefore -x^2 - 2x - 1 + 4 = 0$$

$$\therefore x^2 + 2x - 3 = 0$$

$$\therefore (x+3)(x-1) = 0$$

$$\therefore \underline{x = -3} \checkmark \text{ of } \underline{x = 1} \checkmark \quad (4)$$

b) Max value: 4 ✓ (1)

c) Max distance = $(-x^2 - 2x + 3) - (x^2 - 4x)$

$$= -2x^2 + 2x + 3$$

$$= -2(x^2 - x + \frac{1}{4} - \frac{1}{4}) + 3$$

$$= -2(x - \frac{1}{2})^2 + \frac{1}{2} + 3$$

$$= -2(x - \frac{1}{2})^2 + 3\frac{1}{2}$$

Max length $3\frac{1}{2}$ where $x = \frac{1}{2}$

OR d(Distance) = 0 ✓

$$\therefore -4x + 2 = 0$$

$$\therefore -4x = -2$$

$$\therefore \underline{x = \frac{1}{2}} \checkmark \quad (2)$$

d) $x \in (-3; 0) \cup (1; 4)$ (4)

e) $f'(x) = g'(x) \checkmark$

$$-2x - 2 = 2x - 4 \checkmark$$

$$\therefore -4x = -2$$

$$\therefore \underline{x = \frac{1}{2}} \checkmark \quad (3)$$

[14]

Question 8

ANSWER SHEET

Question 9

a) $y = - (x-2)(x-2)(x+3)$
 $= - (x^2 - 4x + 4)(x+3)$
 $= - (x^3 - 4x^2 + 4x + 3x^2 - 12x + 12)$
 $= -x^3 + x^2 + 8x - 12$

$a = 1$ ✓ ; $b = 8$ ✓ ; $c = -12$ ✓

(6)

b) DE = $14 - (-12) = \underline{26}$ ✓ (2)

c) $f'(x) = 0 \checkmark$

$$-3x^2 + 2x - 8 = 0$$

$$\therefore 3x^2 - 2x - 8 = 0$$

$$\therefore (x-2)(3x+4) = 0$$

$$x = -\frac{4}{3} \checkmark$$

$$y = - \left(-\frac{4}{3}\right)^3 + \left(-\frac{4}{3}\right)^2 + 8\left(-\frac{4}{3}\right) - 12$$

$$= -\frac{500}{27} \checkmark$$

A $(-\frac{4}{3}; -\frac{500}{27})$ (6)

d) $f''(x) = 0 \checkmark$

$$-6x + 2 = 0 \quad \left(\frac{1}{3}; -\frac{250}{27}\right)$$

$$x = \frac{1}{3} \checkmark$$

$$y = -\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) - 12 = -\frac{250}{27}$$

(4)

Question 3

[13 marks]

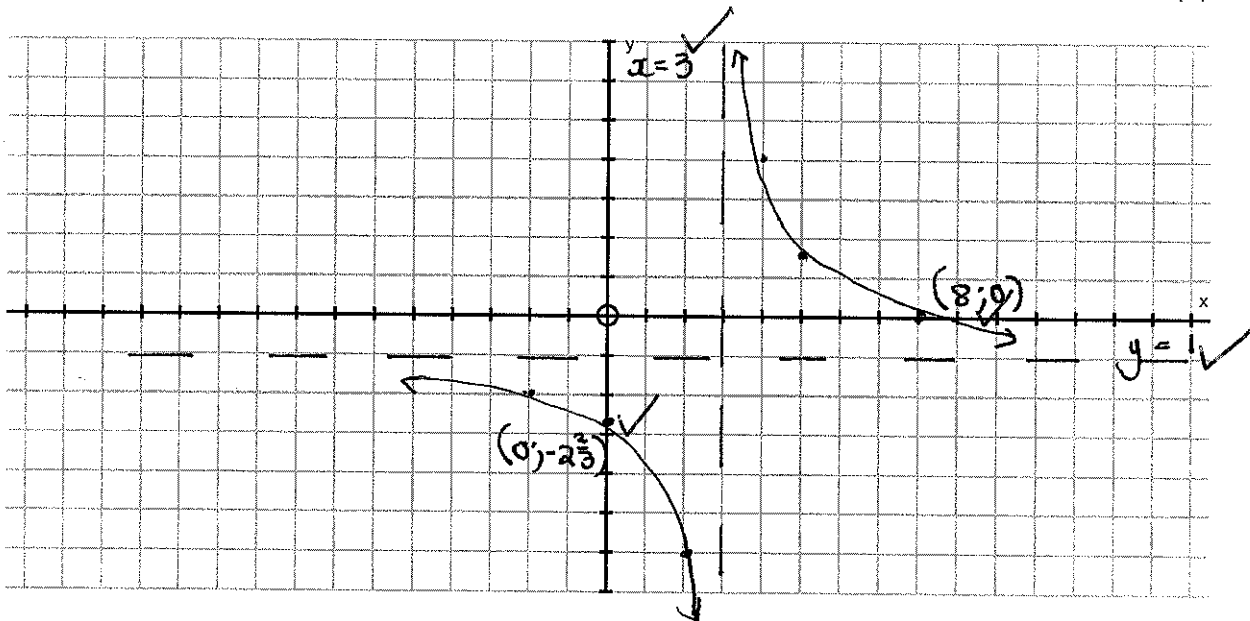
Given: $f(x) = \frac{5}{x-3} - 1$

a) What kind of graph does f represent? hyperbola ✓ (1)

b) Write down the equations of the asymptotes of f . $y = -1$ ✓
 $x = 3$ ✓ (2)

c) Determine the intercepts with the axes. X-intercept: ($y=0$)
Y-intercept: ($x=0$)
 $0 = \frac{5}{x-3} - 1$
 $y = \frac{5}{0-3} - 1$ $\therefore x-3 = 5$
 $= -2\frac{2}{3}$ ✓ $\therefore x = 8$ ✓ (2)

d) Sketch the graph of f , clearly showing all relevant features of this graph. (4)



e) Give the new equations after the following transformations:

1) $f(x)$ reflected about the y-axis

$y = \frac{-5}{x+3} - 1$ ✓

(2) +1

2) $f(x)$ translated 3 units to the right and 2 units up

$y = \frac{5}{x-6} + 1$ ✓

(2)

Question 6

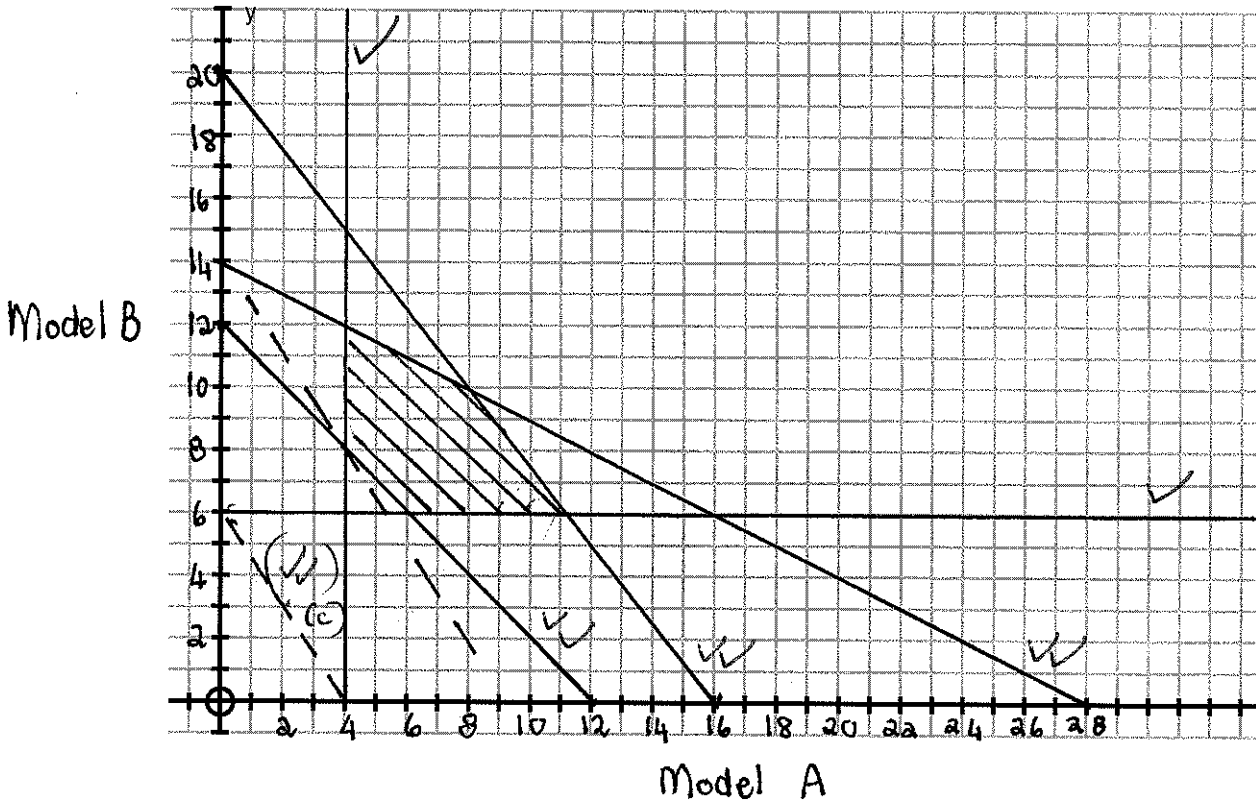
[19 marks]

Let x be the number of articles of model A and y be the number of articles of model B which can be manufactured daily by a factory, subject to the following constraints:

$$\begin{aligned} x &\geq 4 \\ y &\geq 6 \\ x + y &\geq 12 \\ 5x + 4y &\leq 80 \\ 2y + x &\leq 28 \end{aligned}$$

- a) Represent all the constraints on the graph paper provided. Clearly indicate the feasible region.

(8)



- b) If it costs R300 to make each article of model A and R200 to make each article of model B, write down an equation to represent the total cost, T , to manufacture x articles of model A and y articles of model B.

$$T = 300x + 200y$$

$$m = -3/2$$

(2)

- c) Draw on the graph a straight line that you would use to minimize the total production cost.

_____ (2)

- d) Give the number of articles of each model that should be manufactured to ensure a minimum cost, and determine the minimum cost.

Model A - 4 articles ; Model B - 8 articles

$$T = 300(4) + 200(8)$$
$$= R 2800$$

_____ (3)

- e) If the manufacturing cost is adjusted and it now costs the same to manufacture models A and B, but it is not desirable to make more of model A than of model B, determine how many of each should be manufactured to ensure minimum expenditure.

$$T = Ax + Ay ; m = -1$$

line will lie on the line $x + y = 12$

Any point between $(4, 8)$ and $(6, 6)$, including

them would minimize the cost. (4)

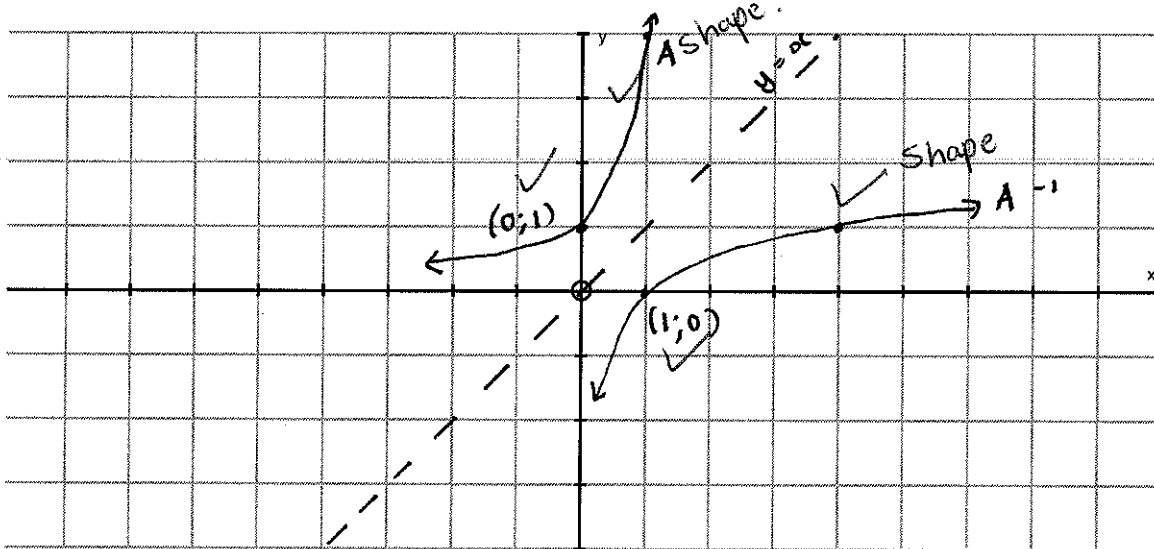
[19]

Question 8

[10 marks]

Given: $A: f(x) = 4^x$

a) Sketch A and A^{-1} on the same set of axes. Label all relevant points. (4)



b) Determine a if $f(a) = 8$.

$$8 = 4^a$$

$$\therefore 2^3 = 2^{2a}$$

$$\therefore 3 = 2a$$

$$\therefore \frac{3}{2} = a$$

(3)

c) Explain how you can use coordinates and transformation rules to determine b if $f^{-1}(8) = b$?

f^{-1} is the reflection of f in the line $y = x$.

$\therefore f^{-1}(8) = b$ and $f(a) = 8$ refers to the same point $(\frac{3}{2}; 8)$

(3)

[10]